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Extremally Disconnectedness in Ideal Bitopological Spaces

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Abstract

*The concept of (i,j)- *extremally disconnected ideal bitopological space have been introduced and studied in this paper.*

Keywords: *Ideal bitopological spaces, (i,j)- *extremally disconnected, (i,j)-extremally disconnected.*

1 Introduction

Pervin [24] introduced the concept of connectedness in bitopological spaces in 1967. And it was further studied by Birsan [2] in 1968, Reilly [27] in 1971 and by Ekici and Noiri [7] in 2008. Extremally disconnected topological spaces were studied by Gillman and Jerison [9] in 1960. Extremally disconnected spaces play an important role in Set-theoretical topology, Boolean algebra and Functional analysis. Extremally disconnectedness in bitopological spaces has been studied by Balasubramaniam [1] in 1991 and *- extremally disconnected ideal topological spaces were studied by Ekici & Noiri [8] in 2009. The purpose of this paper is to introduce and study (i,j)*- extremally disconnected ideal bitopological space.

Bitopological spaces were introduced by Kelly [16] in 1963 as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2. The concept of ideal topological spaces was initiated by Kuratowski [17] and Vaidyanathaswamy [29]. An Ideal **I** on a topological space (X, τ) is a non empty collection of subsets of X which satisfies: (i) A $\in$ **I** and B $\subset$ A then B $\in$ **I** and (ii) A $\in$ **I** and B $\in$ **I** then A $\cup$ B $\in$ **I**. If $\mathcal{P}$(X) is the set of all subsets of X, in a topological space (X, τ) a set operator $(.)^*:\mathcal{P}(X) \to \mathcal{P}(X)$ called the local function of A with respect to τ and **I** and is defined as $A^*(\tau, I) = \{x \in X \mid U \cap A \notin I, \forall\ U \in \tau(x)\}$, where $\tau(x) = U \in \tau \mid x \in U\}$ ($A^*(\tau, I)$ is written in short A^*). A Kuratowski closure operator Cl* (.) for a topology τ^* (τ, **I**), called the *-topology [12], and is defined by Cl*(A) = A $\cup$ A* (τ, **I**). (τ^* is written for τ^* (τ, **I**).

2 Preliminaries

Definition 2.1: *A subset A of a topological space (X, τ) is said to be*

i. *[18] semiopen if for some τ- open set O in X, O $\subseteq$ A $\subseteq$ Cl(O)*
   Also since int(A) is τ- open then A $\subseteq$ Cl(int(A))
ii. *[20] preopen open if A $\subseteq$ int(Cl(A)*
iii. *[21] strongly β- open if A $\subseteq$ Cl(int(Cl(A)))*
iv. *[24] α- open if A $\subseteq$ int(Cl(int(A)))*

Definition 2.2: *A subset A of an ideal topological (X, τ, **I**) space is said to be*

i. *[22] semi-**I** - open if for some τ- open set O in X, O $\subseteq$ A $\subseteq$ Cl*(O)*
   Also since int(A) is τ- open then A $\subseteq$ Cl(int(A))*
ii. *[6] pre-**I** - open if A $\subseteq$ int(Cl*(A)*
iii. *[10] strongly β-**I** - open if A $\subseteq$ Cl*(int(Cl*(A)))*
iv. *[11] α-**I** - open if A $\subseteq$ int(Cl*(int(A)))*

Definition 2.3: *A subset A of a bitopological space (X, τ_1, τ_2); i j = 1,2, i ≠ j is said to be*

i. *[19] (i,j)- semiopen; i, j = 1, 2, i ≠ j if for some τ_i- open set O in X, O $\subseteq$ A $\subseteq \tau_j$-Cl(O).*
   Also If int(A) is τ_i- open then A $\subseteq$ τ_j- Cl(τ_i- int(A))
ii. *[25] (i,j)- preopen; i, j = 1, 2, i ≠ j if A $\subseteq \tau_i$- int(τ_j- Cl(A)*
iii. *[14] (i,j)- β- open; i, j = 1, 2, i ≠ j if A $\subseteq \tau_j$- Cl(τ_i- int(τ_j - Cl(A)))*
iv. *[13] (i,j)- α- open; i, j = 1, 2, i ≠ j if A $\subseteq \tau_i$- int(τ_j- Cl(τ_i-int(A))*

