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Research Article

Novel Criteria on Global Robust Exponential Stability to a Class of Reaction-Diffusion Neural Networks with Delays

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The global exponential robust stability is investigated to a class of reaction-diffusion Cohen-Grossberg neural network (CGNNs) with constant time-delays, this neural network contains time invariant uncertain parameters whose values are unknown but bounded in given compact sets. By employing the Lyapunov-functional method, several new sufficient conditions are obtained to ensure the global exponential robust stability of equilibrium point for the reaction diffusion CGNN with delays. These sufficient conditions depend on the reaction-diffusion terms, which is a preeminent feature that distinguishes the present research from the previous research on delayed neural networks with reaction-diffusion. Two examples are given to show the effectiveness of the obtained results.

1. Introduction

In recent years, considerable attention has been paid to study the dynamics of artificial neural networks with fixed parameters because of their potential applications in the areas such as signal and image processing, pattern recognition, parallel computations, and optimization problems [1–10]. However, during the implementation on very scale integration chips, the stability of a well-designed system may often be destroyed by its unavoidable uncertainty due to the existence of modelling error, external disturbance, and parameter fluctuation. In general, on other hand, a mathematical description is only an approximation of the actual physical system and deals with fixed nominal parameters. Usually, these parameters are not known exactly due to the imperfect identification or measurement, aging of components and/or changes in the environmental condition. Thus, it is almost impossible to get an exact model for the system due to the existence of various parameter uncertainties. So it is essential to introduce the robust technique to design a system with such uncertainty [11, 12]. If the uncertainty of a system is only due to the deviations and perturbations of its parameters, and if those deviations and perturbations are all bounded, then the system is called an interval system [13–23]. Recently, Chen and Rong [23] considered a class of Cohen-Grossberg neural networks (CGNNs) with time-varying delays. Several sufficient conditions were given to ensure global exponential robust stability.

In a real world, strictly speaking, the diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a nonuniform electromagnetic field. Hence, it is essential to consider the state variables varying with the time and space variables. The neural networks with diffusion terms can commonly be expressed by partial differential equations. Recently, some authors have devoted to the study of reaction-diffusion neural networks, for instance see [24–29] and references therein. In particular, more recently, Liu et al. [27] and Wang et al. [28], considered the global exponential robust stability of a class of reaction-diffusion Hopfield neural networks with distributed delays and time-varying delay, respectively. Song and Cao [29] have obtained the criteria to guarantee the global exponential robust stability of a class of reaction-diffusion CGNNs with time-varying delays and Neumann boundary condition. In [27–29], unfortunately, owing to the divergence theorem employed, a negative integral term with gradient is left out in their deduction. As a result, the global exponential robust stability criteria acquired by them do not contain a diffusion term. In other words, the diffusion term does not take effect in their deduction and sufficient conditions. The same case appears also in the other literatures [24–26].

Motivated by the above discussions, in this paper we will consider a class of reaction-diffusion CGNNs with constant time delays and a boundary condition. We will construct a appropriate Lyapunov functional to derive some new criteria ensuring the global exponential robust stability for an equilibrium point of the delayed reaction-diffusion CGGNs with the boundary condition. The present work differs from the paper [27–29] since (i) the diffusion terms play an important role in the global exponential robust stability criteria in the paper, (ii) the boundary condition of CGNNs model considered includes the Neumann type and the Dirichlet type while the boundary condition of model in [27, 28] is the Neumann type. The work will have significance impact on the design and applications of globally exponentially robustly stable reaction-diffusion neural network with delays and is of great interest in many applications.

The rest of this paper is organized as follows. In Section 2, model description and preliminaries are given. In Section 3, several criteria are derived for the global exponential robust stability for an equilibrium point of reaction-diffusion CGNNs with delays and the boundary condition. Then, we give two examples and comparison to illustrate our criteria in Section 4. Finally, in Section 5, some conclusions are made.

2. Model Description and Preliminaries

To begin with, we introduce some notations.

(i) Ω is an open bounded domain in $\mathbb{R}^m$ with smooth boundary $\partial\Omega$, and $\text{mes}\Omega > 0$ as $\text{mes}\Omega$ denotes the measure of Ω. $\overline{\Omega} = \Omega \cup \partial\Omega$.

(ii) $L^2(\Omega)$ is the space of real Lebesgue measurable functions on Ω which is a Banach space. Define the inner product $\langle u, v\rangle = \int_\Omega uv dx$, for any $u, v \in L^2(\Omega)$ and the L^2-norm $\|u\|_2 := \langle u, u\rangle^{1/2}$, for $u \in L^2(\Omega)$.

(iii) $H^1(\Omega) := \{w \in L^2(\Omega), D_i w \in L^2(\Omega)\}$, where $D_i w = \partial w/\partial x_i$, $1 \le i \le m$. $H_0^1(\Omega) :=$ the closure of $C_0^\infty(\Omega)$ in $H^1(\Omega)$.

(iv) Let $\mathcal{C} := \mathcal{C}([0,+\infty)\times\Omega,\mathbb{R}^n)$ be the Banach space of continuous functions which map $[0,+\infty)\times\Omega$ into $\mathbb{R}^n$ with the norm $\|u\|_2 = (\sum_{i=1}^n \|u_i\|_2^2)^{1/2}$ for $u = (u_1,\ldots,u_n)^T \in \mathcal{C}$, where $\|u_i\|_2 = (\int_\Omega |u_i|^2 dx)^{1/2}$, $i = 1,\ldots,n$.

