

Research Article

Global Attractivity and Periodic Solution of a Discrete Multispecies Cooperation and Competition Predator-Prey System

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We propose a discrete multispecies cooperation and competition predator-prey systems. For general nonautonomous case, sufficient conditions which ensure the permanence and the global stability of the system are obtained; for periodic case, sufficient conditions which ensure the existence of a globally stable positive periodic solution of the system are obtained.

1. Introduction

In this paper, we consider the dynamic behavior of the following non-autonomous discrete $n + m$ -species cooperation and competition predator-prey systems

$$\begin{aligned} x_i(k+1) &= x_i(k) \exp \left[r_{1i}(k) \left(1 - \frac{x_i(k)}{a_i(k) + \sum_{l=1, l \neq i}^n b_{il}(k)x_l(k)} - c_i(k)x_i(k) \right) \right. \\ &\quad \left. - \sum_{l=1}^m d_{il}(k)y_l(k) \right], \\ y_j(k+1) &= y_j(k) \exp \left[r_{2j}(k) + \sum_{l=1}^n e_{jl}(k)x_l(k) - \sum_{l=1}^m p_{jl}(k)y_l(k) \right], \end{aligned} \quad (1.1)$$

where $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$; $x_i(k)$ is the density of prey species i at k th generation. $y_j(k)$ is the density of predator species j at k th generation.

Dynamic behaviors of population models governed by difference equations had been studied by a number of papers, see [1–4] and the references cited therein. It has been found that the autonomous discrete systems can demonstrate quite rich and complicated dynamics, see [5, 6]. Recently, more and more scholars paid attention to the non-autonomous discrete population models, since such kind of model could be more appropriate.

May [7] suggested the following set of equations to describe a pair of mutualists:

$$\begin{aligned} \dot{u} &= r_1 u \left(1 - \frac{u}{a_1 + b_1 v} - c_1 u \right), \\ \dot{v} &= r_2 v \left(1 - \frac{v}{a_2 + b_2 u} - c_2 v \right), \end{aligned} \quad (1.2)$$

where u, v are the densities of the species U, V at time t , respectively. $r_i, a_i, b_i, c_i, i = 1, 2$ are positive constants. He showed that system (1.2) has a globally asymptotically stable equilibrium point in the region $u > 0, v > 0$.

Bai et al. [8] argued that the discrete case of cooperative system is more appropriate, and they proposed the following system:

$$\begin{aligned} x_1(k+1) &= x_1(k) \exp \left\{ r_1(k) \left[1 - \frac{x_1(k)}{a_1(k) + b_1(k)x_2(k)} - c_1(k)x_1(k) \right] \right\}, \\ x_2(k+1) &= x_2(k) \exp \left\{ r_2(k) \left[1 - \frac{x_2(k)}{a_2(k) + b_2(k)x_1(k)} - c_2(k)x_2(k) \right] \right\}. \end{aligned} \quad (1.3)$$

Chen [9] investigated the dynamic behavior of the following discrete $n + m$ -species Lotka-Volterra competition predator-prey systems

$$\begin{aligned} x_i(k+1) &= x_i(k) \exp \left[b_i(k) - \sum_{l=1}^n a_{il}(k)x_l(k) - \sum_{l=1}^m c_{il}(k)y_l(k) \right], \\ y_j(k+1) &= y_j(k) \exp \left[r_j(k) + \sum_{l=1}^n d_{jl}(k)x_l(k) - \sum_{l=1}^m e_{jl}(k)y_l(k) \right], \end{aligned} \quad (1.4)$$

he investigated the dynamic behavior of the system (1.4).

The aim of this paper is, by further developing the analysis technique of Huo and Li [10] and Chen [9], to obtain a set of sufficient condition which ensure the permanence and the global stability of the system (1.1); for periodic case, sufficient conditions which ensure the existence of a globally stable positive periodic solution of the system (1.1) are obtained.