Definition 2.4: *A subset A of an ideal bitopological space (X, τ_1, τ_2, **I**); i j = 1,2, i ≠ j is said to be*

i. [4] (i,j)-$\boldsymbol{I}$- semiopen if for some τ_i- open set O, $O \subseteq A \subseteq \tau_j^*$- Cl(O). Also int(A) is τ_i- open therefore $A \subseteq \tau_j^*$- Cl(τ_i- int(A))

ii. [3] (i,j)-$\boldsymbol{I}$- preopen if $A \subseteq \tau_i$- int(τ_j^*- Cl(A))

iii. [4] (i,j)-$\boldsymbol{I}$- α- open if $A \subseteq \tau_i$- int(τ_j^*- Cl(τ_i- int(A)))

Definition 2.5: [9] *A topological space (X, τ) is said to be extremally disconnected if for every open set A in X, Cl(A) is also open.*

Definition 2.6: [8] *An ideal topological space (X, τ, I) is said to be *- extremally disconnected if for every open set A in X, Cl*(A) is also open.*

Definition 2.7: [1] *A bitopological space (X, τ_1, τ_2) is said to be (i,j)- extremally disconnected if τ_j closure of each τ_i- open set is τ_i- open where; i,j = 1,2, $i \neq j$.*

Definition 2.8: [23] *A topological space (X, τ) is said to be normal if, for any two disjoint open sets A and B, there exist two disjoint closed sets U and V such that $A \subset U$, and $B \subset V$.*

Definition 2.9: [8] *An ideal topological space (X, τ, I) is said to be *- normal if for any two disjoint open set and *- open set A and B, respectively there exists disjoint *- closed and closed sets U and V such that $A \subset U$, and $B \subset V$.*

Definition 2.10: [28] *A bitopological space (X, τ_1, τ_2)); i,j = 1,2, $i \neq j$ is said to be binormal if for any two disjoint τ_i - open set and τ_j - open set A and B, respectively there exists disjoint τ_j- closed and τ_i- closed sets U and V such that $A \subset U$, and $B \subset V$.*

Definition 2.11: [8] *A subset A of an ideal topological space (X, τ, I) is said to be R-I- open if A = Int(Cl*(A)). The complement of a R-I- open set is R-I- closed.*

Definition 2.12: [8] *A subset A of an ideal topological space (X, τ, I) is said to be semi*-I- open if $A \subseteq$ Cl(int*(A)). The complement of a semi*-I- open set is semi*-I- closed.*

Definition 2.13: [15] *An ideal bitopological space is a quadruple (X, τ_1, τ_2, I) where I is an ideal defined on a bitopological space (X, τ_1, τ_2)*

Throughout this paper, τ_i- Cl(A) (resp. τ_j- Cl(A)) and τ_i- int(A) {resp. τ_j- int(A)} denote the closure and interior of a subset A of X with respect to topology τ_i (resp. τ_j}and τ_i^*- Cl(A) (resp. τ_j^*- Cl(A)) and τ_i^*- int(A) {resp. τ_j^*- int(A)} denote the closure and interior of a subset A of X with respect to *- topology τ_i^* (resp. τ_j^*)

3 (i,j)*- Extremally Disconnected Ideal Bitopological Spaces

Definition 3.1: *An ideal bitopological space (X, τ_1, τ_2, I) is said to be (i,j)*- extremally disconnected if τ_j^*- closure of each τ_i- open set is τ_i open for; i,j = 1,2, i ≠ j.*

Theorem 3.1: *In an ideal bitopological space (X, τ_1, τ_2, I) if A is a τ_i- open set and B is a τ_j^*- open set such that A ∩ B = ∅ then (τ_j^*- Cl(A)) ∩ (τ_i - Cl(B)) = ∅ if and only if the space is (i,j)*- extremally disconnected; i,j = 1,2, i ≠ j.*

Proof:

Necessary Part: Let A be any τ_i- open set and B = (X–(τ_j^*- Cl(A)). Obviously B is τ_j^* open and A ∩ B = ∅. Given (τ_j^*- Cl(A)) ∩ (τ_i - Cl(B)) = ∅, thus (X–(τ_i- Cl(B))) = (τ_j^*- Cl(A)). This implies (τ_j^*- Cl(A)) is τ_i- open. Similarly (τ_i^*- Cl(B)) is τ_j- open for a τ_j- open set B. Therefore (X, τ_1, τ_2, I) is (i,j)*- extremally disconnected.

