

Research Article

On the Behavior of Solutions of the System of Rational Difference Equations:

$$x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1), \quad y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1), \\ \text{and } z_{n+1} = z_{n-1}/(y_n z_{n-1} - 1)$$

Abdullah Selçuk Kurbanli

Department of Mathematics, Faculty of Education, Selçuk University, 42090 Konya, Turkey

Correspondence should be addressed to Abdullah Selçuk Kurbanli, agurban@selcuk.edu.tr

Received 23 December 2010; Accepted 26 January 2011

Academic Editor: Ibrahim Yalcinkaya

Copyright © 2011 Abdullah Selçuk Kurbanli. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We investigate the solutions of the system of difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1)$, $y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$, $z_{n+1} = z_{n-1}/(y_n z_{n-1} - 1)$, where $y_0, y_{-1}, x_0, x_{-1}, z_{-1}, z_0 \in \mathbb{R}$.

1. Introduction

Recently, there has been great interest in studying difference equation systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economic, probability theory, genetics, psychology, and so forth. There are many papers related to the difference equations system, for example, the following papers.

In [1], Çinar studied the solutions of the systems of the difference equations

$$x_{n+1} = \frac{1}{y_n}, \quad y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}. \quad (1.1)$$

In [2] Pappaschinopoulos and Schinas studied the oscillatory behavior, the boundedness of the solutions, and the global asymptotic stability of the positive equilibrium of the system of nonlinear difference equations

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \quad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \quad n = 0, 1, \dots, p, q. \quad (1.2)$$

In [3] Pappaschinopoulos and Schinas proved the boundedness, persistence, the oscillatory behavior, and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = \sum_{i=0}^k \frac{A_i}{y_{n-i}^{p_i}}, \quad y_{n+1} = \sum_{i=0}^k \frac{B_i}{x_{n-i}^{q_i}}. \quad (1.3)$$

In [4, 5] Özban studied the positive solutions of the system of rational difference equations

$$\begin{aligned} x_n &= \frac{a}{y_{n-3}}, & y_n &= \frac{by_{n-3}}{x_{n-q}y_{n-q}}, \\ x_{n+1} &= \frac{1}{y_{n-k}}, & y_{n+1} &= \frac{y_n}{x_{n-m}y_{n-m-k}}. \end{aligned} \quad (1.4)$$

In [6, 7] Clark and Kulenović investigate the global asymptotic stability

$$x_{n+1} = \frac{x_n}{a + cy_n}, \quad y_{n+1} = \frac{y_n}{b + dx_n}. \quad (1.5)$$

In [8] Camouzis and Pappaschinopoulos studied the global asymptotic behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = 1 + \frac{x_n}{y_{n-m}}, \quad y_{n+1} = 1 + \frac{y_n}{x_{n-m}}. \quad (1.6)$$

In [9] Yang et al. considered the behavior of the positive solutions of the system of the difference equations

$$x_n = \frac{a}{y_{n-p}}, \quad y_n = \frac{by_{n-p}}{x_{n-q}y_{n-q}}. \quad (1.7)$$

In [10] Kulenović and Nurkanović studied the global asymptotic behavior of solutions of the system of difference equations

$$x_{n+1} = \frac{a + x_n}{b + y_n}, \quad y_{n+1} = \frac{c + y_n}{d + z_n}, \quad z_{n+1} = \frac{e + z_n}{f + x_n}. \quad (1.8)$$

In [11] Zhang et al. investigated the behavior of the positive solutions of the system of difference equations

$$x_n = A + \frac{1}{y_{n-p}}, \quad y_n = A + \frac{y_{n-1}}{x_{n-r}y_{n-s}}. \quad (1.9)$$

In [12] Zhang et al. studied the boundedness, the persistence, and global asymptotic stability of the positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{y_{n-m}}{x_n}, \quad y_{n+1} = A + \frac{x_{n-m}}{y_n}. \quad (1.10)$$

In [13] Yalcinkaya studied the global asymptotic behavior of a system of two nonlinear difference equations.

