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Research Article

On p-Adic Analogue of q-Bernstein Polynomials and Related Integrals

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Recently, Kim's work (in press) introduced q-Bernstein polynomials which are different Phillips' q-Bernstein polynomials introduced in the work by (Phillips, 1996; 1997). The purpose of this paper is to study some properties of several type Kim's q-Bernstein polynomials to express the p-adic q-integral of these polynomials on $\mathbb{Z}_p$ associated with Carlitz's q-Bernoulli numbers and polynomials. Finally, we also derive some relations on the p-adic q-integral of the products of several type Kim's q-Bernstein polynomials and the powers of them on $\mathbb{Z}_p$.

1. Introduction

Let $C[0,1]$ denote the set of continuous functions on $[0,1]$. For $0 < q < 1$ and $f \in C[0,1]$, Kim introduced the q-extension of Bernstein linear operator of order n for f as follows:

$$\mathbb{B}_{n,q}(f \mid x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}[x]_q^k[1-x]_{1/q}^{n-k} = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)B_{k,n}(x,q), \tag{1.1}$$

where $[x]_q = (1-q^x)/(1-q)$ (see [1]). Here $\mathbb{B}_{n,q}(f \mid x)$ is called Kim's q-Bernstein operator of order n for f. For $k, n \in \mathbb{Z}_+(=\mathbb{N}\cup\{0\})$, $B_{k,n}(x,q) = \binom{n}{k}[x]_q^k[1-x]_{1/q}^{n-k}$ are called the Kim's q-Bernstein polynomials of degree n (see [2–6]).

In [7], Carlitz defined a set of numbers $\xi_k = \xi_k(q)$ inductively by

$$\xi_0 = 1, \quad (q\xi + 1)^k - \xi_k = \begin{cases} 1 & \text{if } k = 1, \\ 0 & \text{if } k > 1, \end{cases} \tag{1.2}$$

with the usual convention of replacing ξ^k by ξ_k. These numbers are q-analogues of ordinary Bernoulli numbers B_k, but they do not remain finite for $q = 1$. So he modified the definition as follows:

$$\beta_{0,q} = 1, \quad q(q\beta + 1)^k - \beta_{k,q} = \begin{cases} 1 & \text{if } k = 1, \\ 0 & \text{if } k > 1, \end{cases} \tag{1.3}$$

with the usual convention of replacing β^k by $\beta_{k,q}$ (see [7]). These numbers $\beta_{n,q}$ are called the nth Carlitz q-Bernoulli numbers. And Carlitz's q-Bernoulli polynomials are defined by

$$\beta_{k,q}(x) = \left(q^x\beta + [x]_q\right)^k = \sum_{i=0}^{k} \binom{k}{i} \beta_{i,q} q^{ix} [x]_q^{k-i}. \tag{1.4}$$

As $q \to 1$, we have $\beta_{k,q} \to B_k$ and $\beta_{k,q}(x) \to B_k(x)$, where B_k and $B_k(x)$ are the ordinary Bernoulli numbers and polynomials, respectively.

Let p be a fixed prime number. Throughout this paper, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ will denote the ring of rational integers, the field of rational numbers, the ring of p-adic integers, the field of p-adic rational numbers and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ such that $|p|_p = p^{-\nu_p(p)} = 1/p$.

Let q be regarded as either a complex number $q \in \mathbb{C}$ or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, we assume $|q| < 1$, and if $q \in \mathbb{C}_p$, we normally assume $|1 - q|_p < 1$.

We say that f is a uniformly differentiable function at a point $a \in \mathbb{Z}_p$ and denote this property by $f \in \mathrm{UD}(\mathbb{Z}_p)$ if the difference quotient $F_f(x, y) = (f(x) - f(y))/(x - y)$ has a limit $f'(a)$ as $(x, y) \to (a, a)$ (see [1, 3, 8–13]).

For $f \in \mathrm{UD}(\mathbb{Z}_p)$, let us begin with the expression

$$\frac{1}{[p^N]_q} \sum_{0 \le x < p^N} q^x f(x) = \sum_{0 \le x < p^N} f(x)\mu_q\left(x + p^N\mathbb{Z}_p\right), \tag{1.5}$$

representing a q-analogue of the Riemann sums for f (see [11]). The integral of f on $\mathbb{Z}_p$ is defined as the limit as $n \to \infty$ of the sums (if exists). The p-adic q-integral on a function $f \in \mathrm{UD}(\mathbb{Z}_p)$ is defined by

$$I_q(f) = \int_{\mathbb{Z}_p} f(x) d\mu_q(x) = \lim_{N \to \infty} \frac{1}{[p^N]_q} \sum_{x=0}^{p^N - 1} f(x) q^x, \tag{1.6}$$

(see [11]).