(v) Let $\mathcal{C}_1 := \mathcal{C}([-\tau,0]\times\overline{\Omega},\mathbb{R}^n)$ be the Banach space of bounded continuous functions which map $[-\tau,0]\times\overline{\Omega}$ into $\mathbb{R}^n$ with the following norm: $\|\phi\|_\tau := \sup_{s\in[-\tau,\,0]}\|\phi(s)\|_2$, for any $\phi(s) = (\phi_1(s),\ldots,\phi_n(s))^T \in \mathcal{C}_1$, where $\|\phi(s)\|_{\mathcal{C}_1} = (\sum_{i=1}^n \int_\Omega |\phi_i(s)|^2 dx)^{1/2}$.

Consider the following reaction-diffusion CGNNs with interval coefficients and delays on Ω:

$$\frac{\partial u_i(t,x)}{\partial t} = \sum_{l=1}^m \frac{\partial}{\partial x_l}\left(d_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right) - a_i(u_i(t,x)) \times\left[b_i(u_i(t,x)) - \sum_{j=1}^n s_{ij} f_j(u_j(t,x)) - \sum_{j=1}^n t_{ij} g_j(g_j(t-\tau_{ij},x)) + I_i\right], \quad (t,x)\in[0,+\infty)\times\Omega, \tag{2.1}$$

for $i = 1,\ldots,n$, $x = (x_1,\ldots,x_m)^T \in \Omega$ is space variable, $u_i(t,x)$ corresponds to the state of the ith unit at time t and in space x; $d_{il} > 0$, for $i = 1,\ldots,n$, $l = 1,\ldots,m$, corresponds to the transmission diffusion coefficient along the ith neuron, $d_i = \min_{1\le l\le m}\{d_{il}\}$ for $i = 1,\ldots,n$; $a_i(u_i(t,x))$ represents an amplification function; $b_i(u_i(t,x))$ is an appropriate behavior function; s_{ij}, t_{ij} denote the connection strengths of the jth neuron on the ith neuron, respectively; $g_j(u_j(t,x))$, $f_j(u_j(t,x))$ denote the activation functions of jth neuron at time t and in space x; $\tau_{ij}(0 < \tau_{ij} \le \tau)$ corresponds to the transmission delay along the axon of the jth unit from the ith unit. I_i is the constant input from outside of the network.

Throughout this paper, we assume the following.

(H1) Each function $a_i(\xi)$ is positive, continuous, and bounded, that is, there exist constants $\underline{a}_i$, $\overline{a}_i$ such that $0 < \underline{a}_i \le a_i(\xi) \le \overline{a}_i < \infty$, for $\xi \in \mathbb{R}$, $i = 1,\ldots,n$.

(H2) Each function $b_i(\xi) \in \mathcal{C}^1(\mathbb{R},\mathbb{R})$ and $b_i'(\xi) \ge b_i \ge 0$ is locally Lipschitz continuous.

(H3) The activation functions $f_j(\xi)$ and $g_j(\xi)$ satisfy Lipschitz condition, that is, there exist two positive diagonal matrices $F = \operatorname{diag}(F_1,\ldots,F_n)$ and $G = \operatorname{diag}(G_1,\ldots,G_n)$ such that

$$\left|f_j(\xi_1) - f_j(\xi_2)\right| \le F_j|\xi_1 - \xi_2|, \qquad \left|g_j(\xi_1) - g_j(\xi_2)\right| \le G_j|\xi_1 - \xi_2|, \tag{2.2}$$

for all $\xi_1,\xi_2 \in R(\xi_1 \ne \xi_2)$, $j = 1,\ldots,n$.

Remark 2.1. The activation functions f_j and g_j, $j = 1,\ldots,n$, are typically assumed to be sigmoidal which implies that they are monotone, bounded, and smooth. However, in this paper, we only need the previous weaker assumptions.

We assume that the nonlinear delayed systems (2.1) are supplemented with the boundary condition:

$$B[u_i(t,x)] = 0, \quad \text{for } (t,x) \in [-\tau, +\infty) \times \partial\Omega, \quad i = 1, \dots, n, \tag{2.3}$$

where $B[u_i(t,x)] = u_i(t,x)$ is said the Dirichlet boundary condition, $B[u_i(t,x)] = \partial u_i(t,x)/\partial m$ is said the Neumann boundary condition, where $\partial u_i(t,x)/\partial m = (\partial u_i(t,x)/\partial x_1, \dots, \partial u_i(t,x)/\partial x_m)^T$ denotes the outward normal derivative on $\partial\Omega$.

Systems (2.1) are equipped with the initial condition:

$$u_i(s,x) = \phi_i(s,x), \quad \text{for } (s,x) \in [-\tau, 0] \times \overline{\Omega},\ i = 1, \dots, n, \tag{2.4}$$

where $\phi := (\phi_1, \dots, \phi_n)^T \in \mathcal{C}$.