We say that system (1.1) is permanent if there are positive constants M and m such that for each positive solution $(x_1(k), \dots, x_n(k), y_1(k), \dots, y_m(k))$ of system (1.1) satisfies

$$\begin{aligned} m &\leq \liminf_{k \rightarrow +\infty} x_i(k) \leq \limsup_{k \rightarrow +\infty} x_i(k) \leq M, \\ m &\leq \liminf_{k \rightarrow +\infty} y_j(k) \leq \limsup_{k \rightarrow +\infty} y_j(k) \leq M, \end{aligned} \quad (1.5)$$

for all $i = 1, 2, \dots, n; j = 1, 2, \dots, m$.

Throughout this paper, we assume that $r_{1i}(k), b_{il}(k), a_i(k), c_i(k), r_{2j}(k), d_{il}(k), e_{jl}(k), p_{jl}(k)$ are all bounded nonnegative sequence, and use the following notations for any bounded sequence $\{a(k)\}$

$$a^u = \sup_{k \in \mathbb{N}} a(k), \quad a^l = \inf_{k \in \mathbb{N}} a(k). \quad (1.6)$$

For biological reasons, we only consider solution $(x_1(k), \dots, x_n(k), y_1(k), \dots, y_m(k))$ with

$$x_i(0) > 0; \quad i = 1, 2, \dots, n, \quad y_j(0) > 0, \quad j = 1, 2, \dots, m. \quad (1.7)$$

Then system (1.1) has a positive solution $(x_1(k), \dots, x_n(k), y_1(k), \dots, y_m(k))_{k=0}^{\infty}$ passing through $(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0))$.

The organization of this paper is as follows. In Section 2, we obtain sufficient conditions which guarantee the permanence of the system (1.1). In Section 3, we obtain sufficient conditions which guarantee the global stability of the positive solution of system (1.1). As a consequence, for periodic case, we obtain sufficient conditions which ensure the existence of a globally stable positive solution of system (1.1).

2. Permanence

In this section, we establish permanence results for system (1.1).

Lemma 2.1 (see [11]). *Let $k \in N_{k_0}^+ = \{k_0, k_0 + 1, \dots, k_0 + l, \dots\}, r \geq 0$. For any fixed $k, g(k, r)$ is a non-decreasing function with respect to r , and for $k \geq k_0$, the following inequalities hold:*

$$\begin{aligned} y(k+1) &\leq g(k, y(k)), \\ u(k+1) &\geq g(k, u(k)). \end{aligned} \quad (2.1)$$

If $y(k_0) \leq u(k_0)$, then $y(k) \leq u(k)$ for all $k \geq k_0$.

Now let one consider the following single species discrete model:

$$N(k+1) = N(k) \exp\{a(k) - b(k)N(k)\}, \quad (2.2)$$

where $\{a(k)\}$ and $\{b(k)\}$ are strictly positive sequences of real numbers defined for $k \in N = \{0, 1, 2, \dots\}$ and $0 < a^l \leq a^u, 0 < b^l \leq b^u$.

Lemma 2.2 (see [12]). *Any solution of system (2.5) with initial condition $N(0) > 0$ satisfies*

$$m \leq \liminf_{k \rightarrow +\infty} N(k) \leq \limsup_{k \rightarrow +\infty} N(k) \leq M, \quad (2.3)$$

where

$$M = \frac{1}{b^l} \exp\{a^u - 1\}, \quad m = \frac{a^l}{b^u} \exp\{a^l - b^u M\}. \quad (2.4)$$

Proposition 2.3. Assume that

$$-r_{2j}^u + \sum_{l=1}^n e_{jl}^l M_l > 0, \quad j = 1, 2, \dots, m \quad (2.5)$$

holds, then

$$\begin{aligned} \limsup_{k \rightarrow +\infty} x_i(k) &\leq M_i, \quad i = 1, 2, \dots, n, \\ \limsup_{k \rightarrow +\infty} y_i(k) &\leq Q_i, \quad i = 1, 2, \dots, m, \end{aligned} \quad (2.6)$$

where

$$\begin{aligned} M_i &= \frac{1}{c_i^l r_{1i}^l} \exp\{r_{1i}^u - 1\}, \\ Q_i &= \frac{1}{p_{ii}^l} \exp\left\{\sum_{l=1}^n e_{il}^u M_l - r_{2i}^l - 1\right\}. \end{aligned} \quad (2.7)$$