As was shown in [3], Carlitz's q-Bernoulli numbers can be represented by p-adic q-integral on $\mathbb{Z}_p$ as follows:

$$\int_{\mathbb{Z}_p} [x]_q^m d\mu_q(x) = \beta_{m,q}, \quad \text{for } m \in \mathbb{Z}_+. \tag{1.7}$$

Also, Carlitz's q-Bernoulli polynomials $\beta_{k,q}(x)$ can be represented

$$\beta_{m,q}(x) = \int_{\mathbb{Z}_p} [x+y]_q^m d\mu_q(y), \quad \text{for } m \in \mathbb{Z}_+, \tag{1.8}$$

(see [3]).

In this paper, we consider the p-adic analogue of Kim's q-Bernstein polynomials on $\mathbb{Z}_p$ and give some properties of the several type Kim's q-Bernstein polynomials to represent the p-adic q-integral on $\mathbb{Z}_p$ of these polynomials. Finally, we derive some relations on the p-adic q-integral of the products of several type Kim's q-Bernstein polynomials and the powers of them on $\mathbb{Z}_p$.

2. q-Bernstein Polynomials Associated with p-Adic q-Integral on $\mathbb{Z}_p$

In this section, we assume that $q \in \mathbb{C}_p$ with $|1-q|_p < 1$.

From (1.5), (1.7) and (1.8), we note that

$$\begin{aligned}\int_{\mathbb{Z}_p} [1-x+x_1]_{1/q}^n d\mu_{1/q}(x_1) &= \frac{q^n}{(q-1)^{n-1}} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \frac{l+1}{q^{l+1}-1}, \\ \int_{\mathbb{Z}_p} [x+x_1]_q^n d\mu_q(x_1) &= \frac{1}{(1-q)^{n-1}} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \frac{l+1}{1-q^{l+1}}.\end{aligned} \tag{2.1}$$

By (2.1), we get

$$(-1)^n q^n \int_{\mathbb{Z}_p} [x+x_1]_q^n d\mu_q(x_1) = \int_{\mathbb{Z}_p} [1-x+x_1]_{1/q}^n d\mu_{1/q}(x_1). \tag{2.2}$$

Therefore, we obtain the following theorem.

Theorem 2.1. *For $n \in \mathbb{Z}_+$, one has*

$$\int_{\mathbb{Z}_p} [1-x+x_1]_{1/q}^n d\mu_{1/q}(x_1) = (-1)^n q^n \int_{\mathbb{Z}_p} [x+x_1]_q^n d\mu_q(x_1). \tag{2.3}$$

By the definition of Carlitz's q-Bernoulli numbers and polynomials, we get

$$q^2\beta_{n,q}(2) - (n+1)q^2 + q = q\left(q\beta+1\right)^n = \beta_{n,q} \quad \text{if } n > 1. \tag{2.4}$$

Thus, we have the following proposition.

Proposition 2.2. *For $n \in \mathbb{N}$ with $n > 1$, one has*

$$\beta_{n,q}(2) = \frac{1}{q^2}\beta_{n,q} + n + 1 - \frac{1}{q}. \tag{2.5}$$

It is easy to show that

$$[1-x]_{1/q}^n = \left(1 - [x]_q\right)^n = (-1)^n q^n [x-1]_q^n. \tag{2.6}$$

Hence, we have

$$\int_{\mathbb{Z}_p} [1-x]_{1/q}^n d\mu_q(x) = (-1)^n q^n \int_{\mathbb{Z}_p} [x-1]_q^n d\mu_q(x). \tag{2.7}$$

By (1.8), we get

$$\int_{\mathbb{Z}_p} [1-x]_{1/q}^n d\mu_q(x) = (-1)^n q^n \beta_{n,q}(-1). \tag{2.8}$$

By Theorem 2.1 and (2.8), we see that

$$\int_{\mathbb{Z}_p} [1-x]_{1/q}^n d\mu_q(x) = (-1)^n q^n \beta_{n,q}(-1) = \beta_{n,1/q}(2). \tag{2.9}$$

From (2.9) and Proposition 2.2, we have

$$\int_{\mathbb{Z}_p} [1-x]_{1/q}^n d\mu_q(x) = \beta_{n,1/q}(2) = q^2\beta_{n,1/q} + n + 1 - q. \tag{2.10}$$

By (1.7) and (2.10), we obtain the following theorem.