Given boundary condition (2.3) and initial function (2.4), the existence on the solutions of systems (2.1), the reader can refer to [18]. We denote the solution by $u(t,\phi,x) := (u_1(t,\phi,x), \dots, u_n(t,\phi,x))^T$ and sometimes it is denoted by $u(t,x)$, $u(t)$ or u for short when there is no risk of confusion.

Lemma 2.2. *Under assumptions (H1)–(H3), system* (2.1) *has a unique equilibrium point, if*

(H4) $b_i > \Sigma_{j=1}^{n}(s_{ij}^* F_j + t_{ij}^* G_j)$, *for* $i = 1, \dots, n$.

As for the proof of Lemma 2.2, the reader can refer to [21, 28]. Here, we omit it.

Definition 2.3. An equilibrium point u^* of system (2.1)–(2.4) is said to be globally exponentially stable on L^2-norm, if there exist constant $\eta > 0$ and $M \geq 1$ such that

$$\|u(t,x) - u^*\|_2 \leq M \|\phi - u^*\|_\tau e^{-\eta t} \quad \forall t \geq 0, \tag{2.5}$$

where $\|\phi - u^*\|_\tau = \sup_{-\tau \leq s \leq 0} \|\phi(s,x) - u^*\|_2$.

Definition 2.4. Let $\underline{s}_{ij} \leq s_{ij} \leq \overline{s}_{ij}$, $s_{ij}^* = \max(|\underline{s}_{ij}|, |\overline{s}_{ij}|)$, $\underline{t}_{ij} \leq t_{ij} \leq \overline{t}_{ij}$, $t_{ij}^* = \max(|\underline{t}_{ij}|, |\overline{t}_{ij}|)$, $\underline{I}_i \leq I_i \leq \overline{I}_i$, $I_i^* = \max(|\underline{I}_i|, |\overline{I}_i|)$, $\tau_i = \max(|\overline{\tau}_{ij}|)$. An equilibrium point u^* of system (2.1)–(2.4) is said to be globally exponentially robustly stable if its equilibrium point u^* is globally exponentially stable for all $\underline{s}_{ij} \leq s_{ij} \leq \overline{s}_{ij}$, $\underline{t}_{ij} \leq t_{ij} \leq \overline{t}_{ij}$, $\underline{I}_i \leq I_i \leq \overline{I}_i$, $\underline{\tau}_{ij} \leq \tau_{ij} \leq \overline{\tau}_{ij}$, for $i, j = 1, \dots, n$.

Lemma 2.5 (Poincaré inequality [30–32]). *Let Ω be a bounded domain of $\mathbb{R}^m$ with a smooth boundary $\partial\Omega$ of class $\mathcal{C}^2$ by Ω. $v(x)$ is a real-valued function belonging to $H_0^1(\Omega)$ and $B[v(x)]|_{\partial\Omega} = 0$. Then*

$$\lambda_1 \int_\Omega |v(x)|^2 dx \leq \int_\Omega |\nabla v(x)|^2 dx, \tag{2.6}$$

which λ_1 *is the lowest positive eigenvalue of the Laplacian with boundary condition*

$$
\begin{aligned}
-\Delta\varphi(x) &= \lambda\varphi(x), \quad x \in \Omega, \\
B[\varphi(x)] &= 0, \quad x \in \partial\Omega.
\end{aligned}
\tag{2.7}
$$

Regarding the proof of Lemma 2.5, we refer to any textbook on partial differential equations. For example, [30, 31] or [32] are good standard references.

Remark 2.6. (i) When Ω is bounded or at least bounded in one direction, not only limited to a rectangle domain, inequality (2.6) holds. (ii) The lowest positive eigenvalue λ_1 of the Laplacian is sometimes known as the first eigenvalue. Determining the lowest eigenvalue λ_1 is, in general, a very hard task that depends upon the geometry of the domain Ω. Certain special cases are tractable, however. For example, let the Laplacian on $\Omega = \{(x_1, x_2)^T \in \mathbb{R}^2 \mid 0 < x_1 < a,\ 0 < x_2 < b\}$, if $B[v(x)] = v(x)$ or $B[v(x)] = \partial v(x)/\partial m$, then $\lambda_1 = (\pi/a)^2 + (\pi/b)^2$ or $\lambda_1 = \min\{(\pi/a)^2, (\pi/b)^2\}$, respectively. (iii) Although the eigenvalue λ_1 of the laplacian with the Dirichlet boundary condition on a generally bounded domain Ω cannot be determined exactly, a lower bound of it may nevertheless be estimated by $\lambda_1 \geq (m^2/(m+2))((2\pi)^2/\omega_{m-1})(1/V)^{2/m}$, where ω_{m-1} is a surface area of the unit ball in $\mathbb{R}^m$, V is a volume of domain Ω [33].

3. Main Results

Theorem 3.1. *Let hypotheses (H1)–(H4) hold. Assume further that*

(A1) $2(d_i\lambda_1 + \underline{a}_i b_i) > \overline{a}_i \sum_{j=1}^{n}(s_{ij}^* F_i + t_{ij}^* G_i) + \sum_{j=1}^{n} \overline{a}_j (s_{ji}^* F_j + t_{ji}^* G_j)$, *for* $i = 1, \ldots, n$, *then equilibrium point* u^* *of system* (2.1) *with* (2.3) *and* (2.4) *is globally exponentially robust stable for each constant input* $I \in \mathbb{R}^n$.

