Gen. Math. Notes, Vol. 19, No. 2, December, 2013, pp. 104-110
ISSN 2219-7184; Copyright © ICSRS Publication, 2013
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Numerical Algorithm for the Stochastic Present Value of Aggregate Claims in the Renewal Risk Model

Deyi Ma

College of Science, China Three Gorges University
Yichang - 443002, China
E-mail: mdysave@163.com

(Received: 15-9-13 / Accepted: 24-10-13)

Abstract

For the stochastic present value of aggregate claims in the renewal risk model, a numerical algorithm is constructed based on the Monte Carlo and random process principle. The basic idea and design process of this algorithm is detailed. The numerical simulation results show that it is consistence with the theory analysis result under different parameter different distribution. The numerical algorithm results can directly see the ruin probability of this renewal risk model, to develop support in the actual decision.

Keywords: *Renewal risk model, Numerical algorithm, Monte Carlo, Exponential lévy process.*

1 Introduction

Risk free and risky investment model is a classical model to an insurer. Suppose the price process of the investment model is a geometric lévy process $\{e^{R(t)}, t \geq 0\}$. In other words, $\{R(t), t \geq 0\}$ is a lévy process which starts with 0, has independent and stationary increments, and is stochastically continuous. In mathematical finance, this assumption on price process is widely used [1, 2, 3, 4]. For the general theory of lévy process, we refer the reader to see Sato[5], and Applebaum [6].

To the renewal risk model [7] with successive claims, X_1, X_2, Λ , are independent identically distributed (i.i.d.) random variables. The random variable distribution F on $[0,\infty]$. At the same time, their arrival times, $0 \le \tau_1 \le \tau_2 \le \Lambda$, constitute a renewal counting process $N(t) = \#\{n = 1,2,\Lambda \mid \tau_n \le t\}, t \ge 0$. Then the amount of aggregate claims up to time t as follows:

$$S(t) = \sum_{k=1}^{N(t)} X(k), \quad t \ge 0 \tag{1.1}$$

For convenience, suppose that all sources of randomness, $\{X_1, X_2, \Lambda\}$, $\{N(t), \quad t \ge 0\}$ and $\{R(t), \quad t \ge 0\}$ are mutually independent. Then the stochastic present value of future aggregate claims up to time t can be expressed as

$$D(t) = \int_0^t e^{-R(s)} ds = \sum_{k=1}^{\infty} X(k) e^{-R(\tau(k))} 1_{(\tau(k) \le t)}, \quad t \ge 0 \tag{1.2}$$

In this paper, we shall focus on the numerical simulation of $D(t)$, and compare to the theory value. We shall also consider numerical simulation to ruin probability.

The rest of paper consists of fours sections. Section 2 shows the numerical simulation model of this paper; Section 3 presents numerical simulation algorithm, is the key contents of this paper; Section 4 gives the results of numerical simulation; The last section concludes the paper.

2 Numerical Simulation Scheme of the Renewal Risk Model

Suppose an insurance business commencing at time 0 with initial wealth $x > 0$, then the model of the cash flow of premiums less claims is considered as a compound renewal process with the form

$$C(t) = ct - S(t), \quad t \ge 0 \tag{1.3}$$

Where $c \ge 0$ is a fixed rate of premium payment and $\{S(t), \quad t \ge 0\}$ is a compound renewal process defined in (1.1). Because the price process of the investment model is the geometric lévy process $\{e^{R(t)}, t \ge 0\}$, the wealth process of the insurer is defined as [8]

$$U(t) = e^{R(t)}\left(x + \int_0^t e^{-R(t)} dC(t)\right) \quad t \ge 0 \tag{1.4}$$

This paper discusses the model:

$$\overline{U(t)} = x + \int_0^t e^{-R(t)} dC(t) \quad t \ge 0 \tag{1.5}$$

The model (1.5) is too compound to do numerical simulation. In order to construct a numerical simulation scheme, put (1.1), (1.2), (1.3) to (1.5),

$$\overline{U(t)} = x + \int_0^t e^{-R(t)} dC(t) \quad t \ge 0 = x + \int_0^t e^{-R(t)} d(t - S(t))$$
$$= x + c\int_0^t e^{-R(t)} dt - \int_0^t e^{-R(t)} dS(t)$$

$$
\begin{aligned}
&= x + c\int_0^t e^{-R(t)}dt - \sum_{k=1}^{\infty} X(k)e^{-R(\tau(k))}1_{\tau(k)\le t} \\
&= x + c\int_0^t e^{-R(t)}dt - \sum_{k=1}^{N(t)} e^{-R(\tau(k))}X(k) \qquad (1.6)
\end{aligned}
$$

Where $x \ge 0$ is the initial wealth, $c \ge 0$ is a fixed rate of premium payment, $\{R(t), t \ge 0\}$ is a lévy process, $\tau(k)$ is the kth claim times, $N(t) = \#\{n = 1,2,\Lambda \mid \tau(n) \le t\}$, $X(k)$ is the kth claim sizes.