Proof. Let $u(k) = (x_1(k), \dots, x_n(k), y_1(k), \dots, y_m(k))$ be any positive solution of system (1.1), from the i th equation of (1.1) we have

$$\begin{aligned} x_i(k+1) &\leq x_i(k) \exp\{r_{1i}(k)[1 - c_i(k)x_i(k)]\} \\ &= x_i(k) \exp\{r_{1i}(k) - r_{1i}(k)c_i(k)x_i(k)\}. \end{aligned} \quad (2.8)$$

By applying Lemmas 2.1 and 2.2, it immediately follows that

$$\limsup_{k \rightarrow +\infty} x_i(k) \leq \frac{1}{c_i^l r_{1i}^l} \exp\{r_{1i}^u - 1\} := M_i. \quad (2.9)$$

For any positive constant ε small enough, it follows from (2.9) that there exists enough large K_0 such that

$$x_i(k) \leq M_i + \varepsilon, \quad i = 1, 2, \dots, n, \quad \forall k \geq K_0. \quad (2.10)$$

From the $n + j$ th equation of the system (1.1) and (2.10), we can obtain

$$y_j(k+1) \leq y_j(k) \exp\left\{-r_{2j}(k) + \sum_{l=1}^n e_{jl}(k)(M_l + \varepsilon) - p_{jj}(k)y_j(k)\right\}. \quad (2.11)$$

Condition (2.5) shows that Lemmas 2.1 and 2.2 could be applied to (2.11), and so, by applying Lemmas 2.1 and 2.2, it immediately follows that

$$\lim_{k \rightarrow +\infty} \sup y_j(k) \leq \frac{1}{p_{jj}^l} \exp \left\{ \sum_{l=1}^n e_{jl}^u (M_l + \varepsilon) - r_{2j}^l - 1 \right\}, \quad 1 \leq j \leq m. \quad (2.12)$$

Setting $\varepsilon \rightarrow 0$ in the above inequality leads to

$$\lim_{k \rightarrow +\infty} \sup y_j(k) \leq \frac{1}{p_{jj}^l} \exp \left\{ \sum_{l=1}^n e_{jl}^u M_l - r_{2j}^l - 1 \right\} := Q_j, \quad 1 \leq j \leq m. \quad (2.13)$$

This completes the proof of Proposition 2.3. \square

Now we are in the position of stating the permanence of the system (1.1).

Theorem 2.4. *In addition to (2.5), assume further that*

$$\begin{aligned} r_{1i}^l - \sum_{l=1}^m d_{il}^u Q_l &> 0, \quad i = 1, 2, \dots, n, \\ -r_{2j}^u + \sum_{l=1}^n e_{jl}^l m_l - \sum_{l=1, l \neq j}^m p_{jl}^u Q_l &> 0, \quad j = 1, 2, \dots, m, \end{aligned} \quad (2.14)$$

then system (1.1) is permanent, where

$$\begin{aligned} m_i &= \frac{r_{1i}^l - \sum_{l=1}^m d_{il}^u Q_l}{r_{1i}^u (1/a_i^l + c_i^u)} \exp \left\{ r_{1i}^l - \sum_{l=1}^m d_{il}^u Q_l - r_{1i}^u \left(\frac{1}{a_i^l} + c_i^u \right) M_i \right\}, \\ Q_i &= \frac{1}{p_{ii}^l} \exp \left\{ \sum_{l=1}^n e_{il}^u M_l - r_{2i}^l - 1 \right\}. \end{aligned} \quad (2.15)$$

Proof. By applying Proposition 2.3, we see that to end the proof of Theorem 2.4, it is enough to show that under the conditions of Theorem 2.4,

$$\begin{aligned} \lim_{k \rightarrow +\infty} \inf x_j(k) &\geq m_i, \quad 1 \leq i \leq n, \\ \lim_{k \rightarrow +\infty} \inf y_j(k) &\geq q_j, \quad 1 \leq j \leq m. \end{aligned} \quad (2.16)$$