Sufficient Part: Let (X, τ_1, τ_2, I) be (i,j)*- extremally disconnected. Let A be a τ_i- open set and B be a τ_j^*- open set s. t. A ∩ B = ∅. So A ⊆ X–B, then τ_j^*- Cl(A) ⊆ τ_j^*- Cl(X-B) = X-B (as B is τ_j^* open). Since X is (i,j)*- extremally disconnected τ_j^*- Cl(A) is τ_i open. Therefore τ_j^*- Cl(A) = τ_i- int(τ_j^*- Cl(A)) ⊆ τ_i - int(X-B) or τ_j^*- Cl(A) ⊆ X- (τ_i - Cl(B)) Hence, τ_j^*- Cl(A) ∩ τ_i - Cl(B) = ∅

Theorem 3.2: *For an ideal bitopological space (X, τ_1, τ_2, I) the following properties are equivalent:*

(a) *X is (i,j)*- extremally disconnected*

(b) *(τ_j^*- Cl(A)) ∩ (τ_i - Cl(B)) ⊂ τ_i- Cl(A ∩ B) where A is a τ_i- open set and B is a τ_j^*- open set*

(c) *(τ_j^*- Cl(τ_i - int(τ_j^*- Cl(A)))) ∩ τ_i- Cl(B) = ∅ where A is any subset of X and*

(d) *B a τ_j^*- open set with A ∩ B = ∅*

Proof:

(a) ⇒ ***(b)*** Let A be a τ_i- open set and B be a τ_j^*- open set. Since X is (i,j)*- extremally disconnected τ_j^*- Cl(A) is τ_i- open. Then
(τ_j^*- Cl(A)) ∩ (τ_i - Cl(B)) ⊂ τ_i - Cl((τ_j^*- Cl(A)) ∩ B)) ⊂ τ_i - Cl(τ_j^*- Cl(A ∩ B) ⊂ τ_i - Cl(A ∩ B)

(b)* $\Rightarrow$ *(c) Let A be any subset of X. B a τ_j^*- open set with $A \cap B = \emptyset$. Since τ_i- int(τ_j^*- Cl(A)) is τ_i- open and from ***(b)*** and the fact τ_i- int(τ_j^*- Cl(A)) $\cap B = \emptyset$ as $A \cap B = \emptyset$ we get

(τ_j^*- Cl(τ_i- int(τ_j^*- Cl(A)))) $\cap$ (τ_i - Cl(B)) $\subset$ τ_i- Cl(τ_i- int(τ_j^*- Cl(A)) $\cap$ B) $\subset$ τ_i- Cl($\emptyset$) = $\emptyset$

(c)* $\Rightarrow$ *(a) Let B be a τ_j^*- open set and A be any subset of X. Then τ_i- int(τ_j^*- Cl(A)) = U is τ_i- open. Given τ_j^*-Cl(U) $\cap$ τ_i- Cl(B) = $\emptyset$, $U \cap B = \emptyset$ Hence from theorem 3.1(necessary part) X is (i,j)*- extremally disconnected.

Definition 3: *A subset A of an ideal bitopological space (X, τ_1, τ_2, **I**) is said to be (i,j)-**I**- strongly β- open; i,j = 1,2, $i \neq j$ if $A \subseteq \tau_j^*$- Cl(τ_i- int(τ_j^*- Cl(A)))*

Theorem 3.3: For an ideal bitopological space (X, τ_1, τ_2, **I**) the following properties are equivalent:

(a) X is (i,j)*- extremally disconnected
(b) τ_j^*- int(A) is τ_i- closed for every τ_i - closed subset A of X
(c) τ_j^*- Cl(τ_i - int(A)) $\subseteq$ τ_i- int(τ_j^*- Cl(A)) for every subset A of X
(d) Every (i,j)-**I**- semiopen set is (i,j)-**I**- preopen
(e) The τ_j^*- closure of every strongly ((i,j)-**I**- β- open subset of X is τ_i- open
(f) Every (i,j)-**I**- strongly β- open set is (i,j)-**I**- preopen
(g) For every subset A of X, A is (i,j)-**I**- α- open if A is (i,j)-**I**- semiopen

Proof:

(a)* $\Rightarrow$ *(b) Let A be a τ_i- closed subset of X. then (X-A) is τ_i- open. Since X is (i,j)*- extremally disconnected τ_j^*- Cl(X-A) = (X-(τ_j^*- int(A)) is τ_i- open in X.
Therefore τ_j^*- int(A) is τ_i- closed in X.