Proof. Let $y_i(t) = u_i(t) - u^*$. $y_i(t)$ is denoted by y_i for short. From (2.1), we obtain

$$
\frac{\partial y_i}{\partial t} = \sum_{l=1}^{m} \frac{\partial}{\partial x_l}\left(d_{il}\frac{\partial y_i}{\partial x_l}\right) - a_i(u_i)\left[\widetilde{b}_i(y_i) - \sum_{j=1}^{n} s_{ij}\widetilde{f}_j(y_j) - \sum_{j=1}^{n} t_{ij}\widetilde{g}_j(y_j(t-\tau_{ij}))\right],
\tag{3.1}
$$

for $(t,x) \in [0,+\infty) \times \Omega$, $i = 1, \ldots, n$, where

$$
\begin{aligned}
\widetilde{b}_i(y_i) = b_i(y_i + u_i^*) - b_i(u_i^*), \qquad \widetilde{f}_j(y_j) &= f_j\left(y_j + u_j^*\right) - f_j\left(u_j^*\right), \\
\widetilde{g}_j(y_j) &= g_j\left(y_j + u_j^*\right) - g_j\left(u_j^*\right),
\end{aligned}
\tag{3.2}
$$

for $i, j = 1, \ldots, n$.

Taking the inner product of both sides of (3.1) with y_i, we get

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|y_i\|_2^2 = \int_\Omega y_i \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(d_{il}\frac{\partial y_i}{\partial x_l}\right)dx + \int_\Omega y_i a_i(u_i)\sum_{j=1}^{n}s_{ij}\tilde{f}_j(y_j)dx \\ -\int_\Omega y_i a_i(u_i)\tilde{b}_i(y_i)dx + \int_\Omega y_i a_i(u_i)\sum_{j=1}^{n}t_{ij}\tilde{g}_j(y_j(t-\tau_{ij}))dx,\end{aligned} \tag{3.3}$$

for $t \in [0,+\infty)$, $i = 1,\ldots,n$.

From the boundary condition (2.3), Gauss formula and Lemma 2.2, we have

$$\begin{aligned}\int_\Omega y_i \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(d_{il}\frac{\partial y_i}{\partial x_l}\right)dx &= -\int_\Omega \sum_{l=1}^{m} d_{il}\left(\frac{\partial y_i}{\partial x_l}\right)^2 dx \\ &\le -d_i\int_\Omega \sum_{l=1}^{m}\left(\frac{\partial y_i}{\partial x_l}\right)^2 dx = -d_i\int_\Omega |\nabla y_i|^2 dx \\ &\le -\lambda_1 d_i \int_\Omega y_i^2 dx = -\lambda_1 d_i \|y_i\|_2^2.\end{aligned} \tag{3.4}$$

From assumption (H2), we get

$$\int_\Omega y_i a_i(u_i)\tilde{b}_i(y_i)dx \ge \int_\Omega \underline{a}_i b_i |y_i|^2 dx \ge \underline{a}_i b_i \|y_i\|_2^2. \tag{3.5}$$

From assumptions (H1) and (H3), we obtain

$$\int_\Omega y_i \overline{a}_i(u_i)\sum_{j=1}^{n}s_{ij}\tilde{f}_j(y_j)dx \le \sum_{j=1}^{n}\int_\Omega s_{ij}^{*}\overline{a}_i F_j |y_i||y_j|dx \le \sum_{j=1}^{n}s_{ij}^{*}\overline{a}_i F_j \|y_i\|_2\|y_j\|_2. \tag{3.6}$$

By the same way, we have

$$\int_\Omega y_i a_i(u_i)\sum_{j=1}^{n}t_{ij}\tilde{g}_j(y_j(t-\tau_{ij}))dx \le \sum_{j=1}^{n}t_{ij}^{*}\overline{a}_i G_j \|y_i\|_2 \|y_j(t-\tau_{ij})\|_2. \tag{3.7}$$

Combining (3.4)–(3.7) into (3.3), we obtain

$$\frac{d}{dt}\|y_i\|_2^2 \le -2(d_i\lambda_1 + \underline{a}_i b_i)\|y_i\|_2^2 + 2\sum_{j=1}^{n}s_{ij}^{*}\overline{a}_i F_j\|y_i\|_2\|y_j\|_2 + 2\sum_{j=1}^{n}t_{ij}^{*}\overline{a}_i G_j\|y_i\|_2\|y_j(t-\tau_{ij})\|_2, \tag{3.8}$$

for $t \in [0,+\infty)$.