From Proposition 2.3, $\forall \varepsilon > 0$, there exists a $K_1 > 0, K_1 \in \mathbb{N}, \forall k > K_1$,

$$x_i(k) \leq M_i + \varepsilon, \quad i = 1, 2, \dots, n, \quad y_j(k) \leq Q_j + \varepsilon, \quad j = 1, 2, \dots, m. \quad (2.17)$$

From the i th equation of system (1.1) and (2.17), we have

$$\begin{aligned} x_i(k+1) &\geq x_i(k) \exp \left\{ r_{1i}(k) - r_{1i}(k) \left(\frac{1}{a_i^l} + c_i^u \right) x_i(k) - \sum_{l=1}^m d_{il}(k)(Q_l + \varepsilon) \right\} \\ &= x_i(k) \exp \left\{ r_{1i}(k) - \sum_{l=1}^m d_{il}(k)(Q_l + \varepsilon) - r_{1i}(k) \left(\frac{1}{a_i^l} + c_i^u \right) x_i(k) \right\}, \end{aligned} \quad (2.18)$$

for all $k > K_1$.

By applying Lemmas 2.1 and 2.2 to (2.18), it immediately follows that

$$\begin{aligned} \liminf_{k \rightarrow +\infty} x_i(k) &\geq \frac{r_{1i}^l - \sum_{l=1}^m d_{il}^u(Q_l + \varepsilon)}{r_{1i}^u(1/a_i^l + c_i^u)} \\ &\quad \times \exp \left\{ r_{1i}^l - \sum_{l=1}^m d_{il}^u(Q_l + \varepsilon) - r_{1i}^u \left(\frac{1}{a_i^l} + c_i^u \right) M_i \right\}. \end{aligned} \quad (2.19)$$

Setting $\varepsilon \rightarrow 0$ in (2.19) leads to

$$\begin{aligned} \liminf_{k \rightarrow +\infty} x_i(k) &\geq \frac{r_{1i}^l - \sum_{l=1}^m d_{il}^u Q_l}{r_{1i}^u(1/a_i^l + c_i^u)} \\ &\quad \times \exp \left\{ r_{1i}^l - \sum_{l=1}^m d_{il}^u Q_l - r_{1i}^u \left(\frac{1}{a_i^l} + c_i^u \right) M_i \right\} := m_i. \end{aligned} \quad (2.20)$$

Then, for any positive constant ε small enough, from (2.20) we know that there exists an enough large $K_2 > K_1$ such that

$$x_i(k) \geq m_i - \varepsilon, \quad \forall k \geq k_2. \quad (2.21)$$

Equations (2.17), (2.21) combining with the $n + j$ th equation of the system (1.1) leads to

$$\begin{aligned} y_j(k+1) &\geq y_j(k) \exp \left\{ -r_{2j}(k) + \sum_{l=1}^n e_{jl}(k)(m_l - \varepsilon) \right. \\ &\quad \left. - \sum_{l=1, l \neq j}^m p_{jl}(k)(Q_l + \varepsilon) - p_{jj}(k)y_j(k) \right\}, \end{aligned} \quad (2.22)$$

under the condition (2.14), by applying Lemmas 2.1 and 2.2 to (2.22), it immediately follows that

$$\begin{aligned} \liminf_{k \rightarrow +\infty} y_j(k) &\geq \frac{-r_{2j}^u + \sum_{l=1}^n e_{jl}^l (m_l - \varepsilon) - \sum_{l=1, l \neq j}^m p_{jl}^u (Q_l + \varepsilon)}{p_{jj}^u} \\ &\times \exp \left\{ -r_{2j}^u + \sum_{l=1}^n e_{jl}^l (m_l - \varepsilon) - \sum_{l=1, l \neq j}^m p_{jl}^u (Q_l + \varepsilon) - p_{jj}^u Q_j \right\}. \end{aligned} \quad (2.23)$$