(b)* $\Rightarrow$ *(c) Let A be any subset of X. Then (X-(τ_i- int(A)) is τ_i- closed in X and τ_j^*- int(X-(τ_i- int(A)) (by ***(b)***) is τ_i- closed in X. Thus τ_j^*- Cl(τ_i- int(A)) is τ_i- open in X. Hence τ_j^*- Cl(τ_i- int (A)) is a subset of τ_i- int(τ_j^*- Cl(A))

(c)* $\Rightarrow$ *(d) Let A be a (i,j)* semiopen set of X. Then A is a subset of τ_j^*- Cl(τ_i- int(A)) and by ***(c)*** $A \subseteq \tau_i$- int(τ_j^*- Cl(A)). Hence is (i,j)-**I**- preopen.

(d)* $\Rightarrow$ *(e) Let A be a (i,j)* strongly β-**I**- open subset of X. Then τ_j^*- Cl(A) is (i,j)-**I**- semiopen and by ***(d)*** is (i,j)-**I**- preopen. Thus τ_j^*- Cl(A) $\subseteq$ τ_i- int(τ_j^*- Cl(A)). Therefore τ_j^*- Cl(A) is τ_i- open.

(e)* ⇒ *(f) Let A be a (i,j)* strongly β-**I**- open subset of X. then by ***(e)*** τ_j^*- Cl(A) ⊆ τ_i- int(τ_j^*- Cl(A). Hence A is (i,j)-**I**- preopen.

(f)* ⇒ *(g) Let A be a (i,j)-**I**- semiopen set. Since every (i,j)-**I**- semiopen is (i,j)-**I**- strongly β- open, by ***(f)*** A is (i,j)-**I**- preopen. A is (i,j)-**I**- semiopen and (i,j)-**I**- preopen. Hence A is (i,j)-**I**- α- open.

(g)* ⇒ *(a) Let A be a τ_i- open set in X. Then, τ_j^*- Cl(A) is (i,j)-**I**- semiopen by ***(g)***, τ_j^*- Cl(A) is (i,j)-**I**- α- open. Therefore τ_j^*- Cl(A) is the subset of τ_i- int(τ_j^*- Cl(τ_i- int(τ_j^*- Cl(A)))) = τ_i- int(τ_j^*- Cl(A). Thus, τ_j^*- Cl(A) ⊆ τ_i- int(τ_j^*- Cl(A). Hence τ_j^*- Cl(A) is τ_i- open set in X. By definition of (i,j)*- extremally disconnectedness and the fact that τ_j^*- closure of A (which is a τ_i- open) is τ_i open we prove that X is (i,j)*- extremally disconnected.

Definition 3.3: *An ideal bitopological space (X, τ_1, τ_2, **I**) is called (i,j)-**I**- normal if for any τ_i- open set A and τ_j^*- open set B s.t A ∩ B = ∅ ∃ a τ_j^*- closed set M and τ_i- closed set N s.t. A ⊂ M and B ⊂ N and M ∩ N = ∅; i,j = 1,2, i ≠ j*

Theorem 3.2: *For an ideal bitopological space (X, τ_1, τ_2, **I**) the following properties are equivalent:*

*(a) X is (i,j)-**I**- normal*
(b) X is (i,j)- extremally disconnected*

Proof:

(a)* ⇒ *(b) Let (X, τ_1, τ_2, **I**) be (i,j)-**I**- normal and A be a τ_i- open set of X. Put (X- τ_j^*- Cl(A)) = B. (B is τ_j^*- open) then A ∩ B = ∅. Hence ∃ a τ_j^*- closed set M and τ_i- closed set N s.t. A ⊂ M and B ⊂ N and M ∩ N = ∅. Since τ_j^*- Cl(A) ⊂ τ_j^*- Cl(M) = M (because M is τ_j^*- closed) ⊆ (X-N) ⊂ (X-B) = τ_j^*- Cl(A). Hence M = τ_j^*- Cl(A). Also B ⊂ N ⊆ (X-M) = (X- τ_j^*-Cl(A)) = B. Therefore N = B. But N is τ_i- closed so τ_i- Cl(B) = N. Given M ∩ N = ∅ (From Theorem 3.1) X is (i,j)*- extremally disconnected.

