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Research Article

Numerical Analysis for Stochastic Partial Differential Delay Equations with Jumps

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We investigate the convergence rate of Euler-Maruyama method for a class of stochastic partial differential delay equations driven by both Brownian motion and Poisson point processes. We discretize in space by a Galerkin method and in time by using a stochastic exponential integrator. We generalize some results of Bao et al. (2011) and Jacob et al. (2009) in finite dimensions to a class of stochastic partial differential delay equations with jumps in infinite dimensions.

1. Introduction

The theory and application of stochastic differential equations have been widely investigated [1–7]. Liu [2] studied the stability of infinite dimensional stochastic differential equations. For the numerical analysis of stochastic partial differential equations, Gyöngy and Krylov [8] discussed the numerical approximations for linear stochastic partial differential equations in whole space. Jentzen et al. [9] studied the numerical simulations of nonlinear parabolic stochastic partial differential equations with additive noise. Kloeden et al. [10] gave the error analysis for the pathwise approximation of a general semilinear stochastic evolution equations.

By contrast, stochastic partial differential equations with jumps have begun to gain attention [11–15]. Röckner and Zhang [15] considered the existence, uniqueness, and large deviation principles of stochastic evolution equation with jump. In [12], the successive approximation of neutral SPDEs was studied. There are few papers on the convergence rate of numerical solutions for stochastic partial differential equations with jump, although there are some papers on the convergence rate of numerical solutions for stochastic differential equations with jump in finite dimensions [16, 17].

Being motivated by the papers [16, 17], we will discuss the convergence rate of Euler-Maruyama scheme for a class of stochastic partial delay equations with jump, where the numerical scheme is based on spatial discretization by Galerkin method and time discretization by using a stochastic exponential integrator. In consequence, we generalize some results of Bao et al. (2011) and Jacob et al. (2009) in finite dimensions to a class of stochastic partial delay equations with jump in infinite dimensions. The rest of this paper is arranged as follows. We give some preliminary results of Euler-Maruyama scheme in Section 2. The convergence rate is discussed in Section 3.

2. Preliminary Results

Throughout this paper, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with some filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Let $(H, \langle\cdot,\cdot\rangle_H, \|\cdot\|_H)$ and $(K, \langle\cdot,\cdot\rangle_K, \|\cdot\|_K)$ be two real separable Hilbert spaces. We denote by $(\mathcal{L}(K,H), \|\cdot\|)$ the family of bounded linear operators. Let $\tau > 0$ and $D([-\tau, 0], H)$ denote the family of right-continuous function and left-hand limits φ from $[-\tau, 0]$ to H with the norm $\|\varphi\|_D = \sup_{-\tau\leq\theta\leq 0}\|\varphi(\theta)\|_H$. $D^b_{\mathcal{F}_0}([-\tau, 0], H)$ denotes the family of almost surely bounded, $\mathcal{F}_0$-measurable, $D([-\tau, 0], H)$-valued random variables. For all $t \geq 0$, $X_t = \{X(t+\theta) : -\tau \leq \theta \leq 0\}$ is regarded as $D([-\tau, 0], H)$-valued stochastic process.

Let T be a positive constant. For given $\tau \geq 0$, consider the following stochastic partial differential delay equations with jumps:

$$\begin{aligned} dX(t) = & \left[AX(t) + f(X(t), X(t-\tau))\right] dt \\ & + g(X(t), X(t-\tau)) dW(t) \\ & + \int_{\mathbb{Z}} h(X(t), X(t-\tau), u) N(dt, du) \end{aligned} \tag{1}$$

on $t \in [0,T]$ with initial datum $X(t) = \xi(t) \in D^b_{\mathcal{F}_0}([-\tau,0],H)$, $-\tau \leq t \leq 0$. Here $(A, D(A))$ is a self-adjoint operator on H. $\{W(t), t \geq 0\}$ is K-valued $\{\mathcal{F}_t\}_{t\geq 0}$-Wiener process defined on the probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\}$ with covariance operator Q. We assume that $-A$ and the covariance operator Q of the Wiener process have the same eigenbasis $\{e_m\}_{m\geq 1}$ of H; that is,

$$\begin{aligned} -Ae_m &= \lambda_m e_m, \\ Qe_m &= \alpha_m e_m, \quad m = 1, 2, 3, \ldots, \end{aligned} \tag{2}$$

where $\{\lambda_m, m \in \mathbb{N}\}$ are the discrete spectrum of $-A$ and $0 \leq \lambda_1 \leq \lambda_2 \cdots \leq \lim_{m\to\infty}\lambda_m = \infty$, $\{\alpha_m, m \in \mathbb{N}\}$ are the eigenvalues of Q. Then, $W(t)$ is defined by

$$W(t) = \sum_{n=1}^{\infty} \sqrt{\alpha_m}\beta_m(t) e_m, \quad t \geq 0, \tag{3}$$

where $\beta_m(t)$ $(m = 1, 2, 3, \ldots)$ is a sequence of real-valued standard Brownian motions mutually independent of the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$.

According to Da Prato and Zabczyk [1], we define stochastic integrals with respect to the Q-Wiener process $W(t)$. Let $K_0 = Q^{1/2}(K)$ be the subspace of K with the inner product $\langle u, v\rangle_{K_0} = \langle Q^{-1/2}u, Q^{-1/2}v\rangle_K$. Obviously, K_0 is a Hilbert space. Denote by $\mathcal{L}^0_2 = \mathcal{L}(K_0, H)$ the family of Hilbert-Schmidt operators from K_0 into H with the norm $\|\Psi\|^2_{\mathcal{L}^0_2} = \operatorname{tr}((\Psi Q^{1/2})(\Psi Q^{1/2})^*)$.