Setting $\varepsilon \rightarrow 0$ in (2.23) leads to

$$\begin{aligned} \liminf_{k \rightarrow +\infty} y_j(k) &\geq \frac{-r_{2j}^u + \sum_{l=1}^n e_{jl}^l m_l - \sum_{l=1, l \neq j}^m p_{jl}^u Q_l}{p_{jj}^u} \\ &\times \exp \left\{ -r_{2j}^u + \sum_{l=1}^n e_{jl}^l m_l - \sum_{l=1, l \neq j}^m p_{jl}^u Q_l - p_{jj}^u Q_j \right\} := q_j. \end{aligned} \quad (2.24)$$

This ends the proof of Theorem 2.4. \square

It should be noticed that, under the assumption of Theorem 2.4, the set

$$[m_1, M_1] \times \cdots [m_n, M_n] \times [q_1, Q_1] \times \cdots [q_m, Q_m] \quad (2.25)$$

is an invariant set of system (1.1).

3. Global Stability

Now we study the stability property of the positive solution of system (1.1).

Theorem 3.1. *Assume that*

$$\begin{aligned} \lambda_i &= \max \left\{ \left| 1 - \left(c_i^u + \frac{1}{a_i^l} \right) r_{1i}^u M_i \right|, \left| 1 - \left(c_i^l + \frac{1}{a_i^u} \right) r_{1i}^l m_i \right| \right\} + \frac{r_{1i}^u M_i}{(a_i^l)^2} \sum_{l=1, l \neq i}^n b_{il}^u M_l \\ &+ \sum_{l=1}^m d_{il}^u Q_l < 1, \end{aligned} \quad (3.1)$$

$$\delta_j = \max \left\{ \left| 1 - p_{jj}^u Q_j \right|, \left| 1 - p_{jj}^l q_j \right| \right\} + \sum_{l=1}^n e_{ji}^u M_l + \sum_{l=1}^m p_{ji}^u Q_l < 1.$$

Then for any two positive solution $(x_1(k), \dots, x_n(k), y_1(k), \dots, y_m(k))$ and $(\tilde{x}_1(k), \dots, \tilde{x}_n(k), \tilde{y}_1(k), \dots, \tilde{y}_m(k))$ of system (1.1), one has

$$\lim_{k \rightarrow +\infty} (\tilde{x}_i(k) - x_i(k)) = 0, \quad \lim_{k \rightarrow +\infty} (\tilde{y}_j(k) - y_j(k)) = 0. \quad (3.2)$$

Proof. Let

$$x_i(k) = \tilde{x}_i(k) \exp\{u_i(k)\}, \quad y_j(k) = \tilde{y}_j(k) \exp\{v_j(k)\}. \quad (3.3)$$

Then system (1.1) is equivalent to

$$\begin{aligned} u_i(k+1) = & u_i(k) + \frac{r_{1i}(k)\tilde{x}_i(k)}{a_i(k) + \sum_{l=1, l \neq i}^n b_{il}(k)\tilde{x}_l(k)} - \frac{r_{1i}(k)\tilde{x}_i(k) \exp\{u_i(k)\}}{a_i(k) + \sum_{l=1, l \neq i}^n b_{il}(k)\tilde{x}_l(k) \exp\{u_l(k)\}} \\ & - r_{1i}(k)c_i(k)\tilde{x}_i(k)(\exp\{u_i(k)\} - 1) - \sum_{l=1}^m d_{il}(k)\tilde{y}_l(k)(\exp\{v_l(k)\} - 1), \end{aligned} \quad (3.4)$$

$$\begin{aligned} v_j(k+1) = & v_j(k) - \sum_{l=1}^n e_{jl}(k)\tilde{x}_l(k)(\exp\{u_l(k)\} - 1) \\ & - \sum_{l=1}^m p_{jl}(k)\tilde{y}_l(k)(\exp\{v_l(k)\} - 1). \end{aligned}$$