In [14] Yalcinkaya et al. investigated the solutions of the system of difference equations

$$x_{n+1}^{(1)} = \frac{x_n^{(2)}}{x_n^{(2)} - 1}, x_{n+1}^{(2)} = \frac{x_n^{(3)}}{x_n^{(3)} - 1}, \dots, x_{n+1}^{(k)} = \frac{x_n^{(1)}}{x_n^{(1)} - 1}. \quad (1.11)$$

In [15] Yalcinkaya studied the global asymptotic stability of the system of difference equations

$$z_{n+1} = \frac{t_n z_{n-1} + a}{t_n + z_{n-1}}, \quad t_{n+1} = \frac{z_n t_{n-1} + a}{z_n + t_{n-1}}. \quad (1.12)$$

In [16] Irićanin and Stević studied the positive solutions of the system of difference equations

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)}}{x_{n-1}^{(3)}}, x_{n+1}^{(2)} = \frac{1 + x_n^{(3)}}{x_{n-1}^{(4)}}, \dots, x_{n+1}^{(k)} = \frac{1 + x_n^{(1)}}{x_{n-1}^{(2)}}, \quad (1.13)$$

$$x_{n+1}^{(1)} = \frac{1 + x_n^{(2)} + x_{n-1}^{(3)}}{x_{n-2}^{(4)}}, x_{n+1}^{(2)} = \frac{1 + x_n^{(3)} + x_{n-1}^{(4)}}{x_{n-2}^{(5)}}, \dots, x_{n+1}^{(k)} = \frac{1 + x_n^{(1)} + x_{n-1}^{(2)}}{x_{n-2}^{(3)}}.$$

In [17] Kurbanli et al. studied the behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1}. \quad (1.14)$$

Also see references.

In this paper, we investigate the behavior of the solutions of the difference equations system

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \quad z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1}, \quad (1.15)$$

where the initial conditions are arbitrary real numbers.

Theorem 1.1. Let $y_0, y_{-1}, x_0, x_{-1}, z_{-1}, z_0 \in \mathbb{R}$ be arbitrary real numbers and $y_0 = a$, $y_{-1} = b$, $x_0 = c$, $x_{-1} = d$, $z_0 = e$, $z_{-1} = f$ and let $\{x_n, y_n, z_n\}$ be a solution of the system (1.15). Also, assume that $af \neq 1$ and $bc \neq 1$ then all solutions of (1.15) are

$$\begin{aligned}
 x_n &= \begin{cases} \frac{d}{(ad-1)^n}, & n\text{-odd}, \\ c(bc-1)^n, & n\text{-even}, \end{cases} \\
 y_n &= \begin{cases} \frac{b}{(bc-1)^n}, & n\text{-odd}, \\ a(ad-1)^n, & n\text{-even}, \end{cases} \\
 z_n &= \begin{cases} \frac{f}{(-1)^0 \binom{n}{0} a^n f d^{n-1} + (-1)^1 \binom{n}{1} a^{n-1} f d^{n-2} + \dots + (-1)^{n-1} \binom{n}{n-1} a^1 f d^0 + (-1)^n \binom{n}{n}}, & n\text{-odd} \\ (-1)^n \frac{(bc-1)^n e}{(-1)^n \binom{n}{1} b^1 c^0 e + \dots + (-1)^1 \binom{n}{n} b^n c^{n-1} e + (-1)^0 \binom{n}{0} b^n c^n + \dots + (-1)^n \binom{n}{n} b^0 c^0}, & n\text{-even}. \end{cases}
 \end{aligned} \tag{1.16}$$