Let $\Phi : (0,\infty) \to \mathcal{L}^0_2$ be a predictable, $\mathcal{F}_t$-adapted process such that

$$\int_0^t \mathbb{E}\|\Phi(s)\|_{\mathcal{L}^0_2} ds \leq \infty, \quad \forall t > 0. \tag{4}$$

Then, the H-valued stochastic integral $\int_0^t \Phi(s)dW(s)$ is a continuous square martingale. Let $N(dt, du)$ be the Poisson measure which is independent of the Q-Wiener process $W(t)$. Denote the compensated or centered Poisson measure as

$$\widetilde{N}(dt, du) = N(dt, du) - \rho dt\pi(du), \tag{5}$$

where $0 < \rho < \infty$ is known as the jump rate and $\pi(\cdot)$ is the jump distribution (a probability measure). Let $\mathbb{Z} \in \mathcal{B}(K - \{0\})$ be the measurable set. Denote by $\mathcal{P}^2([0,T]\times\mathbb{Z}, H)$ the space of all predictable mappings $h : [0,T]\times\mathbb{Z} \to H$ for which

$$\int_0^T \int_{\mathbb{Z}} \mathbb{E}\|h(t,u)\|^2_H dt\pi(du) < \infty. \tag{6}$$

Then, the H-valued stochastic integral

$$\int_0^T \int_{\mathbb{Z}} h(t,u)\widetilde{N}(dt, du) \tag{7}$$

is a centred square-integrable martingale.

We recall the definition of the mild solution to (1) as follows.

Definition 1. A stochastic process $\{X(t) : t \in [0,T]\}$ is called a mild solution of (1) if

(i) $X(t)$ is adapted to $\mathcal{F}_t$, $t \geq 0$, and has *càdlàg* path on $t \geq 0$ almost surely,

(ii) for arbitrary $t \in [0,T]$, $\mathbb{P}\{w : \int_0^t \|X(s)\|^2_H ds < \infty\} = 1$, and almost surely

$$\begin{aligned} X(t) = & e^{tA}\xi(0) + \int_0^t e^{(t-s)A} f(X(s), X(s-\tau)) ds \\ & + \int_0^t e^{(t-s)A} g(X(s), X(s-\tau)) dW(s) \\ & + \int_0^t \int_{\mathbb{Z}} e^{(t-s)A} h(X(s), X(s-\tau), u) N(ds, du) \end{aligned} \tag{8}$$

for any $X(t) = \xi(t) \in D^b_{\mathcal{F}_0}([-\tau,0],H)$, $-\tau \leq t \leq 0$.

For the existence and uniqueness of the mild solution to (1) (see [11]), we always make the following assumptions.

(H1) $(A, D(A))$ is a self-adjoint operator on H such that $-A$ has discrete spectrum $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lim_{m\to\infty}\lambda_m = \infty$ with corresponding eigenbasis $\{e_m\}_{m\geq 1}$ of H. In this case A generates a compact C_0-semigroup e^{tA}, $t \geq 0$, such that $\|e^{tA}\| \leq e^{-\alpha t}$.

(H2) The mappings $f : H \times H \to H$, $g : H \times H \to \mathcal{L}(K, H)$, and $h : H \times H \times \mathbb{Z} \to H$ are Borel measurable and satisfy the following Lipschitz continuity condition for some constant $L_1 > 0$ and arbitrary $x, y, x_1, y_1, x_2, y_2 \in H$ and $u \in \mathbb{Z}$:

$$\begin{aligned} & \|f(x_1, y_1) - f(x_2, y_2)\|^2_H \\ & \quad \vee \|g(x_1, y_1 - g(x_2, y_2))\|^2_{\mathcal{L}^2_0} \\ & \quad \leq L_1\left(\|x_1 - x_2\|^2_H + \|y_1 - y_2\|^2_H\right), \\ & \|h(x_1, y_1, u) - h(x_2, y_2, u)\|^2_H \\ & \quad \leq L_1\left(\|x_1 - x_2\|^2_H + \|y_1 - y_2\|^2_H\right). \end{aligned} \tag{9}$$

This further implies the linear growth condition; that is,

$$\|f(x,y)\|^2_H + \|g(x,y)\|^2_{\mathcal{L}^0_2} \leq L_0\left(1 + \|x\|^2_H + \|y\|^2_H\right), \tag{10}$$

where

$$L_0 := 2\left(L_2 \vee \|f(0,0)\|^2_H \vee \|g(0,0)\|^2_{\mathcal{L}^0_2}\right). \tag{11}$$

(H3) There exists $L_2 > 0$ satisfying

$$\|h(x, y, u)\|_H^2 \le L_2 \left(\|x\|_H^2 + \|y\|_H^2\right), \tag{12}$$

for each $x, y \in H$ and $u \in \mathbb{Z}$.

(H4) For $\xi \in D^b_{\mathcal{F}_0}([-\tau, 0], H)$, there exists a constant $L_3 > 0$ such that

$$\mathbb{E}\left(|\xi(s) - \xi(t)|^2\right) \le L_3 |t - s|^2, \quad t, s \in [-\tau, 0]. \tag{13}$$

We now describe our Euler-Maruyama scheme for the approximation of (1). For any $n \ge 1$, let $\pi_n : H \to H_n = \operatorname{span}\{e_1, e_2, \ldots, e_n\}$ be the orthogonal projection; that is, $\pi_n x = \sum_{i=1}^n \langle x, e_i\rangle_H e_i$, $x \in H$, $A_n = \pi_n A$, $f_n = \pi_n f$, $g_n = \pi_n g$, and $h_n = \pi_n h$.

Consider the following stochastic differential delay equations with jumps on H_n:

$$\begin{aligned} dX^n(t) = {} & \left[A_n X^n(t) + f_n\left(X^n(t), X^n(t-\tau)\right)\right] dt \\ & + g_n\left(X^n(t), X^n(t-\tau)\right) dW(t) \\ & + \int_{\mathbb{Z}} h_n\left(X^n(t), X^n(t-\tau), u\right) N(dt, du), \\ & X^n(\theta) = \pi_n \xi(\theta), \quad \theta \in [-\tau, 0]. \end{aligned} \tag{14}$$

This spatial approximation (14) is called the Galerkin approximation of (1). Due to the fact that $\pi_n A x = \pi_n A(\sum_{i=1}^n \langle x, e_i\rangle_H e_i) = -\sum_{i=1}^n \lambda_i \langle x, e_i\rangle_H e_i$, $x \in H_n$, it follows that for $x \in H_n$, $A_n x = Ax$, $e^{tA_n x} = e^{tAx}$.