According to (A1), we can choose a sufficiently small $\mu > 0$ such that

$$2\left(d_i\lambda_1 + \underline{a}_i b_i\right) - \mu - \sum_{j=1}^{n} \overline{a}_i \left(s_{ij}^{*} F_i + t_{ij}^{*} G_i\right) - \sum_{j=1}^{n} \overline{a}_j \left(s_{ji}^{*} F_j + t_{ji}^{*} G_j e^{\mu\tau}\right) > 0. \tag{3.9}$$

Now consider the Lyapunov functional $V(t)$ defined by

$$V(t) = \sum_{i=1}^{n} \left[\left\|y_i\right\|_2^2 e^{\mu t} + \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j \int_{t-\tau_{ij}}^{t} e^{\mu(s+\tau_{ij})} \left\|y_j(s)\right\|_2^2 ds \right]. \tag{3.10}$$

By calculating the upper right Dini derivative $D^{+}V(t)$ of $V(t)$ along the solutions of (3.1), we get

$$\begin{aligned}
D^{+}V(t) = e^{\mu t} \sum_{i=1}^{n} \Bigg\{ & \mu \left\|y_i\right\|_2^2 + \frac{d}{dt}\left\|y_i\right\|_2^2 + \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j e^{\mu\tau_{ij}} \left\|y_j(t)\right\|_2^2 \\
& - \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j \left\|y_j(t-\tau_{ij})\right\|_2^2 \Bigg\} \\
\leq e^{\mu t} \sum_{i=1}^{n} \Bigg\{ & \left[-2\left(d_i\lambda_1 + \underline{a}_i b_i\right) + \mu\right] \left\|y_i\right\|_2^2 \\
& + 2\sum_{j=1}^{n} s_{ij}^{*} \overline{a}_i F_j \left\|y_i\right\|_2 \left\|y_j\right\|_2 + 2\sum_{j=1}^{n} t_{ij}^{*} \overline{a}_i G_j \left\|y_i\right\|_2 \left\|y_j(t-\tau_{ij})\right\|_2^2 \\
& + \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j e^{\mu\tau_{ij}} \left\|y_j\right\|_2^2 - \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j \left\|y_j(t-\tau_{ij})\right\|_2^2 \Bigg\} \\
\leq e^{\mu t} \sum_{i=1}^{n} \Bigg\{ & \left[-2\left(d_i\lambda_1 + \underline{a}_i b_i\right) + \mu\right] \left\|y_i\right\|_2^2 \\
& + \sum_{j=1}^{n} s_{ij}^{*} \overline{a}_i F_j \left\|y_i\right\|_2^2 + \sum_{j=1}^{n} s_{ij}^{*} \overline{a}_i F_j \left\|y_j\right\|_2^2 \\
& + \sum_{j=1}^{n} t_{ij}^{*} \overline{a}_i G_j \left\|y_i\right\|_2^2 + \sum_{j=1}^{n} t_{ij}^{*} \overline{a}_i G_j \left\|y_j(t-\tau_{ij})\right\|_2 \\
& + \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j e^{\mu\tau_{ij}} \left\|y_j\right\|_2^2 - \overline{a}_i \sum_{j=1}^{n} t_{ij}^{*} G_j \left\|y_j(t-\tau_{ij})\right\|_2^2 \Bigg\}
\end{aligned}$$

$$\begin{aligned}
&\leq e^{\mu t}\sum_{i=1}^{n}\left\{\left[-2\left(d_i\lambda_1+\underline{a}_i b_i\right)+\mu+\overline{a}_i\left(\sum_{j=1}^{n}s_{ij}^{*}F_j+\sum_{j=1}^{n}t_{ij}^{*}G_j\right)\right]\left\|y_i\right\|_2^2\right.\\
&\qquad\left.+\left(\sum_{j=1}^{n}s_{ij}^{*}\overline{a}_iF_j+\sum_{j=1}^{n}t_{ij}^{*}\overline{a}_iG_je^{\mu\tau_{ij}}\right)\left\|y_j\right\|_2^2\right\}\\
&\leq e^{\mu t}\sum_{i=1}^{n}\left\{\left[-2\left(d_i\lambda_1+\underline{a}_i b_i\right)+\mu+\overline{a}_i\left(\sum_{j=1}^{n}s_{ij}^{*}F_j+\sum_{j=1}^{n}t_{ij}^{*}G_j\right)\right]\left\|y_i\right\|_2^2\right.\\
&\qquad\left.+\left(\sum_{j=1}^{n}s_{ji}^{*}\overline{a}_jF_i+\sum_{j=1}^{n}t_{ji}^{*}\overline{a}_jG_ie^{\mu\tau}\right)\left\|y_i\right\|_2^2\right\}\\
&=e^{\mu t}\sum_{i=1}^{n}\left\{-2\left(d_i\lambda_1+\underline{a}_i b_i\right)+\mu+\overline{a}_i\sum_{j=1}^{n}\left(s_{ij}^{*}F_j+\sum_{j=1}^{n}t_{ij}^{*}G_j\right)\right.\\
&\qquad\left.+\sum_{j=1}^{n}\overline{a}_j\left(s_{ji}^{*}F_i+t_{ji}^{*}G_ie^{\mu\tau}\right)\right\}\left\|y_i\right\|_2^2,
\end{aligned}\tag{3.11}$$

for $t\in[0,+\infty)$. Hence

$$\left\|y(t)\right\|_2^2e^{\mu t}\leq V(t)\leq V(0),\quad \text{for } t\in[0,+\infty).\tag{3.12}$$