Proof. For $n = 0, 1, 2, 3$, we have

$$\begin{aligned}
 x_1 &= \frac{x_{-1}}{y_0 x_{-1} - 1} = \frac{d}{ad-1}, \\
 y_1 &= \frac{y_{-1}}{x_0 y_{-1} - 1} = \frac{b}{bc-1}, \\
 z_1 &= \frac{z_{-1}}{y_0 z_{-1} - 1} = \frac{f}{af-1}, \\
 x_2 &= \frac{x_0}{y_1 x_0 - 1} = \frac{c}{(b/(bc-1))c - 1} = \frac{c(bc-1)}{bc - bc + 1} = c(bc-1), \\
 y_2 &= \frac{y_0}{x_1 y_0 - 1} = \frac{a}{(d/(ad-1))a - 1} = \frac{a(ad-1)}{ad - ad + 1} = a(ad-1), \\
 z_2 &= \frac{z_0}{y_1 z_0 - 1} = \frac{e}{(b/(bc-1))e - 1} = \frac{e(bc-1)}{be - bc + 1} = -\frac{e(bc-1)}{-1 \cdot be + bc - 1}, \\
 x_3 &= \frac{x_1}{y_2 x_1 - 1} = \frac{d/(ad-1)}{a(ad-1) \cdot d/(ad-1) - 1} = \frac{d}{(ad-1)^2}, \\
 y_3 &= \frac{y_1}{x_2 y_1 - 1} = \frac{b/(bc-1)}{c(bc-1) \cdot b/(bc-1) - 1} = \frac{b/(bc-1)}{bc-1} = \frac{b}{(bc-1)^2}, \\
 z_3 &= \frac{z_1}{y_2 z_1 - 1} = \frac{f/(af-1)}{a(ad-1) \cdot f/(af-1) - 1} = \frac{f/(af-1)}{(a^2 df - af - af + 1)/(af-1)} = \frac{f}{a^2 df - 2af + 1}
 \end{aligned} \tag{1.17}$$

for $n = k$ assume that

$$x_k = \begin{cases} \frac{d}{(ad-1)^k}, & k\text{-odd}, \\ c(bc-1)^k, & k\text{-even}, \end{cases}$$

$$y_k = \begin{cases} \frac{b}{(bc-1)^k}, & k\text{-odd}, \\ a(ad-1)^k, & k\text{-even}, \end{cases}$$

$$z_{2k-1} = \frac{f}{(-1)^0 \binom{k}{0} a^k f d^{k-1} + (-1)^1 \binom{k}{1} a^{k-1} f d^{k-2} + \dots + (-1)^{k-1} \binom{k}{k-1} a^1 f d^0 + (-1)^k \binom{k}{k}},$$

$k = 1, 2, \dots,$

$$z_{2k} = (-1)^k \frac{(bc-1)^k e}{(-1)^k \binom{k}{1} b^1 c^0 e + \dots + (-1)^1 \binom{k}{k} b^k c^{k-1} e + (-1)^0 \binom{k}{0} b^k c^k + \dots + (-1)^k \binom{k}{k} b^1 c^1},$$

$k = 1, 2, \dots$
(1.18)

are true. Then for $n = k + 1$ we will show that (1.16) is true. From (1.15), we have

$$x_{2k+1} = \frac{x_{2k-1}}{y_{2k} x_{2k-1} - 1} = \frac{d/(ad-1)^k}{a(ad-1)^k \cdot d/(ad-1)^k - 1} = \frac{d}{(ad-1)^{k+1}}. \quad (1.19)$$

Also, similarly from (1.15), we have

$$y_{2k+1} = \frac{y_{2k-1}}{x_{2k} y_{2k-1} - 1} = \frac{b/(bc-1)^k}{c(bc-1)^k \cdot b/(bc-1)^k - 1} = \frac{b}{(bc-1)^{k+1}},$$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!},$$

(1.20)

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k},$$

$$\binom{n}{0} = \binom{n}{n} = \binom{n+1}{n+1}$$

from properties of Binomial coefficients,

$$\begin{aligned}
 z_{2k+1} &= \frac{z_{2k-1}}{y_{2k}z_{2k-1} - 1} \\
 &= \frac{f / \left((-1)^0 \binom{k}{0} a^k f d^{k-1} + \dots + (-1)^{k-1} \binom{k}{k-1} a^1 f d^0 + (-1)^k \binom{k}{k} \right)}{a(ad-1)^k f / \left((-1)^0 \binom{k}{0} a^k f d^{k-1} + \dots + (-1)^{k-1} \binom{k}{k-1} a^1 f d^0 + (-1)^k \binom{k}{k} \right) - 1} \\
 &= \frac{f}{(-1)^0 \binom{k+1}{0} a^{k+1} f d^k + (-1)^1 \binom{k+1}{1} a^k f d^{k-1} + \dots + (-1)^k \binom{k+1}{k} a^1 f d^0 + (-1)^{k+1} \binom{k+1}{k+1}} \tag{1.21}
 \end{aligned}$$

written and accurate.