By (H2) and (H3) and the property of the projection operator, we have that

$$\begin{aligned} \|A_n x - A_n y\|_H^2 &= \|A_n(x - y)\|_H^2 \le \lambda_n^2 \|x - y\|_H, \\ \|f_n(x_1, y_1) &- f_n(x_2, y_2)\|_H^2 \\ &\vee \|g(x_1, y_1) - g(x_2, y_2)\|_{\mathcal{L}_2^0}^2 \\ &= \|f(x_1, y_1) - f(x_2, y_2)\|_H^2 \\ &\vee \|g(x_1, y_1) - g(x_2, y_2)\|_{\mathcal{L}_2^0}^2 \\ &\le L_1\left(\|x_1 - x_2\|_H^2 + \|y_1 - y_2\|_H^2\right), \\ \|h_n(x_1, y_1, u) &- h_n(x_2, y_2, u)\|_H^2 \\ &= \|h(x_1, y_1, u) - h(x_2, y_2, u)\|_H^2 \\ &\le L_1\left(\|x_1 - x_2\|_H^2 + \|y_1 - y_2\|_H^2\right), \\ \|h_n(x, y, u)\|_H^2 &= \|h(x, y, u)\|_H^2 \le L_2\left(\|x\|_H^2 + \|y\|_H^2\right) \end{aligned} \tag{15}$$

for arbitrary $x, y, x_1, x_2, y_1, y_2 \in H_n$ and $u \in \mathbb{Z}$. Hence, (14) admits a unique solution $X^n(t)$ on H_n.

We introduce a time discretization scheme for (14) by using a stochastic exponential integrator. For given $T \ge 0$ and $\tau > 0$, the time-step size $\Delta \in (0, 1)$ is defined by $\Delta := \tau/N$, for some sufficiently large integer $N > \tau$. For any integer $k \ge 0$, the time discretization scheme applied to (14) produces approximations $\overline{Y}^n(k\Delta) \approx X^n(k\Delta)$ by forming

$$\begin{aligned} & \overline{Y}^n((k+1)\Delta) \\ & \quad = e^{\Delta A_n}\Big\{\overline{Y}^n(k\Delta) + f_n\left(\overline{Y}^n(k\Delta), \overline{Y}^n(k\Delta - \tau)\right)\Delta \\ & \qquad + g_n\left(\overline{Y}^n(k\Delta), \overline{Y}^n(k\Delta - \tau)\right)\Delta W_k \\ & \qquad + \int_{\mathbb{Z}} h_n\left(\overline{Y}^n(k\Delta), \overline{Y}^n(k\Delta - \tau), u\right)\Delta N_k(u)\Big\}, \\ & \qquad \overline{Y}^n(\theta) = \pi_n \xi(\theta), \quad \theta \in [-\tau, 0], \end{aligned} \tag{16}$$

where $\Delta W_k = W((k+1)\Delta) - W(k\Delta)$ and $\Delta N_k(du) = N((0, (k+1)\Delta], du) - N((0, k\Delta], du)$.

The continuous-time version of this scheme associated with (14) is defined by

$$\begin{aligned} & Y^n(t) \\ & \quad = e^{tA_n} Y^n(0) + \int_0^t e^{(t - \lfloor s \rfloor)A_n} f_n\left(Y^n(\lfloor s \rfloor), Y^n(\lfloor s \rfloor - \tau)\right) ds \\ & \qquad + \int_0^t e^{(t - \lfloor s \rfloor)A_n} g_n\left(Y^n(\lfloor s \rfloor), Y^n(\lfloor s \rfloor - \tau)\right) dW(s) \\ & \qquad + \int_0^t \int_{\mathbb{Z}} e^{(t - \lfloor s \rfloor)A_n} h_n\left(Y^n(\lfloor s \rfloor), Y^n(\lfloor s \rfloor - \tau), u\right) \\ & \qquad\quad \times N(ds, du), \\ & \qquad Y^n(\theta) = \pi_n \xi(\theta), \quad \theta \in [-\tau, 0], \end{aligned} \tag{17}$$

where $\lfloor t \rfloor = [t/\Delta]\Delta$ with $[t/\Delta]$ denotes the integer of t/Δ.

From (16) and (17), we have $Y^n(k\Delta) = \overline{Y}^n(k\Delta)$ for every $k \ge 0$. That is, the discrete-time and continuous-time schemes coincide at the grid points.

3. Convergence Rate

In this section, we shall investigate the convergence rate of the Euler-Maruyama method. In what follows, $C > 0$ is a generic constant whose values may change from line to line.

Lemma 2. *Let (H1)–(H4) hold; then there is a positive constant $C > 0$ which depends on T, ξ, L_1, L_2, and L_3 but is independent of Δ, such that*

$$\sup_{0 \le t \le T}\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2} \vee \sup_{0 \le t \le T}\left(\mathbb{E}\|Y^n(t)\|_H^2\right)^{1/2} \le C. \tag{18}$$

Proof. Due to the fact that $(\mathbb{E}\|\cdot\|_H^2)^{1/2}$ is a norm, we have from (8) that

$$\begin{aligned}
&\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2} \\
&\quad\le \left(\mathbb{E}\left\|e^{tA_n}\xi(0)\right\|_H^2\right)^{1/2} \\
&\qquad+\left(\mathbb{E}\left\|\int_0^t e^{(t-s)A} f(X(s),X(s-\tau))\,ds\right\|_H^2\right)^{1/2} \\
&\qquad+\left(\mathbb{E}\left\|\int_0^t e^{(t-s)A} g(X(s),X(s-\tau))\,dW(s)\right\|_H^2\right)^{1/2} \\
&\qquad+\left(\mathbb{E}\left\|\int_0^t e^{(t-s)A} h(X(s),X(s-\tau),u)\,N(ds,du)\right\|_H^2\right)^{1/2} \\
&\quad=\sum_{i=1}^4 I_i(t).
\end{aligned} \tag{19}$$

Recall the property of the operator A (see [18]):

$$\begin{gathered}
\left\|(-A)^{\delta_1}e^{At}\right\| \le Ct^{-\delta_1}, \\
\left\|(-A)^{\delta_2}\left(1-e^{At}\right)\right\| \le Ct^{\delta_2}, \quad \delta_1\ge 0,\ \delta_2\in[0,1], \\
(-A)^{\alpha+\beta}x=(-A)^{\alpha}(-A)^{\beta}x, \quad x\in D\left((-A)^r\right),
\end{gathered} \tag{20}$$

for $\alpha,\beta\in\mathbb{R}$, where $r=\max\{\alpha,\beta,\alpha+\beta\}$.