So,

$$\begin{aligned} |u_i(k+1)| \leq & \left| 1 - \left(c_i(k) + \frac{1}{a_i(k)} \right) r_{1i}(k)\tilde{x}_i(k) \exp\{\theta_i(k)u_i(k)\} \right| |u_i(k)| \\ & + \frac{r_{1i}(k)\tilde{x}_i(k)}{a_i^2(k)} \sum_{l=1, l \neq i}^n b_{il}(k)\tilde{x}_l(k) \exp\{\theta_l(k)u_l(k)\} |u_l(k)| \\ & + \sum_{l=1}^m d_{il}(k)\tilde{y}_l(k) \exp\{\xi_l(k)v_l(k)\} |v_l(k)| \end{aligned} \quad (3.5)$$

$$\begin{aligned} |v_i(k+1)| \leq & \left| 1 - p_{ij}(k)\tilde{y}_j(k) \exp\{\xi_j(k)v_j(k)\} \right| |v_j(k)| \\ & + \sum_{l=1}^n e_{jl}(k)\tilde{x}_l(k) \exp\{\theta_l(k)u_l(k)\} |u_l(k)| \\ & + \sum_{l=1}^m p_{il}(k)\tilde{y}_l(k) \exp\{\xi_l(k)v_l(k)\} |v_l(k)|, \end{aligned}$$

where $\theta_l(k), \xi_l(k) \in [0, 1]$, to complete the proof, it suffices to show that

$$\lim_{k \rightarrow +\infty} u_i(k) = 0, \quad \lim_{k \rightarrow +\infty} v_j(k) = 0. \quad (3.6)$$

In view of (3.1), we can choose $\varepsilon > 0$ small enough such that

$$\begin{aligned} \lambda_i^\varepsilon &= \max \left\{ \left| 1 - \left(c_i^u + \frac{1}{a_i^l} \right) r_{1i}^u(M_i + \varepsilon) \right|, \left| 1 - \left(c_i^l + \frac{1}{a_i^u} \right) r_{1i}^l(m_i - \varepsilon) \right| \right\} \\ &\quad + \frac{r_{1i}^u(M_i + \varepsilon)}{(a_i^l)^2} \sum_{l=1, l \neq i}^n b_{il}^u(M_l + \varepsilon) + \sum_{l=1}^m d_{il}^u(Q_l + \varepsilon) < 1, \\ \delta_j^\varepsilon &= \max \left\{ \left| 1 - p_{jj}^u(Q_j + \varepsilon) \right|, \left| 1 - p_{jj}^l(q_j - \varepsilon) \right| \right\} + \sum_{l=1}^n e_{jl}^u(M_l + \varepsilon) \\ &\quad + \sum_{l=1}^n p_{jl}^u(Q_l + \varepsilon) < 1. \end{aligned} \quad (3.7)$$

For the above $\varepsilon > 0$, according to Theorem 2.4 in Section 2, there exists a $k^* \in N$ such that

$$\begin{aligned} m_i - \varepsilon &\leq x_i(k), \quad \tilde{x}_i(k) \leq M_i + \varepsilon, \\ q_j - \varepsilon &\leq y_j(k), \quad \tilde{y}_j(k) \leq Q_j + \varepsilon, \end{aligned} \quad (3.8)$$

for all $k \geq k^*$.

Noticing that $\theta_l(k), \xi_l(k) \in [0, 1]$ implies that $\tilde{x}_l(k) \exp\{\theta_l(k)u_l(k)\}$ lies between $\tilde{x}_l(k)$ and $x_l(k)$, $\tilde{y}_l(k) \exp\{\xi_l(k)v_l(k)\}$ lies between $\tilde{y}_l(k)$ and $y_l(k)$. From (3.5), we get