Also, we have

$$x_{2k+2} = \frac{x_{2k}}{y_{2k+1}x_{2k} - 1} = \frac{c(bc-1)^k}{\left(\frac{b}{(bc-1)^{k+1}} \right) c(bc-1)^k - 1} = \frac{c(bc-1)^k}{bc/(bc-1) - 1} = c(bc-1)^{k+1},$$

$$y_{2k+2} = \frac{y_{2k}}{x_{2k+1}y_{2k} - 1} = \frac{a(ad-1)^k}{\left(\frac{d}{(ad-1)^{k+1}} \right) \cdot a(ad-1)^k - 1} = \frac{a(ad-1)^k}{ad/(ad-1) - 1} = a(ad-1)^{k+1},$$

$$\begin{aligned}
 z_{2k+2} &= \frac{z_{2k}}{y_{2k+1}z_{2k} - 1} \\
 &= \frac{(-1)^k (bc-1)^k e / \left(\mathcal{A} + (-1)^0 \binom{k}{0} b^k c^k + \dots + (-1)^k \binom{k}{k} b^0 c^0 \right)}{\left(\frac{b}{(bc-1)^{k+1}} \right) \cdot (-1)^k (bc-1)^k e / \left(\mathcal{A} + (-1)^0 \binom{k}{0} b^k c^k + \dots + (-1)^k \binom{k}{k} b^0 c^0 \right) - 1} = (-1)^{k+1} \\
 &\quad \times \frac{(bc-1)^{k+1} e}{(-1)^{k+1} \binom{k+1}{1} b^1 c^0 e + \dots + (-1)^1 \binom{k+1}{k+1} b^{k+1} c^k e + (-1)^0 \binom{k+1}{0} b^{k+1} c^{k+1} + \dots + (-1)^{k+1} \binom{k+1}{k+1} b^1 c^1} \tag{1.22}
 \end{aligned}$$

where \mathcal{A} denotes $(-1)^k \binom{k}{1} b^1 c^0 e + \dots + (-1)^1 \binom{k}{k} b^k c^{k-1} e$. □

Corollary 1.2. *Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system (1.15). If $ad \neq 1$, $bc \neq 1$, $0 < ad < 1$ and $0 < bc < 1$ then one has*

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} y_{2n-1} = \infty, \tag{1.23}$$

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} y_{2n} = 0.$$

Proof. From $ad \neq 1$, $bc \neq 1$, $0 < ad < 1$ and $0 < bc < 1$ we have $-1 < ad - 1 < 0$ and $-1 < bc - 1 < 0$. Hence, we have

$$\begin{aligned} x_{2n-1} &= \frac{d}{(ad-1)^n}, & y_{2n-1} &= \frac{b}{(bc-1)^n}, \\ x_{2n} &= c(bc-1)^n, & y_{2n} &= a(ad-1)^n. \end{aligned} \quad (1.24)$$

Then,

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} \frac{d}{(ad-1)^n} = \begin{cases} +\infty, & d > 0, n\text{-even} \\ +\infty, & d < 0, n\text{-odd} \\ -\infty, & d > 0, n\text{-odd} \\ -\infty, & d < 0, n\text{-even}, \end{cases} \quad (1.25)$$

$$\lim_{n \rightarrow \infty} y_{2n-1} = \lim_{n \rightarrow \infty} \frac{b}{(bc-1)^n} = \begin{cases} +\infty, & b > 0, n\text{-even} \\ +\infty, & b < 0, n\text{-odd} \\ -\infty, & b > 0, n\text{-odd} \\ -\infty, & b < 0, n\text{-even}. \end{cases}$$

Also,

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} c(bc-1)^n = 0, \quad (1.26)$$

$$\lim_{n \rightarrow \infty} y_{2n} = \lim_{n \rightarrow \infty} a(ad-1)^n = 0.$$

□

Example 1.3. If $y(0) = -1/2$, $y(-1) = -1/2$, $x(0) = -1/2$, $x(-1) = -1/2$, then the solutions of (1.15) can be represented by Table 1.