By (H1) and (H2), together with the Minkowski integral inequality, we derive that

$$\begin{aligned}
I_2(t) &\le \int_0^t \left(\mathbb{E}\left\|e^{(t-s)A} f(X(s),X(s-\tau))\right\|_H^2\right)^{1/2} ds \\
&\le C\int_0^t \Big\{1+\left(\mathbb{E}\|X(s)\|_H^2\right)^{1/2} \\
&\qquad +\left(\mathbb{E}\|X(s-\tau)\|_H^2\right)^{1/2}\Big\}\,ds \\
&\le C+C\int_0^t \left(\mathbb{E}\|X(s)\|_H^2\right)^{1/2} ds.
\end{aligned} \tag{21}$$

By (H1), (H2), and (H3) and using the Itô isometry, we have

$$\begin{aligned}
&I_3(t)+I_4(t) \\
&\quad\le \left(\int_0^t \mathbb{E}\left\|e^{(t-s)A} g(X(s),X(s-\tau))\right\|_{\mathscr{L}_2^0}^2 ds\right)^{1/2} \\
&\qquad+\Bigg(\mathbb{E}\Bigg\|\int_0^t\int_Z e^{(t-s)A} h(X(s),X(s-\tau),u)\,\widetilde{N}(ds,du) \\
&\qquad\qquad+\rho\int_0^t\int_{\mathbb{Z}} e^{(t-s)A} h \\
&\qquad\qquad\qquad\times(X(s),X(s-\tau),u)\,\pi(du)\Bigg\|_H^2\Bigg)^{1/2} \\
&\quad\le \left(\int_0^t L_0\left(1+\mathbb{E}\|X(s)\|_H^2+\mathbb{E}\|X(s-\tau)\|_H^2\right)ds\right)^{1/2} \\
&\qquad+\Bigg(\mathbb{E}\Bigg\|\int_0^t\int_Z e^{(t-s)A} h \\
&\qquad\qquad\times(X(s),X(s-\tau),u)\,\widetilde{N}(ds,du)\Bigg\|_H^2\Bigg)^{1/2} \\
&\qquad+\rho\Bigg(\mathbb{E}\Bigg\|\int_0^t\int_{\mathbb{Z}} e^{(t-s)A} h \\
&\qquad\qquad\times(X(s),X(s-\tau),u)\,\pi(du)\,ds\Bigg\|_H^2\Bigg)^{1/2}.
\end{aligned} \tag{22}$$

Using Hölder inequality and (H3), for the last term of (22), we have

$$\begin{aligned}
&\rho\left(\mathbb{E}\left\|\int_0^t\int_Z e^{(t-s)A} h(X(s),X(s-\tau),u)\,\pi(du)\,ds\right\|_H^2\right)^{1/2} \\
&\quad\le C\left(\mathbb{E}\int_0^t\int_Z \|h(X(s),X(s-\tau),u)\|_H^2\pi(du)\,ds\right)^{1/2} \\
&\quad\le C\sqrt{L_2}\left(\int_0^t\left(\mathbb{E}\|X(s)\|_H^2+\mathbb{E}\|X(s-\tau)\|_H^2\right)ds\right)^{1/2} \\
&\quad\le C\sqrt{L_2}\sqrt{\tau}\mathbb{E}\|\xi\|_H+pC\sqrt{2L_2}\left(\int_0^t \mathbb{E}\|X(s)\|_H^2 ds\right)^{1/2}.
\end{aligned} \tag{23}$$

Moreover, by using the Itô isometry and (H3), we obtain that

$$\begin{aligned}
&\left(\mathbb{E}\left\|\int_0^t\int_{\mathbb{Z}} e^{(t-s)A} h(X(s),X(s-\tau),u)\,\widetilde{N}(ds,du)\right\|_H^2\right)^{1/2} \\
&\quad\le \left(\int_0^t\int_{\mathbb{Z}} \mathbb{E}\|h(X(s),X(s-\tau),u)\|_H^2\pi(du)\,ds\right)^{1/2} \\
&\quad\le \sqrt{L_2}\left(\int_0^t\left(\mathbb{E}\|X(s)\|_H^2+\mathbb{E}\|X(s-\tau)\|_H^2\right)ds\right)^{1/2} \\
&\quad\le \sqrt{L_2}\sqrt{\tau}\mathbb{E}\|\xi\|_H+\sqrt{2L_2}\left(\int_0^t(\mathbb{E}\|X(s)\|_H^2 ds\right)^{1/2}.
\end{aligned} \tag{24}$$

Substituting (23) and (24) into (22), it follows that

$$I_3(t)+I_4(t)\le C+C\,\mathbb{E}\|\xi\|_H+C\left(\int_0^t \mathbb{E}\|X(s)\|_H^2 ds\right)^{1/2}. \tag{25}$$

Hence,

$$\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2}\le C+C\,\mathbb{E}\|\xi\|_H+C\left(\int_0^t \mathbb{E}\|X(s)\|_H^2 ds\right)^{1/2}. \tag{26}$$

Applying the Gronwall inequality, we have

$$\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2}\le C. \tag{27}$$

Using the similar argument, the second assertion of (18) follows. □

Lemma 3. *Let (H1)–(H4) hold; for sufficiently small* Δ,

$$\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)-X(\lfloor t\rfloor)\|_H^2\right)^{1/2}\le C\Delta^{1/2}, \tag{28}$$

where $C>0$ *is constant dependent on* T,ξ,L_1,L_2,L_3, *and* L_4, *while being independent of* Δ.