(b)* ⇒ *(a) Let X be (i,j)*- extremally disconnected and let A be a τ_i- open set and B be a τ_j^*- open set s.t A ∩ B = ∅. Put M = τ_j^*- Cl(A) and N = τ_i- Cl(B) where B = (X-(τ_j^*- Cl(A))). Then M is a τ_j^* closed set and N is a τ_i closed set s.t. A ⊂ M and B ⊂ N. Clearly M ∩ N = ∅. Hence (X, τ_1, τ_2, **I**) is (i,j)-**I**- normal.

4 R-I- Open Sets

Definition 4.1: *A subset A of an ideal bitopological space (X, τ_1, τ_2, **I**) is said to be (i,j)-R-**I**- open if A = τ_i- Int(τ_j^*- Cl(A)); i,j = 1,2, i ≠ j. The complement of a (i,j)-R-**I**- open set is (i,j)-R-**I**- closed.*

Theorem 4.1: *For an ideal bitopological space (X, τ_1, τ_2, I) the following properties are equivalent:*

(a) *X is (i,j)*- extremally disconnected*
(b) *Every (i,j)-R-I- open subset of X is τ_j^*- closed in X*
(c) *Every (i,j)-R-I- closed subset of X is τ_j^*- open in X*

Proof:

(a) ⇒ *(b)* Let X be (i,j)*- extremally disconnected and A be a (i,j)-R-**I**- open subset of X. Then A = τ_i- Int(τ_j^*- Cl(A) thus, A is τ_i- open. X is (i,j)*- extremally disconnected therefore τ_j^*- Cl(A) is τ_i- open. In other words, A = τ_j^*- Cl(A), hence A is τ_j^*- closed in X.

(b)* ⇒ *(c) The result is obvious as the complement of a (i,j)-R-**I**- open set is (i,j)-R-**I**- closed and complement of a τ_j^*- closed in X is τ_j^*- open in X

(c)* ⇒ *(a) Given every (i,j)-R-**I**- closed subset of X is τ_j^*- open in X that is, every (i,j)-R-**I**- open subset of X is τ_j^*- closed in X. Let A be a τ_i- open subset of X. τ_i- Int(τ_j^*- Cl(A)) = A. Hence is (i,j)-R-**I**- open and thus τ_j^*- closed in X. Therefore τ_j^*- Cl **(**A) ⊆ A = τ_i- Int(τ_j^*- Cl(A)). Hence τ_j^*- Cl(A) is τ_i- open. τ_j^* of a τ_i- open set is τ_i- open .So X is (i,j)*- extremally disconnected.

Theorem 4.2: *For an ideal bitopological space (X, τ_1, τ_2, **I**) the following properties hold:*

(a) *If A, B are (i,j)-R-**I**- closed subsets of X, A ∩ B is also a (i,j)-R-**I**- closed subset of X.*
(b) *If A, B are (i,j)-R-**I**- open subsets of X, A ∪ B is also a (i,j)-R-**I**- open subset of X.*

Proof:

(a) Let X be (i,j)*- extremally disconnected and let A, B be (i,j)-R-**I**- closed subsets of X. Since A, B are τ_i- closed by theorem 3.2 ***(b)*** τ_j^*- int(A) and τ_j^*- int(B) is τ_i- closed. This implies

A ∩ B = (τ_i- Cl(τ_j^*- int(A))) ∩ (τ_i- Cl(τ_j^*- int(B))) =

$(\tau_j^*\text{- int}(A)) \cap (\tau_j^*\text{- int}(B)) = (\tau_j^*\text{- int}(A \cap B)) \subseteq \tau_i\text{- Cl}(\tau_j^*\text{- int}(A \cap B))$
Also,
$\tau_i\text{- Cl}(\tau_j^*\text{- int}(A \cap B)) = \tau_i\text{- Cl}((\tau_j^*\text{- int}(A) \cap (\tau_j^*\text{- int}(B))$
$\subseteq (\tau_i\text{- Cl}(\tau_j^*\text{- int}(A))) \cap (\tau_i\text{- Cl}(\tau_j^*\text{- int}(B))) = A \cap B$
Hence,
$\tau_i\text{- Cl}(\tau_j^*\text{- int}(A \cap B)) = A \cap B$

Therefore A ∩ B is a (i,j)-R-**I**- closed subset of X.