Note that

$$\begin{aligned}
V(0)&=\sum_{i=1}^{n}\left[\left\|y_i(0)\right\|_2^2+\overline{a}_i\sum_{j=1}^{n}t_{ij}^{*}G_j\int_{-\tau_{ij}}^{0}e^{\mu(s+\tau_{ij})}\left\|y_j(s)\right\|_2^2ds\right]\\
&\leq\left[1+\frac{1}{\mu}\max_{1\leq i\leq n}\left\{\sum_{j=1}^{n}\overline{a}_jt_{ji}^{*}G_i\tau_{ji}e^{\mu\tau_{ji}}\right\}\right]\sum_{i=1}^{n}\left\|y_i\right\|_\tau^2.
\end{aligned}\tag{3.13}$$

Denote $M>0$ and

$$M^2=1+\frac{1}{\mu}\max_{1\leq i\leq n}\left\{\sum_{j=1}^{n}\overline{a}_jt_{ji}^{*}G_i\tau_{ji}e^{\mu\tau_{ji}}\right\},\tag{3.14}$$

where $M>0$, then $M\geq1$. So

$$\left\|y(t)\right\|_2^2\leq M^2\left\|\phi-u^{*}\right\|_\tau^2e^{-\mu t}\quad \text{for } t\in[0,+\infty),\tag{3.15}$$

that is,

$$\|u(t) - u^*\|_2 \leq M\|\phi - u^*\|_\tau e^{-(1/2\mu t)} \quad \text{for } t \in [0, +\infty). \tag{3.16}$$

Since all the solutions of system (2.1)–(2.4) tend to u^* exponentially as $t \to +\infty$ for any values of the coefficients in system (2.1) with (2.3)-(2.4), that is, the system described by (2.1) with (2.3)-(2.4) has a unique equilibrium which is globally exponentially robust stable on L^2-norm and the theorem is proved. □

Remark 3.2. In the deduction for Theorem 3.1, by Lemma 2.5, we have obtained $-d_i\int_\Omega|\nabla y_i|^2 dx \leq -\lambda_1 d_i\|y_i(t)\|_2^2$ (see (3.4)). This is an important step. As a result, the condition of Theorem 3.1 includes the diffusion terms.

Changing a little the Lyapunov functional (3.10) by

$$V(t) = \sum_{i=1}^{n}\left[\|y_i\|_2^2 e^{\mu t} + \overline{a}_i\sum_{j=1}^{n} t_{ij}G_j^2\int_{t-\tau_{ij}}^{t} e^{\mu(s+\tau_{ij})}\|y_j(s)\|_2^2 ds\right], \tag{3.17}$$

and using the similar way of the proof of Theorem 3.1, we derive another new criterion.

Theorem 3.3. *Under assumptions (H1)–(H4), if, in addition*

(A2) $2(d_i\lambda_1 + \underline{a}_i b_i) > \sum_{j=1}^n \overline{a}_i(s_{ij}^* + t_{ij}^*) + \sum_{j=1}^n \overline{a}_j(s_{ji}^* F_j^2 + t_{ji}^* G_j^2)$, *for $i = 1, \dots, n$, then equilibrium point u^* of system* (2.1) *with* (2.3)-(2.4) *is globally exponentially robust stable for each constant input $I \in \mathbb{R}^n$.*

For system (2.1), when the strength of the neuron interconnections s_{ij} and t_{ij} $(i, j = 1, \dots, n)$ is fixed constant matrices, the following result is obvious from Theorems 3.1 and 3.3.

Corollary 3.4. *Under assumptions (H1)–(H4), if any one of the following condition is true:*

(A3) $2(d_i\lambda_1 + \underline{a}_i b_i) > \overline{a}_i \sum_{j=1}^n (s_{ij}F_i + t_{ij}G_i) + \sum_{j=1}^n \overline{a}_j(s_{ji}F_j + t_{ji}G_j)$,

(A4) $2(d_i\lambda_1 + \underline{a}_i b_i) > \overline{a}_i \sum_{j=1}^n (s_{ij} + t_{ij}) + \sum_{j=1}^n \overline{a}_j(s_{ji}F_j^2 + t_{ji}G_j^2)$, *for $i = 1, \dots, n$, then equilibrium point u^* of system* (2.1) *with* (2.3)-(2.4) *is globally exponentially stable.*

Remark 3.5. When $a_i(u_i(t,x)) = 1$, $b_i(u_i(t,x)) = b_i u_i(t,x)$, $i = 1, \dots, n$, then system (2.1) reduces to the following reaction-diffusion cellular neural network:

$$\begin{aligned}\frac{\partial u_i(t,x)}{\partial t} &= \sum_{l=1}^{m}\frac{\partial}{\partial x_l}\left(d_{il}\frac{\partial u_i(t,x)}{\partial x_l}\right)\\ &\quad - \left[b_i u_i(t,x) - \sum_{j=1}^{n} s_{ij} f_j(u_j(t,x)) - \sum_{j=1}^{n} t_{ij} g_j(g_j(t-\tau_{ij},x)) + I_i\right], \quad (t,x) \in [0,+\infty)\times\Omega,\end{aligned} \tag{3.18}$$

for $i = 1, \dots, n$.

From Theorems 3.1 and 3.3, we have the following results.