Proof. For any $t\in[0,T]$, we have from (8) that

$$\begin{aligned}
&X(t)-X(\lfloor t\rfloor)\\
&=e^{\lfloor t\rfloor A}\left(e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right)\xi(0)\\
&\quad+\int_0^{\lfloor t\rfloor}\left(e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right)e^{(\lfloor t\rfloor-s)A}f(X(s),X(s-\tau))\,ds\\
&\quad+\int_{\lfloor t\rfloor}^{t}e^{(t-s)A}f(X(s),X(s-\tau))\,ds\\
&\quad+\int_0^{\lfloor t\rfloor}\left(e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right)e^{(\lfloor t\rfloor-s)A}g(X(s),X(s-\tau))\,dW(s)\\
&\quad+\int_0^{\lfloor t\rfloor}\int_{\mathbb{Z}}\left(e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right)e^{(\lfloor t\rfloor-s)A}\\
&\qquad\qquad\times h(X(s),X(s-\tau),u)\,N(ds,du)\\
&\quad+\int_{\lfloor t\rfloor}^{t}e^{(t-s)A}g(X(s),X(s-\tau))\,dW(s)\\
&\quad+\int_{\lfloor t\rfloor}^{t}\int_{\mathbb{Z}}e^{(t-s)A}h(X(s),X(s-\tau),u)\,N(ds,du)\\
&=\sum_{i=1}^{7}J_i(t).
\end{aligned} \tag{29}$$

Since $(\mathbb{E}\|\cdot\|_H^2)^{1/2}$ is a norm, it follows that

$$\left(\mathbb{E}\|X(t)-X(\lfloor t\rfloor)\|_H^2\right)^{1/2}\le\sum_{i=1}^{7}\left(\mathbb{E}\|J_i(t)\|_H^2\right)^{1/2}. \tag{30}$$

Recalling the fundamental inequality $1-e^{-y}\le y$, $y>0$, we get from (H1) that

$$\begin{aligned}
&\left\|\left(e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right)x\right\|_H^2\\
&\quad=\left\|\sum_{i=1}^{\infty}\left(e^{-\lambda_i(t-\lfloor t\rfloor)}-1\right)\langle x,e_i\rangle e_i\right\|_H^2\\
&\quad\le\left(1-e^{-\lambda_1(t-\lfloor t\rfloor)}\right)^2\|x\|_H^2\\
&\quad\le\lambda_1^2\Delta^2\|x\|_H^2.
\end{aligned} \tag{31}$$

Therefore,

$$\begin{aligned}
&\left(\mathbb{E}\|J_1(t)\|_H^2\right)^{1/2}\\
&\quad=\left(\mathbb{E}\left\|e^{\lfloor t\rfloor A}\left\{e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right\}\xi(0)\right\|_H^2\right)^{1/2}\\
&\quad\le\lambda_1\left(\mathbb{E}\|\xi(0)\|_H^2\right)^{1/2}\Delta.
\end{aligned} \tag{32}$$

By (H1), (H2), and the Minkowski integral inequality, we obtain that

$$\begin{aligned}
&\sum_{i=2}^{3}\left(\mathbb{E}\|J_i(t)\|_H^2\right)^{1/2}\\
&\quad\le\int_0^{\lfloor t\rfloor}\left\|e^{(t-\lfloor t\rfloor)A}-\mathbf{1}\right\|\left\|e^{(\lfloor t\rfloor-s)A}\right\|\\
&\qquad\qquad\times\left(\mathbb{E}\|f(X(s),X(s-\tau))\|_H^2\right)^{1/2}ds\\
&\qquad+\int_{\lfloor t\rfloor}^{t}\left(\mathbb{E}\|f(X(s),X(s-\tau))\|_H^2\right)^{1/2}ds.
\end{aligned} \tag{33}$$

Together with (31), we arrive at

$$\begin{aligned}
&\sum_{i=2}^{3}\left(\mathbb{E}\|J_i(t)\|_H^2\right)^{1/2}\\
&\quad\le\left(\lambda_1\Delta\int_0^{\lfloor t\rfloor}ds+\Delta\right)C\sup_{0\le t\le T}\left(\mathbb{E}\|f(X(t),X(t-\tau))\|_H^2\right)^{1/2}\\
&\quad\le C\left(1+\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2}\right)\Delta.
\end{aligned} \tag{34}$$

Following the argument of (22), we derive that

$$\begin{aligned}
&\sum_{i=4}^{7}\left(\mathbb{E}\|J_i(t)\|_H^2\right)^{1/2}\\
&\quad\le\left(\int_0^{\lfloor t\rfloor}\left\|e^{(t-\lfloor t\rfloor)A}-1\right\|^2\left\|e^{(\lfloor t\rfloor-s)A}\right\|^2\right.\\
&\qquad\qquad\left.\times\mathbb{E}\|g(X(s),X(s-\tau))\|_{\mathscr{L}_2^0}^2ds\right)^{1/2}\\
&\qquad+C\left(\int_0^{\lfloor t\rfloor}\int_{\mathbb{Z}}\left\|e^{(t-\lfloor t\rfloor)A}-1\right\|^2\left\|e^{(\lfloor t\rfloor-s)A}\right\|^2\right.\\
&\qquad\qquad\left.\times\mathbb{E}\|h(X(s),X(s-\tau),u)\|_H^2\pi(du)\,ds\right)^{1/2}\\
&\qquad+\left(\int_{\lfloor t\rfloor}^{t}\left\|e^{(t-s)A}\right\|^2\mathbb{E}\|g(X(s),X(s-\tau))\|_{\mathscr{L}_2^0}^2ds\right)^{1/2}\\
&\qquad+C\left(\int_{\lfloor t\rfloor}^{t}\int_{\mathbb{Z}}\left\|e^{(t-s)A}\right\|^2\right.\\
&\qquad\qquad\left.\times\mathbb{E}\|h(X(s),X(s-\tau),u)\|_H^2\pi(du)\,ds\right)^{1/2}\\
&\quad\le C\left(1+\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2}\right)\Delta^{1/2}.
\end{aligned} \tag{35}$$

Substituting (32), (34), and (35) into (30), we arrive at

$$\left(\mathbb{E}\|X(t)-X(\lfloor t\rfloor)\|_H^2\right)^{1/2} \le C\left(1+\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)\|_H^2\right)^{1/2}\right)\Delta^{1/2}. \tag{36}$$

Therefore, by Lemma 2, the required assertion (28) follows. □

Now, we state our main result in this paper as follows.

Theorem 4. *Let (H1)–(H4) hold, and*

$$\sqrt{L_1}\left(2\alpha^{-1}+(\rho+3)(2\alpha)^{-1/2}\right)<1. \tag{37}$$

Then,

$$\sup_{0\le t\le T}\left(\mathbb{E}\|X(t)-Y^n(t)\|_H^2\right)^{1/2} \le C\left\{\lambda_n^{-1/2}+\Delta^{1/2}\right\}, \tag{38}$$

where $C>0$ is a constant dependent on T,ξ,L_1,L_2,L_3, and L_4, while being independent of n and Δ.