The scheme (1.6) is the numerical simulation scheme for the stochastic present value of aggregate claims in the renewal risk model. This paper mainly researches the results in the numerical simulation scheme with different parameter different distribution.

3 Numerical Algorithm of the Renewal Risk Model

In order to numerical simulation one random process of the model (1.6), it needs to assume concrete distribution. Without loss of generality, this paper assumes $R(t) = \mu t + \sigma W(t)$, where μ, σ are constants, $W(t)$ is Brownian motion. $X(k) \sim$ lognormal distribution or weibull distribution, $N(t) \sim$ Poisson distribution. $EX(k) = \lambda$, $EN(t) = \theta$. According to the numerical simulation scheme, the algorithm flowchart is defined as follows.

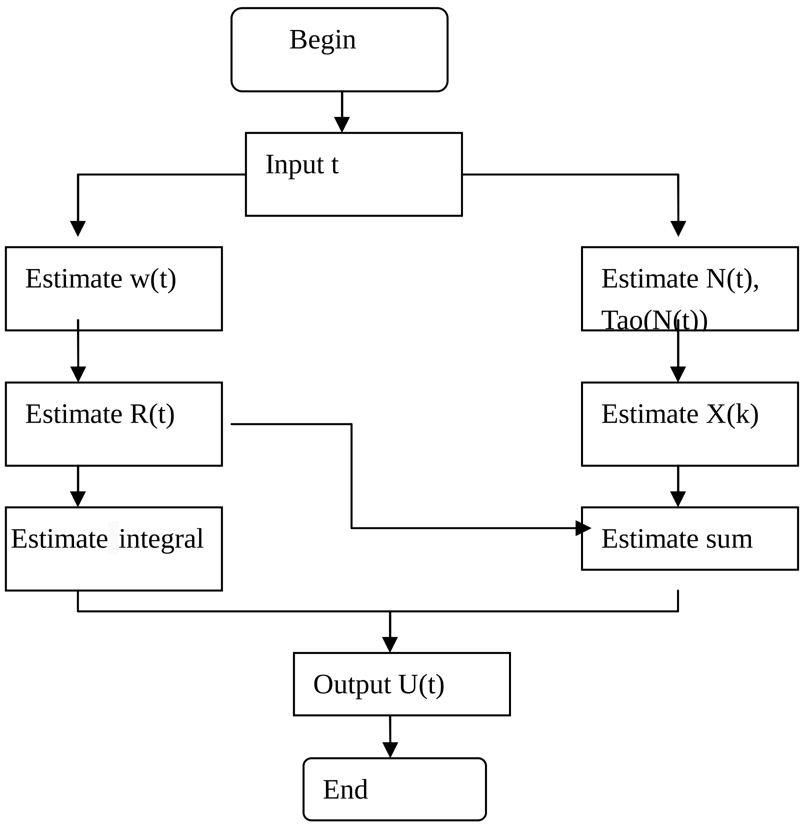

Figure 1: The algorithm flowchart of the renewal risk model

(1) In Estimating w(t) Module

Every time always begin from 0 and end at time t, the time interval is 0.01. Every moment generator one random follows normal distribution, and then accumulate the values to time t which is the value of Brownian motion w(t). Finally w(t) is standardized.

(2) In Estimating R(t) Module

$R(t) = \mu t + \sigma W(t)$, in which μ denotes average income of investment, and σ denotes variance of investment income. So the usual assumption is $\mu > 0$. The parameters μ, σ will directly affect the probability of ruin.

(3) In Estimating Integral Module

R(t) is estimated, which means the values of integrand in [0,t] are all known. So the integral value can be obtained by using trapezoidal integration or fast integration method.

(4) In Estimating N(t) and Tao N(t) Module

N(t) follows the Poisson distribution, but the moment of events can't be recorded if it directly generates a random which follows the Poisson distribution. In this paper, a random can be get which follows the similar Poisson distribution through exponential distribution in [0,t]. The specific process is that every time a random which follows the exponential distribution is obtained. if less than t, again producing a random, then summation, if less than t, repeat last step, otherwise end of the process. So N(t) is the number of random numbers minus one, Tao(N(t)) is the cumulative sum of random number.