Theorem 2.3. *For $n \in \mathbb{N}$ with $n > 1$, one has*

$$\int_{\mathbb{Z}_p} [1-x]_{1/q}^n d\mu_q(x) = q^2 \int_{\mathbb{Z}_p} [x]_{1/q}^n d\mu_{1/q}(x) + n + 1 - q. \tag{2.11}$$

Taking the p-adic q-integral on $\mathbb{Z}_p$ for one Kim's q-Bernstein polynomials, we get

$$\begin{aligned}\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_q(x) &= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_q^k[1-x]_{1/q}^{n-k}\,d\mu_q(x)\\ &= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{k+l}\,d\mu_q(x)\\ &= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\beta_{k+l,q},\end{aligned} \tag{2.12}$$

and, by the q-symmetric property of $B_{k,n}(x,q)$, we see that

$$\begin{aligned}\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_q(x) &= \int_{\mathbb{Z}_p} B_{n-k,n}\left(1-x,\frac{1}{q}\right)d\mu_q(x)\\ &= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\int_{\mathbb{Z}_p}[1-x]_{1/q}^{n-l}\,d\mu_q(x).\end{aligned} \tag{2.13}$$

For $n > k+1$, by Theorem 2.3 and (2.13), one has

$$\begin{aligned}\int_{\mathbb{Z}_p} B_{k,n}(x,q)\,d\mu_q(x) &= \binom{n}{k}\sum_{l=0}^{k}(-1)^{k+l}\binom{k}{l}\left[n-l+1-q+q^2\int_{\mathbb{Z}_p}[x]_{1/q}^{n-l}\,d\mu_{1/q}(x)\right]\\ &= \binom{n}{k}\sum_{l=0}^{k}(-1)^{k+l}\binom{k}{l}\left[n-l+1-q+q^2\beta_{n-l,1/q}\right].\end{aligned} \tag{2.14}$$

Let $m,n,k \in \mathbb{Z}_+$ with $m+n > 2k+1$. Then the p-adic q-integral for the multiplication of two Kim's q-Bernstein polynomials on $\mathbb{Z}_p$ can be given by the following relation:

$$\begin{aligned}\int_{\mathbb{Z}_p} B_{k,n}(x,q)B_{k,m}(x,q)\,d\mu_q(x) &= \binom{n}{k}\binom{m}{k}\int_{\mathbb{Z}_p}[x]_q^{2k}[1-x]_{1/q}^{n+m-2k}\,d\mu_q(x)\\ &= \binom{n}{k}\binom{m}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{l+2k}q\int_{\mathbb{Z}_p}[1-x]_{1/q}^{n+m-l}\,d\mu_q(x).\end{aligned} \tag{2.15}$$

By Theorem 2.3 and (2.15), we get

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n}(x,q)B_{k,m}(x,q)\,d\mu_q(x)\\
&\quad=\binom{n}{k}\binom{m}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{l+2k}\left[n+m-l+1-q+q^2\int_{\mathbb{Z}_p}[x]_{1/q}^{n+m-l}\,d\mu_{1/q}(x)\right]\\
&\quad=\binom{n}{k}\binom{m}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{l+2k}\left[n+m-l+1-q+q^2\beta_{n+m-l,1/q}\right].
\end{aligned}\tag{2.16}$$

By the simple calculation, we easily get

$$\begin{aligned}
\int_{\mathbb{Z}_p} B_{k,n}(x,q)B_{k,m}(x,q)\,d\mu_q(x)&=\binom{n}{k}\binom{m}{k}\int_{\mathbb{Z}_p}[x]_q^{2k}[1-x]_{1/q}^{n+m-2k}\,d\mu_q(x)\\
&=\binom{n}{k}\binom{m}{k}\sum_{l=0}^{n+m-2k}\binom{n+m-2k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{l+2k}\,d\mu_q(x)\\
&=\binom{n}{k}\binom{m}{k}\sum_{l=0}^{n+m-2k}\binom{n+m-2k}{l}(-1)^l\beta_{l+2k,q}.
\end{aligned}\tag{2.17}$$