(b) If A, B are (i,j)-R-**I**- open subsets of X, A^c and B^c are (i,j)-R-**I**- closed subsets of X therefore from ***(a)*** we get $A^c \cap B^c = (A \cup B)^c$ is also a (i,j)-R-**I**- closed subset of X. Therefore A ∪ B is a (i,j)-R-**I**- open subset of X.

Theorem 4.3: *For an ideal bitopological space (X, τ_1, τ_2, **I**) the following properties are equivalent:*

(a) *X is (i,j)*- extremally disconnected*
(b) *The τ_j^*- closure of every (i,j)-**I**- semiopen subset of X is τ_i- open*
(c) *The τ_j^*- closure of every (i,j)-**I**- preopen subset of X is τ_i- open*
(d) *The τ_j^*- closure of every (i,j)-R-**I**- open subset of X is τ_i- open*

Proof:

(a)* ⇒ *(b) Let X be (i,j)*- extremally disconnected and let A be a (i,j)-***I***- semiopen subset of X. hence from Theorem 3.3, A is strongly ((i,j)-***I***- β- open and τ_j^*- closure of every strongly ((i,j)-***I***- β- open subset of X is τ_i- open (by theorem 3.3***(e)***), therefore τ_j^*- closure of every (i,j)-***I***- semiopen subset of X is τ_i- open.

(b)* ⇒ *(c) Let A be (i,j)-***I***- semiopen then A is (i,j)-***I***- preopen. (by theorem 3.3***(d)***). Thus the result follows and we get τ_j^*- closure of every (i,j)-***I***- preopen is τ_i- open.

(c)* ⇒ *(d) Let A be (i,j)-R-***I***- open then $A = \tau_i\text{- int}(\tau_j^*\text{- Cl}(A))$, therefore $A \subseteq (\tau_i\text{- int}(\tau_j^*\text{- Cl}(A)))$ thus A is (i,j)-***I***- preopen. Obviously, τ_j^*- closure of every (i,j)-R-***I***- open subset of X is τ_i- open.

(d)* ⇒ *(a) Let A be a τ_i- open subset of X. Then, $\tau_i\text{- int}(\tau_j^*\text{- Cl}(A))$ is (i,j)-R-***I***- open. Given τ_j^*- closure of every (i,j)-R-***I***- open is τ_i- open. Thus $\tau_j^*\text{- Cl}(\tau_i\text{- int}(\tau_j^*\text{- Cl}(A)))$ is τ_i- open.

$\tau_j^*\text{- Cl}(A) \subset \tau_j^*\text{- Cl}(\tau_i\text{- int}(\tau_j^*\text{- Cl}(A))) = \tau_i\text{- int}(\tau_j^*\text{- Cl}(\tau_i\text{- int}(\tau_j^*\text{- Cl}(A)))) = \tau_i\text{- int}(\tau_j^*\text{- Cl}(A))$

Hence τ_j^*- Cl(A)) is τ_i- open. Thus X is (i,j)*- extremally disconnected.

5 (i,j)-I- Semi*- Open

Definition 5.1: *A subset A of an ideal bitopological space (X, τ_1, τ_2, **I**) is said to be (i,j)-**I**- semi*- open if A ⊆ τ_i- Cl(τ_j^*- int(A)); i,j = 1,2, i ≠ j. Complement of a (i,j)-**I**- semi*- open set is (i,j)-**I**- semi*- closed.*

Theorem 5.1: *A subset A of an ideal bitopological space (X, τ_1, τ_2, **I**) is (i,j)-**I**- semi*- open if and only if τ_i- Cl(A) = τ_i- Cl(τ_j^*- int(A))*

Proof:

Necessary Part: Let A be (i,j)-**I**- semi*- open thus A ⊆ τ_i- Cl(τ_j^*- int(A)). Therefore we have τ_i- Cl(A) ⊆ τ_i- Cl(τ_j^*- int(A)) but, τ_i- Cl(τ_j^*- int(A)) ⊆ τ_i- Cl(A). Thus τ_i- Cl(A) = τ_i- Cl(τ_j^*- int(A))

Sufficient Part: Let τ_i- Cl(A) = τ_i- Cl(τ_j^*- int(A)). But A ⊆ τ_i- Cl(A) hence A is (i,j)-**I**- semi*- open.