Corollary 1.4. Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system (1.15). If $ad \neq 1$, $bc \neq 1$, $1 < ad < 2$ and $1 < bc < 2$ then one has

$$\begin{aligned} \lim_{n \rightarrow \infty} x_{2n-1} &= \lim_{n \rightarrow \infty} y_{2n-1} = \infty, \\ \lim_{n \rightarrow \infty} x_{2n} &= \lim_{n \rightarrow \infty} y_{2n} = 0. \end{aligned} \quad (1.27)$$

Table 1

i	1	2	3	4	5	6	7	8
x_{2i-1}	0.6667	-0.8889	1.1852	-1.5803	2.1070	-2.8093	3.7458	-4.9944
x_{2i}	0.3750	-0.2813	0.2109	-0.1582	0.1187	-0.0890	0.0667	-0.0501
y_{2i-1}	0.6667	-0.8889	1.1852	-1.5803	2.1070	-2.8093	3.7458	-4.9944
y_{2i}	0.3750	-0.2813	0.2109	-0.1582	0.1187	-0.0890	0.0667	-0.0501
z_{2i-1}	0.6667	-0.8889	1.1852	-1.5803	2.1070	-2.8093	3.7458	-4.9944
z_{2i}	0.3750	-0.2813	0.2109	-0.1582	0.1187	-0.0890	0.0667	-0.0501

Proof. From $ad \neq 1$, $bc \neq 1$, $1 < ad < 2$ and $1 < bc < 2$ we have $0 < ad - 1 < 1$ and $0 < bc - 1 < 1$. Hence, we have

$$\begin{aligned}
 x_{2n-1} &= \frac{d}{(ad-1)^n}, & y_{2n-1} &= \frac{b}{(bc-1)^n}, \\
 x_{2n} &= c(bc-1)^n, & y_{2n} &= a(ad-1)^n.
 \end{aligned} \tag{1.28}$$

Then,

$$\begin{aligned}
 \lim_{n \rightarrow \infty} x_{2n-1} &= \lim_{n \rightarrow \infty} \frac{d}{(ad-1)^n} = \begin{cases} +\infty, & d > 0 \\ -\infty, & d < 0, \end{cases} \\
 \lim_{n \rightarrow \infty} y_{2n-1} &= \lim_{n \rightarrow \infty} \frac{b}{(bc-1)^n} = \begin{cases} +\infty, & b > 0 \\ -\infty, & b < 0. \end{cases}
 \end{aligned} \tag{1.29}$$

Also,

$$\begin{aligned}
 \lim_{n \rightarrow \infty} x_{2n} &= \lim_{n \rightarrow \infty} c(bc-1)^n = 0, \\
 \lim_{n \rightarrow \infty} y_{2n} &= \lim_{n \rightarrow \infty} a(ad-1)^n = 0.
 \end{aligned} \tag{1.30}$$

□

Example 1.5. If $y(0) = 3/2$, $y(-1) = 3/4$, $x(0) = 3/2$, $x(-1) = 3/4$, $z(0) = 3/2$, $z(-1) = 3/4$ then the solutions of (1.15) can be represented by Table 2.

Corollary 1.6. Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system (1.15).

If $-\infty < ad < 0$ and $-\infty < bc < 0$ then one has

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} y_{2n-1} = 0, \quad (1.31)$$

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} y_{2n} = \infty.$$

Proof. From $-\infty < ad < 0$ and $-\infty < bc < 0$ we have $-\infty < ad - 1 < -1$ and $-\infty < bc - 1 < -1$. Hence, we have

$$x_{2n-1} = \frac{d}{(ad-1)^{n'}}, \quad y_{2n-1} = \frac{b}{(bc-1)^{n'}}, \quad (1.32)$$

$$x_{2n} = c(bc-1)^n, \quad y_{2n} = a(ad-1)^n.$$

Then,

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} \frac{d}{(ad-1)^n} = 0, \quad (1.33)$$

$$\lim_{n \rightarrow \infty} y_{2n-1} = \lim_{n \rightarrow \infty} \frac{b}{(bc-1)^n} = 0.$$

Also,

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} c(bc-1)^n = \begin{cases} +\infty, & c > 0, n\text{-even} \\ +\infty, & c < 0, n\text{-odd} \\ -\infty, & c > 0, n\text{-odd} \\ -\infty, & c < 0, n\text{-even,} \end{cases} \quad (1.34)$$

$$\lim_{n \rightarrow \infty} y_{2n} = \lim_{n \rightarrow \infty} a(ad-1)^n = \begin{cases} +\infty, & a > 0, n\text{-even} \\ +\infty, & a < 0, n\text{-odd} \\ -\infty, & a > 0, n\text{-odd} \\ -\infty, & a < 0, n\text{-even.} \end{cases}$$

□

Example 1.7. If $y(0) = -3/2$, $y(-1) = -3/4$, $x(0) = 3/2$, $x(-1) = 3/4$, $z(0) = 3/2$, $z(-1) = 3/4$ then the solutions of (1.15) can be represented by Table 3.