Corollary 3.6. *Under assumptions (H3) and (H4), if, in addition, any one of the following condition is true:*

(A3) $2(d_i\lambda_1 + b_i) > \sum_{j=1}^{n}(s_{ij}^{*}F_i + t_{ij}^{*}G_i) + \sum_{j=1}^{n}(s_{ji}^{*}F_j + t_{ji}^{*}G_j)$,

(A4) $2(d_i\lambda_1 + b_i) > \sum_{j=1}^{n}(s_{ij}^{*} + t_{ij}^{*}) + \sum_{j=1}^{n}(s_{ji}^{*}F_j^2 + t_{ji}^{*}G_j^2)$, *for* $i = 1,\ldots,n$, *then equilibrium point* u^* *of system* (3.18) *with* (2.3) *and* (2.4) *is globally exponentially robust stable for each constant input* $I \in \mathbb{R}^n$.

Remark 3.7. When $d_{il} = 0$, then system (2.1) reduces to the following system without diffusive terms:

$$\frac{\partial u_i(t)}{\partial t} = -a_i(u_i(t))\left[b_i(u_i(t)) - \sum_{j=1}^{n} s_{ij} f_j(u_j(t)) \sum_{j=1}^{n} t_{ij} g_j(g_j(t-\tau_{ij})) + I_i\right], \tag{3.19}$$

for $t \geq 0$, $i = 1,\ldots,n$.

From Theorems 3.1 and 3.3, we have the following results.

Corollary 3.8. *Under assumptions (H1)–(H4), if, in addition, any one of the following condition holds:*

(A5) $2\underline{a}_i b_i > \sum_{j=1}^{n} \overline{a}_i(s_{ij}F_i + t_{ij}G_i) + \sum_{j=1}^{n} \overline{a}_j(\overline{s}_{ji}F_j + \overline{t}_{ji}G_j)$,

(A6) $2\underline{a}_i b_i > \sum_{j=1}^{n} \overline{a}_i(s_{ij} + t_{ij}) + \sum_{j=1}^{n} \overline{a}_j(\overline{s}_{ji}F_j^2 + \overline{t}_{ji}G_j^2)$, *for* $i = 1,\ldots,n$, *then equilibrium point* u^* *of system* (3.19) *with* (2.4) *is globally exponentially robust stable for each constant input* $I \in \mathbb{R}^n$.

Remark 3.9. From Theorems 3.1 and 3.3, Corollary 3.8, we see that condition (A5) or condition (A6) imply (A1) and (A2), respectively, conversely, if conditions (A1) and (A2) hold, (A5) and (A6) do not certainly hold. This show that the reaction-diffusion terms have play an important role in the globally exponentially robust stability to a reaction-diffusion neural network.

4. Examples and Comparison

In order to illustrate the feasibility of the previous established criteria in the preceding sections, we provide concrete two examples. Although the selection of the coefficients and functions in the examples is somewhat artificial, the possible application of our theoretical theory is clearly expressed.

Example 4.1. Consider the following reaction-diffusion CGNNs on $\Omega = \{x = (x_1, x_2, x_3)^T \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 < 1\}$:

$$\begin{aligned}
\frac{\partial}{\partial t}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} &= \begin{bmatrix} 0.65\dfrac{\partial u_1(t)}{\partial x_1} & 0.72\dfrac{\partial u_1(t)}{\partial x_2} & 0.65\dfrac{\partial u_1(t)}{\partial x_3} \\ 0.82\dfrac{\partial u_2(t)}{\partial x_1} & 0.65\dfrac{\partial u_2(t)}{\partial x_2} & 0.71\dfrac{\partial u_2(t)}{\partial x_3} \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \\ \dfrac{\partial}{\partial x_3} \end{bmatrix} \\
&\quad - \begin{bmatrix} 1 + 0.2\cos u_1(t,x) & 0 \\ 0 & 1 + 0.2\sin u_2(t,x) \end{bmatrix} \\
&\quad \times \left\{ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} - \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{bmatrix} \sin u_1(t) \\ \cos u_2(t) \end{bmatrix} \right. \\
&\qquad \left. - \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\begin{bmatrix} \tanh(u_1(t-\tau_{1j})) \\ \tanh(u_2(t-\tau_{2j})) \end{bmatrix} \right\}, \quad (t,x) \in [0,+\infty) \times \Omega, \\
u_i(t) &= 0, \quad (t,x) \in [0,+\infty) \times \partial\Omega, \quad i = 1,2, \\
u_i(s) &= \phi_i(s), \quad (s,x) \in [-1,0] \times \overline{\Omega}, \quad i = 1,2,
\end{aligned} \tag{4.1}$$

where $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$, $s_{11} \in [1/5, 1/4]$, $s_{12} \in [1/20, 1/16]$, $s_{21} \in [-1/4, 1/17]$, $s_{22} \in [-1/4, -1/12]$, $t_{11} \in [1/6, 1/4]$, $t_{12} \in [1/20, 1/6]$, $t_{21} \in [-1/4, -1/12]$, $t_{22} \in [1/3, 1/2]$, $I_1 \in [-1/5, 1]$, $I_1 \in [-1/5, 3/5]$, $\tau_{11} \in [0.3, 0.8]$, $\tau_{12} \in [0.4, 1]$, $\tau_{21} \in [0.1, 0.6]$, and $\tau_{22} \in [0.2, 0.9]$.