$$\begin{aligned} |u_i(k+1)| &\leq \max \left\{ \left| 1 - \left(c_i^u + \frac{1}{a_i^l} \right) r_{1i}^u(M_i + \varepsilon) \right|, \left| 1 - \left(c_i^l + \frac{1}{a_i^u} \right) r_{1i}^l(m_i - \varepsilon) \right| \right\} |u_i(k)| \\ &\quad + \frac{r_{1i}^u(M_i + \varepsilon)}{(a_i^l)^2} \sum_{l=1, l \neq i}^n b_{il}^u(M_l + \varepsilon) |u_l(k)| + \sum_{l=1}^m d_{il}^u(Q_l + \varepsilon) |v_l(k)|, \\ |v_j(k+1)| &\leq \max \left\{ \left| 1 - p_{jj}^u(Q_j + \varepsilon) \right|, \left| 1 - p_{jj}^l(q_j - \varepsilon) \right| \right\} |v_j(k)| \\ &\quad + \sum_{l=1}^n e_{jl}^u(M_l + \varepsilon) |u_l(k)| + \sum_{l=1}^n p_{jl}^u(Q_l + \varepsilon) |v_l(k)|. \end{aligned} \quad (3.9)$$

Let $\gamma = \max\{\lambda_i^\varepsilon, \delta_j^\varepsilon\}$, then $\gamma < 1$. In view of (3.9), for $k \geq k^*$, we get

$$\max\{|u_i(k+1)|, |v_j(k+1)|\} \leq \gamma \max\{|u_i(k)|, |v_j(k)|\}. \quad (3.10)$$

This implies

$$\max\{|u_i(k)|, |v_j(k)|\} \leq \gamma^{k-k^*} \max\{|u_i(k^*)|, |v_j(k^*)|\}. \quad (3.11)$$

From (3.11), we have

$$\lim_{k \rightarrow +\infty} u_i(k) = 0, \quad \lim_{k \rightarrow +\infty} v_j(k) = 0. \quad (3.12)$$

This ends the proof of Theorem 3.1. \square

4. Existence and Stability of Periodic Solution

In this section, we further assume that the coefficients of the system (1.1) satisfies (4.1).

There exists a positive integer ω such that for $k \in N$,

$$\begin{aligned} 0 < r_{1i}(k + \omega) &= r_{1i}(k), & 0 < b_{il}(k + \omega) &= b_{il}(k), \\ 0 < a_i(k + \omega) &= a_i(k), & 0 < c_i(k + \omega) &= c_i(k), \\ 0 < r_{2j}(k + \omega) &= r_{2j}(k), & 0 < d_{il}(k + \omega) &= d_{il}(k), \\ 0 < e_{jl}(k + \omega) &= e_{jl}(k), & 0 < p_{jl}(k + \omega) &= p_{jl}(k). \end{aligned} \quad (4.1)$$

Our first result concerned with the existence of a positive periodic solution of system (1.1).

Theorem 4.1. *Assume that (2.5) and (2.14) hold, then system (1.1) admits at least one positive ω -periodic solution which ones denotes by $(\tilde{x}_1(k), \dots, \tilde{x}_n(k), \tilde{y}_1(k), \dots, \tilde{y}_m(k))$.*

Proof. As noted at the end of Section 2,

$$D^{n+m} := [m_1, M_1] \times \dots \times [m_n, M_n] \times [q_1, Q_1] \times \dots \times [q_m, Q_m] \quad (4.2)$$

is an invariant set of system (1.1). Thus, we can define a mapping F on D^{n+m} by

$$F(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0)) = (x_1(\omega), \dots, x_n(\omega), y_1(\omega), \dots, y_m(\omega)), \quad (4.3)$$

for $(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0)) \in D^{n+m}$. Obviously, F depends continuously on $(x_1(0), \dots, x_n(0), y_1(0), \dots, y_m(0))$. Thus, F is continuous and maps the compact set D^{n+m} into itself. Therefore, F has a fixed point. It is easy to see that the solution $(\tilde{x}_1(k), \dots, \tilde{x}_n(k), \tilde{y}_1(k), \dots, \tilde{y}_m(k))$ passing through this fixed point is an ω -periodic solution of the system (1.1). This completes the proof of Theorem 4.1. \square

Theorem 4.2. *Assume that (2.5), (2.14), and (3.1) hold, then system (1.1) has a global stable positive ω -periodic solution.*

Proof. Under the assumption of Theorem 4.2, it follows from Theorem 4.1 that system (1.1) admits at least one positive ω -periodic solution. Also, Theorem 3.1 ensures the positive solution to be globally stable. This ends the proof of Theorem 4.2. \square

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