(5) In Estimating X(k) Module

X(k) denotes the size of every claim amount which may follows different distributions. This paper mainly considers two types of distribution: lognormal distribution and weibull distribution. Different distribution will leads to different probability of ruin.

(6) In Estimating Summation Module

After obtaining N(t), R(Tao(k)) and X(k), the results can be achieved by cumulative sum.

4 Numerical Simulation Results

In order to observe the results of different parameter different distribution, firstly assume $X(k) \sim$ lognormal distribution.

a) $x = 1000$, $c = 50$, $\lambda = 0.5$, $\theta = 0.5$, $\mu = 0.5$, $\sigma = 1$, $W(t)$ is normal Brownian motion, $t = 10$, step length 0.1.

b) $x = 1000$, $c = 5$, $\lambda = 10$, $\theta = 10$, $\mu = 0.5$, $\sigma = 1$, $W(t)$ is normal Brownian motion, $t = 10$, step length 0.1.

c) $x=100000$, $c=50$, $\lambda=0.5$, $\theta=0.5$, $\mu=0.5$, $\sigma=1$, $W(t)$ is normal Brownian motion, $t=10$, step length 0.1.

d) $x=100000$, $c=5$, $\lambda=10$, $\theta=10$, $\mu=0.5$, $\sigma=1$, $W(t)$ is normal Brownian motion, $t=10$, step length 0.1.

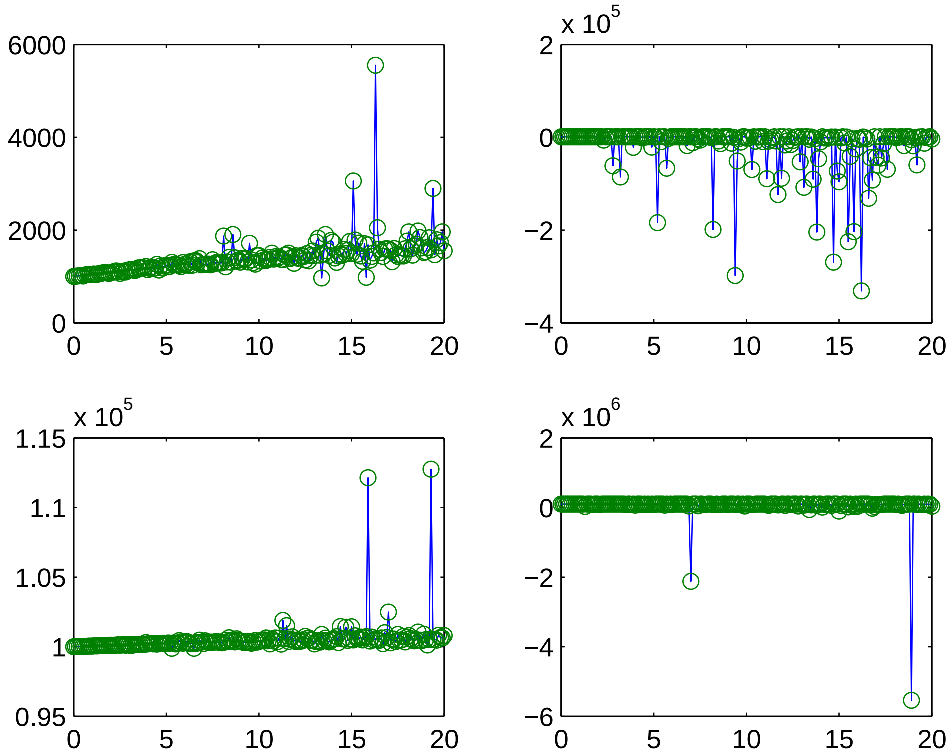

Figure 2: X(k) is the lognormal distribution

From the fig.2, it can be seen that when $c-\lambda\theta >> 0$, On average, don't lose money, or $c-\lambda\theta << 0$, it may be ruin. But fig. 1 only presents the one random process. In order to further discuss the relation between c and $\lambda\theta$, under the same condition, repeat 10000 times.

Table 1: Repeat 10000 times

	a)	b)	c)	d)
Ruin probability	0.6893	1	0	1

From table 1, if the initial wealth is too little and X(k) is the lognormal distribution, ruin probability relatively high without the relation between c and $\lambda\theta$.