This model satisfies assumptions (H1)–(H4) in this paper with $\lambda_1 \geq 0.5387$, $d_1 = d_2 = 0.65$, $\overline{a}_1 = \overline{a}_2 = 1.2$, $\underline{a}_1 = \underline{a}_2 = 0.8$, $b_1 = b_2 = 1.2$, $F_1 = F_2 = G_1 = G_2 = 1$, $S^* = (s_{ij}^*)_{n\times n} = \begin{bmatrix} 1/4 & 1/16 \\ 1/4 & 1/4 \end{bmatrix}$, $T^* = (t_{ij}^*)_{n\times n} = \begin{bmatrix} 1/4 & 1/16 \\ 1/4 & 1/2 \end{bmatrix}$, $\tau = 1$, $I_1^* = 1$, $I_2^* = 3/5$. It is easily computed that

$$\left.\begin{matrix} 2.6204 \\ 2.6204 \end{matrix}\right\} = 2(d_i\lambda_1 + \underline{a}_i b_i) > \overline{a}_i \sum_{j=1}^{n}\left(s_{ij}^* F_i + t_{ij}^* G_i\right) + \sum_{j=1}^{n} \overline{a}_j \left(s_{ji}^* F_j + t_{ji}^* G_j\right) = \begin{cases} 1.9500, & i = 1, \\ 2.5500, & i = 2. \end{cases} \tag{4.2}$$

From Theorem 3.1, we know that model (4.1) has a unique equilibrium point which is globally exponentially robustly stable.

Remark 4.2. It should be noted that

$$2.4 = 2\underline{a}_2 b_2 \not> \overline{a}_2 \sum_{j=1}^{n}\left(s_{2j}^* F_2 + t_{2j}^* G_2\right) + \sum_{j=1}^{n} \overline{a}_j \left(s_{j2}^* F_j + t_{j2}^* G_j\right) = 2.5500. \tag{4.3}$$

From Corollary 3.8, the corresponding delayed differential equation of system (4.1) without reaction-diffusion terms is not certainly robustly stable, as we can see in Example 4.1, reaction-diffusion terms do contribute to the exponentially robust stability of system (4.1).

Example 4.3. For the model in Example 4.1, if the diffusion operator, Ω, and the boundary condition are replaced by, respectively,

$$\begin{bmatrix} 2\dfrac{\partial u_1(t)}{\partial x_1} & 1.2\dfrac{\partial u_1(t)}{\partial x_2} \\ 1.2\dfrac{\partial u_2(t)}{\partial x_1} & 2\dfrac{\partial u_2(t)}{\partial x_2} \end{bmatrix} \begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ \dfrac{\partial}{\partial x_2} \end{bmatrix}, \quad \Omega = \left\{ (x_1, x_2)^T \in \mathbb{R}^2 \mid 0 < x_i < \pi, i = 1, 2 \right\}, \tag{4.4}$$

and the Neumann boundary condition

$$\frac{\partial u_i(t)}{\partial m} = 0, \quad (t, x) \in [0, +\infty) \times \partial\Omega, \; i = 1, 2, \tag{4.5}$$

the remainder parameters unchanged. According to Remark 2.1, we see that $\lambda_1 = 2$. By Theorem 3.1, using the same way with Example 4.1, we see that model (4.1) has a unique equilibrium point which is globally exponentially robustly stable.

Remark 4.4. Song and Cao have considered reaction-diffusion CGNNs with the Neumann boundary condition and obtained the criteria of the globally exponentially robust stability unique equilibrium point for CGNN, that is, [29, Theorem 1]. We notice that [29, Theorem 1] is irrelevant to the reaction-diffusion terms. In principal, [29, Theorem 1] could be applied to analyze the globally exponentially robust stability for the system in Example 4.2. Unfortunately, [29, Theorem 1] is not applicable to ascertain the globally exponentially robust stability for the system in Example 4.2, since (according to the symbols in this paper)

$$\begin{bmatrix} \dfrac{\underline{a}_1 b_1}{\overline{a}_1} & 0 \\ 0 & \dfrac{\underline{a}_2 b_2}{\overline{a}_1} \end{bmatrix} - S^* F - T^* G = \begin{bmatrix} \dfrac{1}{6} & -\dfrac{1}{8} \\ -\dfrac{1}{2} & -\dfrac{1}{12} \end{bmatrix} \tag{4.6}$$

is not an M-matrix, where $F = \operatorname{diag}\{F_1, F_2\}$, $G = \operatorname{diag}\{G_1, G_2\}$.

5. Conclusion

In this paper, we have proposed several sufficient condition for the globally exponentially robustly stability of equilibrium point for the reaction-diffusion CGNNs with constant time delays. All the criteria are established by constructing suitable Lyapunov functionals, without assuming the monotonicity and differentiability of activation functions and the symmetry of connection matrices. The space domain that CGNNs model is on is relatively general, the boundary condition of CGNNs model includes the Dirichlet and the Neumann. In particular, Poincaré inequality is used and all the criteria obtained depend on reaction-diffusion terms, this is a preeminent feature that distinguishes our research from the previous research on delayed neural network with reaction diffusion. Numerical examples are presented to illustrate the feasibility of this method.

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