Proof. By (8) and (17), we obtain

$$\begin{aligned}
X(t)-Y^n(t) &= e^{tA}(1-\pi_n)\xi(0) \\
&\quad+\int_0^t e^{(t-s)A}\left(f(X(s),X(s-\tau)) - f_n(X(s),X(s-\tau))\right)ds \\
&\quad+\int_0^t e^{(t-s)A}\left(f_n(X(s),X(s-\tau)) - f_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau))\right)ds \\
&\quad+\int_0^t e^{(t-s)A}\left(g_n(X(s),X(s-\tau)) - g_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau))\right)dW(s) \\
&\quad+\int_0^t e^{(t-s)A}\left(f_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau)) - f_n(Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau))\right)ds \\
&\quad+\int_0^t e^{(t-s)A}\left(g_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau)) - g_n(Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau))\right)dW(s) \\
&\quad+\int_0^t e^{(t-s)A}\left(g(X(s),X(s-\tau)) - g_n(X(s),X(s-\tau))\right)dW(s) \\
&\quad+\int_0^t e^{(t-s)A}\left(1-e^{(s-\lfloor s\rfloor)A}\right)f_n(Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau))\,ds \\
&\quad+\int_0^t e^{(t-s)A}\left(1-e^{(s-\lfloor s\rfloor)A}\right)\times g_n(Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau))\,dW(s) \\
&\quad+\int_0^t\int_{\mathbb{Z}} e^{(t-s)A}\left\{h(X(s),X(s-\tau),u) - h_n(X(s),X(s-\tau),u)\right\}N(ds,du) \\
&\quad+\int_0^t\int_{\mathbb{Z}} e^{(t-s)A}\left\{h_n(X(s),X(s-\tau),u) - h_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau),u)\right\}N(ds,du) \\
&\quad+\int_0^t\int_{\mathbb{Z}} e^{(t-s)A}\left\{h_n(X(\lfloor s\rfloor),X(\lfloor s\rfloor-\tau),u) - h_n \times (Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau),u)\right\}N(ds,du) \\
&\quad+\int_0^t\int_{Z} e^{(t-s)A}\left(1-e^{(s-\lfloor s\rfloor)A}\right)h_n \times (Y^n(\lfloor s\rfloor),Y^n(\lfloor s\rfloor-\tau),u)\,N(ds,du) \\
&= \sum_{i=1}^{13} K_i(t).
\end{aligned} \tag{39}$$

Noting that $(\mathbb{E}\|\cdot\|_H^2)^{1/2}$ is a norm, we have

$$\left(\mathbb{E}\|X(t)-Y^n(t)\|_H^2\right)^{1/2} \le \sum_{i=1}^{13}\left(\mathbb{E}\|K_i(t)\|_H^2\right)^{1/2}. \tag{40}$$

By (H1) and the nondecreasing spectrum $\{\lambda_m\}_{m\ge1}$, it easily follows that

$$\begin{aligned}
\mathbb{E}\left\|e^{tA}(1-\pi_n)\xi(0)\right\|_H &= \mathbb{E}\left(\sum_{m=n+1}^{\infty} e^{-2\lambda_m t}\langle\xi(0),e_m\rangle_H^2\right)^{1/2} \\
&= \mathbb{E}\left(\sum_{m=n+1}^{\infty} \frac{e^{-2\lambda_m t}}{\lambda_m^2}\lambda_m^2\langle\xi(0),e_m\rangle_H^2\right)^{1/2} \\
&\le \frac{1}{\lambda_n}\mathbb{E}\left\|A\xi(0)\right\|_H.
\end{aligned} \tag{41}$$

By (H2), the Minkowski integral inequality, and Lemma 2, we have

$$\begin{aligned}
&\left(\mathbb{E}\left\|K_2(t)\right\|_H^2\right)^{1/2} \\
&\quad\le \int_0^t \left(\mathbb{E}\left\|e^{(t-s)A}\left(1-\pi_n\right) f\left(X(s), X(s-\tau)\right)\right\|_H^2\right)^{1/2} ds \\
&\quad= \int_0^t \left(\mathbb{E} \sum_{m=n+1}^{\infty} e^{-2\lambda_m(t-s)} \left\langle f\left(X(s), X(s-\tau)\right), e_m\right\rangle_H^2\right)^{1/2} ds \\
&\quad\le \int_0^t e^{-\lambda_n(t-s)} \left(\mathbb{E} \sum_{m=n+1}^{\infty} \left\langle f\left(X(s), X(s-\tau)\right), e_m\right\rangle_H^2\right)^{1/2} ds \\
&\quad\le C \int_0^t e^{-\lambda_n(t-s)} \\
&\qquad\times \left\{1 + \left(\mathbb{E}\left\|X(s)\right\|_H^2\right)^{1/2} + \left(\mathbb{E}\left\|X(s-\tau)\right\|_H^2\right)^{1/2}\right\} ds \\
&\quad\le C\lambda_n^{-1}.
\end{aligned} \tag{42}$$

Applying (H1), (H2), and Lemma 3 and combining the Minkowski integral inequality and the Itô isometry yield