Secondly assume $X(k)\sim$ weibull distribution.

a) $x=1000$, $c=50$, $\lambda=0.5$, $\theta=0.5$, $\mu=0.5$, $\sigma=1$, $W(t)$ is normal Brownian motion, $t=10$, step length 0.1.

b) $x = 1000$, $c = 5$, $\lambda = 10$, $\theta = 10$, $\mu = 0.5$, $\sigma = 1$, $W(t)$ is normal Brownian motion, $t = 10$, step length 0.1.

c) $x = 100000$, $c = 50$, $\lambda = 0.5$, $\theta = 0.5$, $\mu = 0.5$, $\sigma = 1$, $W(t)$ is normal Brownian motion, $t = 10$, step length 0.1.

d) $x = 100000$, $c = 5$, $\lambda = 10$, $\theta = 10$, $\mu = 0.5$, $\sigma = 1$, $W(t)$ is normal Brownian motion, $t = 10$, step length 0.1.

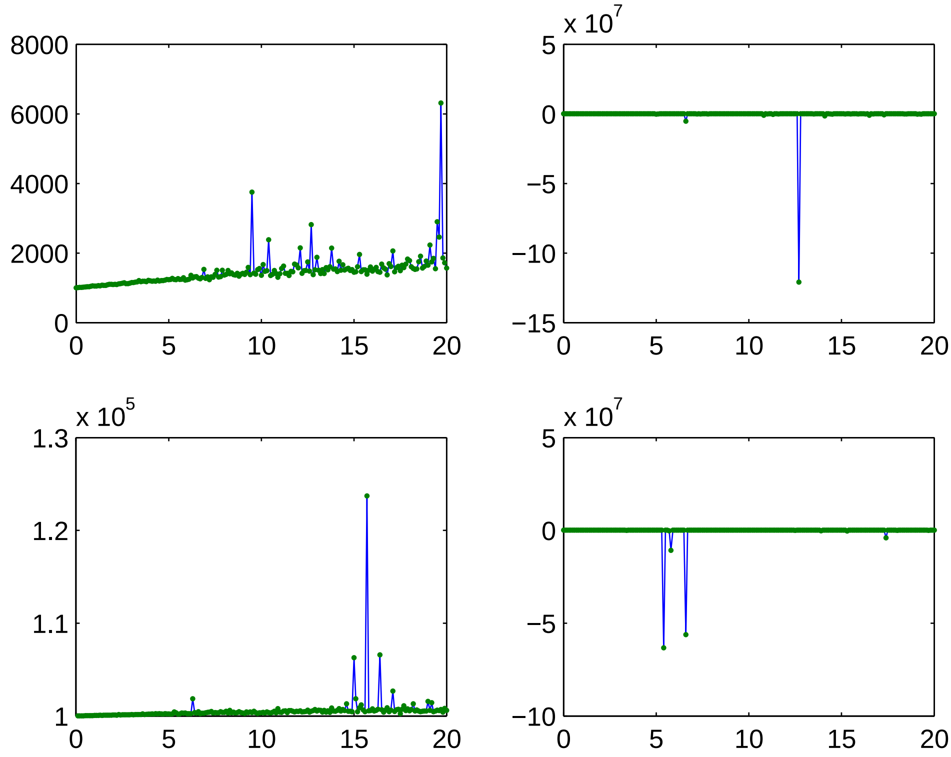

Figure 3: X(k) is weibull distribution

From the fig. 3, it can be seen that the results similar to the fig. 1. Similar, under the same condition, repeat 10000 times.

Table 2: Repeat 10000 times

	a)	b)	c)	d)
probability	0	1	0	1

Note that the results are not happened. In the literature, if $X(k)$ is heavy tailed and $c - \lambda\theta < 0$, the result ruin probability is one when $t \to \infty$. The numerical simulation and theory analysis are consistence. From another perspective, c can be simply think to receive money, λ is the mean of claims size, θ is the mean of claims times. When $c - \lambda\theta < 0$, in other words, the aggregate claims are greater than income, ruin is not avoid.

5 Conclusion

The stochastic present value of aggregate claims in the renewal risk model is widely used in mathematical finance. In the literature, most researchers were interested in theory analysis, but ignore numerical analysis. In this paper, we firstly construct a numerical simulation scheme for the stochastic present value of aggregate claims in the renewal risk model. Based on the numerical simulation scheme, we design a numerical simulation algorithm for it. Different parameter different distribution leads to different numerical simulation results. These results are all consistence with the theory analysis results.

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