Theorem 5.2: *For an ideal bitopological space (X, τ_1, τ_2, **I**) the following properties are equivalent:*

(a) *X is (i,j)*- extremally disconnected*
(b) *If A is (i,j)-**I**- strongly β- open and B is (i,j)-**I**- semi*- open then, (τ_j^*- Cl(A)) ∩ (τ_i- Cl(B)) ⊆ τ_i- Cl(A ∩ B)*
(c) *If A is (i,j)-**I**- semiopen and B is (i,j)-**I**- semi*- open then,(τ_j^*- Cl(A)) ∩ (τ_i- Cl(B)) ⊆ τ_i- Cl(A ∩ B)*
(d) *τ_j^*- Cl(A) ∩ τ_i- Cl(B) = ∅ for every (i,j)-**I**- semiopen set A and (i,j)-**I**- semi*- open set B with A ∩ B = ∅*
(e) *If A is (i,j)-**I**- preopen and B is (i,j)-**I**- semi*- open then, (τ_j^*- Cl(A)) ∩ (τ_i- Cl(B)) ⊆ τ_i- Cl(A ∩ B)*

Proof:

(a) ⇒ ***(b)*** Let A be (i,j)-**I**- strongly β- open hence from Theorem 3.3***(e)*** τ_j^*- Cl(A) is τ_i- open. Also let B be a (i,j)-**I**- semi*- open thus B ⊆ τ_i- Cl(τ_j^*- int(B)). Therefore,

(τ_j^*- Cl(A)) ∩ (τ_i- Cl(B)) ⊆ (τ_j^*- Cl(A)) ∩ (τ_i- Cl(τ_i- Cl(τ_j^*- int(B))))

= (τ_j^*- Cl(A)) ∩ (τ_i- Cl(τ_j^*- int(B))) = τ_i- Cl((τ_j^*- Cl(A)) ∩ (τ_j^*- int(B)))

$\subseteq \boldsymbol{\tau}_{\mathbf{i}}$- Cl(A ∩ ($\boldsymbol{\tau}_j^*$- int(B)) $\subseteq \boldsymbol{\tau}_{\mathbf{i}}$- Cl(A ∩ B)

Hence we have ($\boldsymbol{\tau}_j^*$- Cl(A)) ∩ ($\boldsymbol{\tau}_{\mathbf{i}}$- Cl(B)) $\subseteq \boldsymbol{\tau}_{\mathbf{i}}$- Cl(A ∩ B)

(b) ⇒ ***(c)*** Let A be (i,j)-**I**- semiopen this implies A is (i,j)-**I**- strongly β- open (by Theorem 3.3). Also let B be (i,j)-**I**- semi*- open. Hence from above, ($\boldsymbol{\tau}_j^*$- Cl(A)) ∩ ($\boldsymbol{\tau}_{\mathbf{i}}$- Cl(B)) $\subseteq \boldsymbol{\tau}_{\mathbf{i}}$- Cl(A ∩ B)

(c) ⇒ ***(d)*** Let A be (i,j)-**I**- semiopen and B be (i,j)-**I**- semi*- open with A ∩ B = ∅. From the above result it is obvious that $\boldsymbol{\tau}_j^*$- Cl(A) ∩ $\boldsymbol{\tau}_{\mathbf{i}}$- Cl(B) = ∅.

(d) ⇒ ***(e)*** Let A be (i,j)-**I**- preopen and let B be a (i,j)-**I**- semi*- open. The result follows from Theorem 3.3***(d)***

(e) ⇒ ***(a)*** Let A and B be $\boldsymbol{\tau}_{\mathbf{i}}$- open and τ_j^*- open respectively with A ∩ B = ∅. Obviously A is (i,j)-**I**- preopen and B is (i,j)-**I**- semi*- open. So from ***(e)*** we get ($\boldsymbol{\tau}_j^*$- Cl(A)) ∩ ($\boldsymbol{\tau}_{\mathbf{i}}$- Cl(B)) $\subseteq \boldsymbol{\tau}_{\mathbf{i}}$- Cl(A ∩ B) = ∅. This implies (by Theorem 3.1) X is (i,j)*- extremally disconnected.