$$\begin{aligned}
&\sum_{i=3}^{6}\left(\mathbb{E}\left\|K_i(t)\right\|_H^2\right)^{1/2} \\
&\quad\le \sqrt{L_1} \int_0^t \left\|e^{(t-s)A}\right\| \left(\mathbb{E}\left(\left\|X(s) - X(\lfloor s\rfloor)\right\|_H^2\right.\right. \\
&\qquad\qquad \left.\left. + \left\|X(s-\tau) - X(\lfloor s\rfloor - \tau)\right\|_H^2\right)\right)^{1/2} ds \\
&\qquad + \sqrt{L_1} \int_0^t \left\|e^{(t-s)A}\right\| \left(\mathbb{E}\left(\left\|X(\lfloor s\rfloor) - Y^n(\lfloor s\rfloor)\right\|_H^2\right.\right. \\
&\qquad\qquad \left.\left. + \left\|X(\lfloor s\rfloor - \tau) - Y^n(\lfloor s\rfloor - \tau)\right\|_H^2\right)\right)^{1/2} ds \\
&\qquad + \sqrt{L_1} \left(\int_0^t \left\|e^{(t-s)A}\right\|^2 \left(\mathbb{E}\left(\left\|X(s) - X(\lfloor s\rfloor)\right\|_H^2\right.\right.\right. \\
&\qquad\qquad \left.\left.\left. + \left\|X(s-\tau) - Y^n(\lfloor s\rfloor - \tau)\right\|_H^2\right)\right) ds\right)^{1/2} \\
&\qquad + \sqrt{L_1} \left(\int_0^t \left\|e^{(t-s)A}\right\|^2 \left(\mathbb{E}\left\|X(\lfloor s\rfloor) - Y^n(\lfloor s\rfloor)\right\|_H^2\right.\right. \\
&\qquad\qquad \left.\left. + \left\|X(\lfloor s\rfloor - \tau) - Y^n(\lfloor s\rfloor - \tau)\right\|_H^2\right) ds\right)^{1/2} \\
&\quad\le C\Delta^{1/2} + \sqrt{L_1} \sup_{0\le s\le t}\left(\mathbb{E}\left\|X(s) - Y^n(s)\right\|_H^2\right)^{1/2} \\
&\qquad \times \int_0^t e^{-\alpha(t-s)} ds \\
&\qquad + \sqrt{L_1} \sup_{0\le s\le t}\left(\mathbb{E}\left\|X(s) - Y^n(s)\right\|_H^2\right)^{1/2} \left(\int_0^t e^{-2\alpha(t-s)} ds\right)^{1/2} \\
&\qquad + \sqrt{L_1} \sup_{0\le s\le t}\left(\mathbb{E}\left\|X(s) - Y^n(s)\right\|_H^2\right)^{1/2} \int_{-\tau}^{t-\tau} e^{-\alpha(t-s-\tau)} ds \\
&\qquad + \sqrt{L_1} \sup_{0\le s\le t}\left(\mathbb{E}\left\|X(s) - Y^n(s)\right\|_H^2\right)^{1/2} \left(\int_{-\tau}^{t-\tau} e^{-2\alpha(t-s-\tau)} ds\right)^{1/2} \\
&\quad\le C\Delta^{1/2} + \sqrt{L_1} \sup_{0\le s\le t}\left(\mathbb{E}\left\|X(s) - Y^n(s)\right\|_H^2\right)^{1/2} \\
&\qquad \times \left(2\alpha^{-1} + 2(2\alpha)^{-1/2}\right).
\end{aligned} \tag{43}$$

By the Itô isometry and a similar argument to that of (42), we deduce that

$$\begin{aligned}
&\left(\mathbb{E}\left\|K_7(t)\right\|_H^2\right)^{1/2} \\
&\quad\le \left(\int_0^t \mathbb{E}\left\|e^{(t-s)A}\left(1-\pi_n\right) g\left(X(s), X(s-\tau)\right)\right\|_{\mathscr{L}_2^0}^2 ds\right)^{1/2} \\
&\quad\le C\left(\int_0^t e^{-2\lambda_n(t-s)} \mathbb{E}\left\|g\left(X(s), X(s-\tau)\right)\right\|_{\mathscr{L}_2^0}^2 ds\right)^{1/2} \\
&\quad\le C\lambda_n^{-1/2}.
\end{aligned} \tag{44}$$

Moreover, by (31), (H2), and Lemma 2 and combining the Minkowski integral inequality and the Itô isometry, we have

$$\begin{aligned}
&\sum_{i=8}^{9}\left(\mathbb{E}\left\|K_i(t)\right\|_H^2\right)^{1/2} \\
&\quad\le \int_0^t \left(\mathbb{E}\left\|e^{(t-s)A}\left(1 - e^{(s-\lfloor s\rfloor)A}\right)\right\|^2\right. \\
&\qquad\qquad \left.\times \left\|f_n(Y^n(\lfloor s\rfloor), Y^n(\lfloor s\rfloor - \tau))\right\|_H^2\right)^{1/2} ds \\
&\qquad + \left(\int_0^t \mathbb{E}\left\|e^{(t-s)A}\left(1 - e^{(s-\lfloor s\rfloor)A}\right)\right\|^2\right. \\
&\qquad\qquad \left.\times \left\|g_n\left(Y^n(\lfloor s\rfloor), Y^n(\lfloor s\rfloor - \tau)\right)\right\|_H^2 ds\right)^{1/2} \\
&\quad\le C\Delta \int_0^t \left(\mathbb{E}\left\|f_n\left(Y^n(\lfloor s\rfloor), Y^n(\lfloor s\rfloor - \tau)\right)\right\|_H^2\right)^{1/2} ds \\
&\qquad + C\Delta\left(\int_0^t \mathbb{E}\left\|g_n\left(Y^n(\lfloor s\rfloor), Y^n(\lfloor s\rfloor - \tau)\right)\right\|_{\mathscr{L}_2^0}^2 ds\right)^{1/2} \\
&\quad\le C\Delta.
\end{aligned} \tag{45}$$

By (31) and the Itô isometry, we obtain that

$$\begin{aligned}
&\left(\mathbb{E}\left\|K_{10}(t)\right\|_H^2\right)^{1/2} \\
&\quad\le \left(\mathbb{E}\left\|\int_0^t \int_{\mathbb{Z}} e^{(t-s)A}\left(1-\pi_n\right) h\right.\right. \\
&\qquad\qquad \left.\left.\times \left(X(s), X(s-\tau), u\right) \widetilde{N}(ds, du)\right\|_H^2\right)^{1/2} \\
&\qquad + \rho\left(\mathbb{E}\left\|\int_0^t \int_{\mathbb{Z}} e^{(t-s)A}\left(1-\pi_n\right) h\right.\right. \\
&\qquad\qquad \left.\left.\times \left(X(s), X(s-\tau), u\right) \pi(du)\, ds\right\|_H^2\right)^{1/2} \\
&\quad\le \left(\int_0^t \int_{\mathbb{Z}} \left\|e^{(t-s)A}\left(1-\pi_n\right)\right\|^2 \mathbb{E}\right. \\
&\qquad\qquad \left.\times \left\|h\left(X(s), X(s-\tau), u\right)\right\|_H^2 \pi(du)\, ds\right)^{1/2}
\end{aligned}$$

$$+\rho\left(\int_0^t\int_{\mathbb{Z}}\left\|e^{(t-s)A}\left(1-\pi_n\right)\right\|^2 \times\mathbb{E}\|h\left(X\left(s\right),X\left(s-\tau\right),u\right)\|_H^2\pi(du)ds\right)^{1/2}$$
$$\leq C\left(\int_0^t e^{-2\lambda_n(t-s)}\left(\mathbb{E}\|X\left(s\right)\|_H^2+\mathbb{E}\|X\left(s-\tau\right)\|_H^2\right)ds\right)^{1/2}$$
$$\leq C\lambda_n^{-1/2}. \tag{46}$$