Continuing this process, we obtain

$$\begin{aligned}
\int_{\mathbb{Z}_p}\left(\prod_{i=1}^{s}B_{k,n_i}(x,q)\right)d\mu_q(x)&=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\int_{\mathbb{Z}_p}[x]_q^{sk}[1-x]_{1/q}^{n_1+\cdots+n_s-sk}\,d\mu_q(x)\\
&=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{sk}\binom{sk}{l}(-1)^{sk+l}\int_{\mathbb{Z}_p}[1-x]_{1/q}^{n_1+\cdots+n_s-l}\,d\mu_q(x).
\end{aligned}\tag{2.18}$$

Let $s\in\mathbb{N}$ and $n_1,\ldots,n_s, k\in\mathbb{Z}_+$ with $n_1+n_2+\cdots+n_s>sk+1$. By Theorem 2.3 and (2.18), we get

$$\begin{aligned}
&\int_{\mathbb{Z}_p}\left(\prod_{i=1}^{s}B_{k,n_i}(x,q)\right)d\mu_q(x)\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{sk}\binom{sk}{l}(-1)^{sk+l}\left\{\sum_{i=1}^{s}n_i-l+1-q+q^2\int_{\mathbb{Z}_p}[x]_{1/q}^{n_1+\cdots+n_s-l}\,d\mu_{1/q}(x)\right\}\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{sk}\binom{sk}{l}(-1)^{sk+l}\left\{\sum_{i=1}^{s}n_i-l+1-q+q^2\beta_{n_1+\cdots+n_s-l,1/q}\right\}.
\end{aligned}\tag{2.19}$$

From the definition of binomial coefficient, we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p}\left(\prod_{i=1}^{s}B_{k,n_i}(x,q)\right)d\mu_q(x)\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\int_{\mathbb{Z}_p}[x]_q^{sk}[1-x]_{1/q}^{n_1+\cdots+n_s-sk}d\mu_q(x)\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{n_1+\cdots+n_s-sk}\binom{n_1+\cdots+n_s-sk}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{sk+l}d\mu_q(x)\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{n_1+\cdots+n_s-sk}\binom{n_1+\cdots+n_s-sk}{l}(-1)^l\beta_{sk+l,q},
\end{aligned}\tag{2.20}$$

where $s\in\mathbb{N}$ and $n_1,\ldots,n_s,k\in\mathbb{Z}_+$.

By (2.19) and (2.20), we obtain the following theorem.

Theorem 2.4. (I) *For $s\in\mathbb{N}$ and $n_1,\ldots,n_s,k\in\mathbb{N}$ with $n_1+n_2+\cdots+n_s>sk+1$, one has*

$$\begin{aligned}
&\int_{\mathbb{Z}_p}\left(\prod_{i=1}^{s}B_{k,n_i}(x,q)\right)d\mu_q(x)\\
&\quad=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{sk}\binom{sk}{l}(-1)^{sk+l}\left\{\sum_{i=1}^{s}n_i-l+1-q+q^2\beta_{n_1+\cdots+n_s-l,1/q}\right\}.
\end{aligned}\tag{2.21}$$

(II) *For $s\in\mathbb{N}$ and $n_1,\ldots,n_s,k\in\mathbb{Z}_+$, one has*

$$\int_{\mathbb{Z}_p}\left(\prod_{i=1}^{s}B_{k,n_i}(x,q)\right)d\mu_q(x)=\left(\prod_{i=1}^{s}\binom{n_i}{k}\right)\sum_{l=0}^{n_1+\cdots+n_s-sk}\binom{n_1+\cdots+n_s-sk}{l}(-1)^l\beta_{sk+l,q}.\tag{2.22}$$

By Theorem 2.4, we obtain the following corollary.