References

[1] G. Balasubramanian, Extremally disconnectedness in bitopological spaces, *Bull. Cal. Math. Soc.*, 83(1991), 247-252.

[2] T. Birsan, Surles espaces bitopologiques connexes, *An. Sti. Univ. Al. Cuza Iasi. N Sect. I XIV Face*, 2(1968), 293-96.

[3] M. Caldas, S. Jafari and N. Rajesh, Preopen sets in ideal bitopological spaces, *Bol. Soc. Paran. Mat.*, 3(29) (2) (2011), 61-68.

[4] M. Caldas, S. Jafari and N. Rajesh, Some fundamental properties of β-open sets in ideal bitopological spaces, *European J. of Pure and Applied Mathematics*, 6(2) (2013), 247-255.

[5] M.C. Datta, Contributions to the theory of bitopological spaces, *Ph.D. Thesis*, (1971), B.I.T.S. Pilani, India.

[6] J. Dontchev, Idealization of Ganster-Reilly decomposition theorems, *arXiv:math/9901017v1* [*math.GN*], (1999), 1-13.

[7] E. Ekici and T. Noiri, Connectedness in ideal topological spaces, *Novi Sad J. Math.*, 38(2) (2008), 65-70.

[8] E. Ekici and T. Noiri, *-Extremally disconnected ideal topological spaces, *Acta Math. Hunger*, 122(1-2) (2009), 81-90.

[9] L. Gillman and M. Jerison, Rings of continuous functions, *Univ. Series in Higher Maths*, Van Nostrand, Princeton N.J., (1960).

[10] E. Hatir, A. Keskin and T. Noiri, A note on strong β-I- sets and strongly β-I- continuous functions, *Acta Math. Hungar*, 108(1-2) (2005), 87-94.

[11] E. Hatir and T. Noiri, On decomposition of continuity via idealization, *Acta Math. Hungar,* 96(2002), 341-349.

[12] D. Jankovic and T.R. Hamlett, New topologies from old via ideals, *Amer.*

Math. Monthly, 97(4) (1990), 295-310.
[13] M. Jelic, Feeble P-continuious mappings, *Rend. Circ. Math. Palermo*, 24(1990), 387-395.
[14] F.H. Khedr and S.M. Al-Areefi, Precontinuity and semi-pre-continuity in bitopological spaces, *Indian. J. Pure. Appl. Math.*, 23(9) (1992), 625-633.
[15] M. Kar and S.S. Thakur, Pair-wise open sets in ideal bitopological spaces, *Int. J of Mathematical Sciences and Applications*, 2(2) (2012), 839-842.
[16] J.C. Kelly, Bitopological spaces, *Proc. London Math. Soc.*, 13(1963), 71-89.
[17] K. Kuratowski, *Topology (Vol. 1)*, Academic Press, New York, (1966).
[18] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.
[19] S.N. Maheswari and R. Prasad, Semi open sets and semi continuous functions in bitopological spaces, *Math Notae*, 26(1978), 29-37.
[20] A.S. Mashhour, M.E.A. El. Monsef and S.N. El. Deeb, On precontinuous and weak precontinuous mappings, *Proc. Math and Phys. Soc.*, Egypt, 53(1982), 47-53.
[21] M.E.A. El. Monsef, S.N. El. Deeb and R.A. Mahmoud, β- open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77-90.
[22] M.E.A. El. Monsef, E.F. Lashien and A.A. Nasef, On I-open sets and I-continuous functions, *Kyungpook Math. J.*, 32(1) (1992), 21-30.
[23] J.R. Munkers, *Topology (Second Edition)*, Pearson Education Asia, (2000).
[24] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 15(1965), 961-970.
[25] T. Noiri, Hyperconnetedness and preopen sets, *Rev. Roumania Math. Pure Appl.*, 29(1984), 329-334.
[26] W.J. Pervin, Connectedness in bitopological spaces, *Indag. Math.*, 29(1967), 369-372.
[27] I.L. Reilly, On pairwise connected bitopological spaces, *Kyungpook Math J.*, 11(1971), 25-28.
[28] I.L. Reilly, On bitopological separation properties, *Nanta Math.*, V(2) (1972), 14-25.
[29] R. Vaidyanathaswamy, The localization theory in set topology, *Proc. Indian Acad. Sci.*, 20(1945), 51-61.