Carrying out the similar arguments to those of (43) and (45), we derive that

$$\begin{aligned}\left(\mathbb{E}\left\|K_{11}\left(t\right)\right\|_H^2\right)^{1/2}+\left(\mathbb{E}\left\|K_{12}\left(t\right)\right\|_H^2\right)^{1/2} &\leq C\Delta^{1/2}+(2\alpha)^{-1/2}\left(\rho+1\right)\\ &\quad\times\sqrt{L_1}\sup_{0\leq s\leq t}\left(\mathbb{E}\left\|X\left(s\right)-Y^n\left(s\right)\right\|_H^2\right)^{1/2},\\ \left(\mathbb{E}\left\|K_{13}\left(t\right)\right\|_H^2\right)^{1/2} &\leq C\Delta.\end{aligned} \tag{47}$$

As a result, putting (41)–(47) into (40) gives that

$$\begin{aligned}\sup_{0\leq s\leq t}\left(\mathbb{E}\left\|X\left(s\right)-Y^n\left(s\right)\right\|_H^2\right)^{1/2} &\leq C\lambda_n^{-1/2}+C\Delta^{1/2}+\sqrt{L_1}\left(2\alpha^{-1}+\left(\rho+3\right)(2\alpha)^{-1/2}\right)\\ &\quad\times\sup_{0\leq s\leq t}\left(\mathbb{E}\left\|X\left(s\right)-Y^n\left(s\right)\right\|_H^2\right)^{1/2},\end{aligned} \tag{48}$$

and therefore the desired assertion follows. □

Remark 5. For finite-dimensional Euler-Maruyama method, the condition (37) can be deleted by the Gronwall inequality [16, 17].

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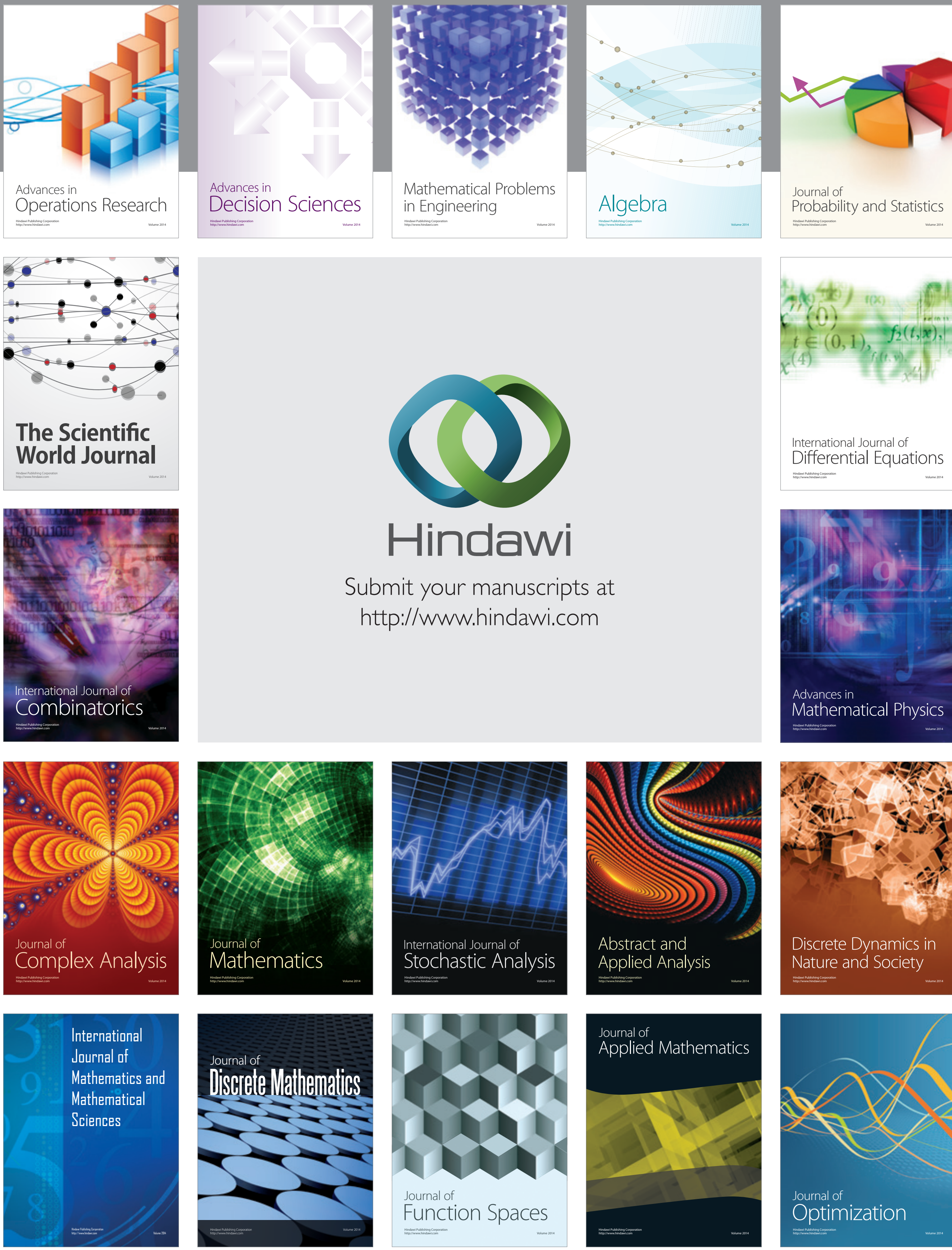
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