Table 2

i	1	2	3	4	5	6	7	8
x_{2i-1}	6	48	384	3072	24576	$1.967 \cdot 10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
x_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$
y_{2i-1}	6	48	384	3072	24576	$1.967 \cdot 10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
y_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$
z_{2i-1}	6	48	384	3072	24576	$1.967 \cdot 10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
z_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$

Table 3

i	1	2	3	4	5	6	7	8
x_{2i-1}	-0.3529	0.1661	-0.0782	0.0368	-0.0173	0.0081	-0.0038	0.0018
x_{2i}	-3.1875	6.7734	-14.3936	30.5863	-64.9959	138.1163	-293.4971	623.6813
y_{2i-1}	0.3529	-0.1661	0.0782	-0.0368	0.0173	-0.0081	0.0038	-0.0018
y_{2i}	3.1875	-6.7734	14.3936	-30.5863	64.9959	-138.1163	293.4971	-623.6813
z_{2i-1}	-0.3529	0.1661	-0.0782	0.0368	-0.0173	0.0081	-0.0038	0.0018
z_{2i}	-3.1875	6.7734	-14.3936	30.5863	-64.9959	138.1163	-293.4971	623.6813

Corollary 1.8. Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system (1.15). If $2 < ad < +\infty$ and $2 < bc < +\infty$ then one has

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} y_{2n-1} = 0, \quad (1.35)$$

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} y_{2n} = \infty.$$

Proof. From $2 < ad < +\infty$ and $2 < bc < +\infty$ we have $1 < ad - 1 < +\infty$ and $1 < bc - 1 < +\infty$. Hence, we have

$$x_{2n-1} = \frac{d}{(ad-1)^n}, \quad y_{2n-1} = \frac{b}{(bc-1)^n}, \quad (1.36)$$

$$x_{2n} = c(bc-1)^n, \quad y_{2n} = a(ad-1)^n.$$

Then,

$$\lim_{n \rightarrow \infty} x_{2n-1} = \lim_{n \rightarrow \infty} \frac{d}{(ad-1)^n} = 0, \quad (1.37)$$

$$\lim_{n \rightarrow \infty} y_{2n-1} = \lim_{n \rightarrow \infty} \frac{b}{(bc-1)^n} = 0.$$

Table 4

i	1	2	3	4	5	6	7	8
x_{2i-1}	0.546	0.198	0.072	0.026	0.010	0.004	0.001	0.0005
x_{2i}	4.125	11.344	31.195	85.787	235.915	648.765	1784.104	4906.285
y_{2i-1}	0.909	0.331	0.120	0.044	0.016	0.006	0.002	0.001
y_{2i}	6.875	18.906	51.992	142.979	393.191	1081.275	2973.506	8177.142
z_{2i-1}	0.546	0.198	0.072	0.026	0.010	0.004	0.001	0.0005
z_{2i}	4.125	11.344	31.195	85.787	235.915	648.765	1784.104	4906.285

Also,

$$\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} c(bc - 1)^n = \begin{cases} +\infty, & c > 0 \\ -\infty, & c < 0, \end{cases} \quad (1.38)$$

$$\lim_{n \rightarrow \infty} y_{2n} = \lim_{n \rightarrow \infty} a(ad - 1)^n = \begin{cases} +\infty, & a > 0 \\ -\infty, & a < 0. \end{cases}$$

□

Example 1.9. If $y(0) = 5/2$, $y(-1) = 5/2$, $x(0) = 3/2$, $x(-1) = 3/2$, $z(0) = 3/2$, $z(-1) = 3/2$ then the solutions of (1.15) can be represented by Table 4.

Acknowledgment

The authors are grateful to the anonymous referees for their valuable suggestions that improved the quality of this study.