Corollary 2.5. *For $s\in\mathbb{N}$ and $n_1,\ldots,n_s,k\in\mathbb{N}$ with $n_1+n_2+\cdots+n_s>sk+1$, one has*

$$\begin{aligned}
&\sum_{l=0}^{sk}\binom{sk}{l}(-1)^{sk+l}\left\{\sum_{i=1}^{s}n_i-l+1-q+q^2\beta_{n_1+\cdots+n_s-l,1/q}\right\}\\
&\quad=\sum_{l=0}^{n_1+\cdots+n_s-sk}\binom{n_1+\cdots+n_s-sk}{l}(-1)^l\beta_{sk+l,q}.
\end{aligned}\tag{2.23}$$

Let $s \in \mathbb{N}$ and $m_1, \ldots, m_s, n_1, \ldots, n_s, k \in \mathbb{Z}_+$ with $m_1 n_1 + \cdots + m_s n_s > (m_1 + \cdots + m_s)\ k + 1$. Then one has

$$\begin{aligned}
\int_{\mathbb{Z}_p} \left(\prod_{i=1}^{s} B_{k,n_i}^{m_i}(x,q) \right) d\mu_q(x) &= \left(\prod_{i=1}^{s} \binom{n_i}{k}^{m_i} \right) \sum_{l=0}^{k\sum_{i=1}^{s} m_i} \binom{k\sum_{i=1}^{s} m_i}{l} (-1)^{k\sum_{i=1}^{s} m_i - l} \\
&\quad \times \int_{\mathbb{Z}_p} [1-x]_q^{\sum_{i=1}^{s} n_i m_i - l} d\mu_q(x) \\
&= \left(\prod_{i=1}^{s} \binom{n_i}{k}^{m_i} \right) \sum_{l=0}^{k\sum_{i=1}^{s} m_i} \binom{k\sum_{i=1}^{s} m_i}{l} (-1)^{k\sum_{i=1}^{s} m_i - l} \\
&\quad \times \left\{ \left(\sum_{i=1}^{s} m_i n_i - l + 1 \right) - q + q^2 \int_{\mathbb{Z}_p} [x]_{1/q}^{\sum_{i=1}^{s} n_i m_i - l} d\mu_{1/q}(x) \right\} \\
&= \left(\prod_{i=1}^{s} \binom{n_i}{k}^{m_i} \right) \sum_{l=0}^{k\sum_{i=1}^{s} m_i} \binom{k\sum_{i=1}^{s} m_i}{l} (-1)^{k\sum_{i=1}^{s} m_i - l} \\
&\quad \times \left\{ \left(\sum_{i=1}^{s} m_i n_i - l + 1 \right) - q + q^2 \beta_{n_1 m_1 + \cdots + n_s m_s - l, 1/q} \right\}.
\end{aligned} \tag{2.24}$$

From the definition of binomial coefficient, one has

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \left(\prod_{i=1}^{s} B_{k,n_i}^{m_i}(x,q) \right) d\mu_q(x) \\
&\quad = \left(\prod_{i=1}^{s} \binom{n_i}{k}^{m_i} \right) \sum_{l=0}^{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i} \binom{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i}{l} (-1)^l \\
&\qquad \times \int_{\mathbb{Z}_p} [x]_q^{(m_1 + \cdots + m_s)k + l} d\mu_q(x) \\
&\quad = \left(\prod_{i=1}^{s} \binom{n_i}{k}^{m_i} \right) \sum_{l=0}^{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i} \binom{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i}{l} \\
&\qquad \times (-1)^l \beta_{(m_1 + \cdots + m_s)k + l, q}.
\end{aligned} \tag{2.25}$$

By (2.24) and (2.25), we obtain the following theorem.

Theorem 2.6. *For* $s \in \mathbb{N}$ *and* $m_1, \ldots, m_s, n_1, \ldots, n_s, k \in \mathbb{Z}_+$ *with* $m_1 n_1 + \cdots + m_s n_s > (m_1 + \cdots + m_s)k + 1$, *one has*

$$\sum_{l=0}^{k\sum_{i=1}^{s} m_i} \binom{k\sum_{i=1}^{s} m_i}{l} (-1)^{k\sum_{i=1}^{s} m_i - l} \left\{ \left(\sum_{i=1}^{s} m_i n_i - l + 1 \right) - q + q^2 \beta_{n_1 m_1 + \cdots + n_s m_s - l, 1/q} \right\}$$
$$= \sum_{l=0}^{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i} \binom{\sum_{i=1}^{s} n_i m_i - k\sum_{i=1}^{s} m_i}{l} (-1)^l \beta_{(m_1 + \cdots + m_s)k + l, q}. \tag{2.26}$$

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