References

- [1] C. Çinar, "On the positive solutions of the difference equation system $x_{n+1} = 1/y_n$, $y_{n+1} = y_n/x_{n-1}y_{n-1}$," *Applied Mathematics and Computation*, vol. 158, no. 2, pp. 303–305, 2004.
- [2] G. Papaschinopoulos and C. J. Schinas, "On a system of two nonlinear difference equations," *Journal of Mathematical Analysis and Applications*, vol. 219, no. 2, pp. 415–426, 1998.
- [3] G. Papaschinopoulos and C. J. Schinas, "On the system of two difference equations," *Journal of Mathematical Analysis and Applications*, vol. 273, no. 2, pp. 294–309, 2002.
- [4] A. Y. Özban, "On the system of rational difference equations $x_n = a/y_{n-3}$, $y_n = by_{n-3}/x_{n-q}y_{n-q}$," *Applied Mathematics and Computation*, vol. 188, no. 1, pp. 833–837, 2007.
- [5] A. Y. Özban, "On the positive solutions of the system of rational difference equations $x_{n+1} = 1/y_{n-k}$, $y_{n+1} = y_n/x_{n-m}y_{n-m-k}$," *Journal of Mathematical Analysis and Applications*, vol. 323, no. 1, pp. 26–32, 2006.
- [6] D. Clark and M. R. S. Kulenović, "A coupled system of rational difference equations," *Computers & Mathematics with Applications*, vol. 43, no. 6-7, pp. 849–867, 2002.
- [7] D. Clark, M. R. S. Kulenović, and J. F. Selgrade, "Global asymptotic behavior of a two-dimensional difference equation modelling competition," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 52, no. 7, pp. 1765–1776, 2003.
- [8] E. Camouzis and G. Papaschinopoulos, "Global asymptotic behavior of positive solutions on the system of rational difference equations $x_{n+1} = 1 + x_n/y_{n-m}$, $y_{n+1} = 1 + y_n/x_{n-m}$," *Applied Mathematics Letters*, vol. 17, no. 6, pp. 733–737, 2004.

- [9] X. Yang, Y. Liu, and S. Bai, "On the system of high order rational difference equations $x_n = a/y_{n-p}$, $y_n = by_{n-p}/x_{n-q}y_{n-q}$," *Applied Mathematics and Computation*, vol. 171, no. 2, pp. 853–856, 2005.
- [10] M. R. S. Kulenović and Z. Nurkanović, "Global behavior of a three-dimensional linear fractional system of difference equations," *Journal of Mathematical Analysis and Applications*, vol. 310, no. 2, pp. 673–689, 2005.
- [11] Y. Zhang, X. Yang, G. M. Megson, and D. J. Evans, "On the system of rational difference equations $x_n = A + 1/y_{n-p}$, $y_n = A + y_{n-1}/x_{n-r}y_{n-s}$," *Applied Mathematics and Computation*, vol. 176, no. 2, pp. 403–408, 2006.
- [12] Y. Zhang, X. Yang, D. J. Evans, and C. Zhu, "On the nonlinear difference equation system $x_{n+1} = A + y_{n-m}/x_n$, $y_{n+1} = A + x_{n-m}/y_n$," *Computers & Mathematics with Applications*, vol. 53, no. 10, pp. 1561–1566, 2007.
- [13] I. Yalcinkaya, "On the global asymptotic behavior of a system of two nonlinear difference equations," *Ars Combinatoria*, vol. 95, pp. 151–159, 2010.
- [14] I. Yalcinkaya, C. Çinar, and M. Atalay, "On the solutions of systems of difference equations," *Advances in Difference Equations*, vol. 2008, Article ID 143943, 9 pages, 2008.
- [15] I. Yalcinkaya, "On the global asymptotic stability of a second-order system of difference equations," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 860152, 12 pages, 2008.
- [16] B. D. Iričanin and S. Stević, "Some systems of nonlinear difference equations of higher order with periodic solutions," *Dynamics of Continuous, Discrete & Impulsive Systems. Series A. Mathematical Analysis*, vol. 13, no. 3-4, pp. 499–507, 2006.
- [17] A. S. Kurbanlı, C. Çinar, and I. Yalcinkaya, "On the behavior of positive solutions of the system of rational difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1}) + 1$, $y_{n+1} = y_{n-1}/(x_n y_{n-1}) + 1$," *Mathematical and Computer Modelling*, vol. 53, no. 5-6, pp. 1261–1267, 2011.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

