*Gen. Math. Notes, Vol. 3, No. 1, March 2011, pp.73-85*
*ISSN 2219-7184;* 
*www.i-csrs.org*
*Available free online at http://www.geman.in*

# Topological Mappings via $\widetilde{g}_\alpha$-Sets

**M. Lellis Thivagar[1] and Nirmala Rebecca Paul[2]**

[1]Department of Mathematics, Arul Anandar College,
Karumathur, Madurai(Dt.)-625514,
Tamilnadu, India
E-mail: mlthivagar@yahoo.co.in
[2]Department of Mathematics, Lady Doak College
Madurai,625002,Tamilnadu, India
E-mail: nimmirebecca@yahoo.com

(Received: 30-11-10/ Accepted: 7-12-10)

**Abstract**

*The aim of this paper is to introduce the different notions of $\widetilde{g}_\alpha$-closed maps and study some of their basic properties.Strongly $\widetilde{g}_\alpha$-closed maps have been defined to find the relationship with $\widetilde{g}_\alpha$-closed maps and as an application $\widetilde{g}_\alpha$-regular spaces have been defined to study its properties in terms of $\widetilde{g}_\alpha$-closed maps.*

**Keywords:** $^{\#}$*gs-closed sets,*$^{\#}$*gs-open sets ,*$\widetilde{g}_\alpha$*-closed and* $\widetilde{g}_\alpha$*-open sets.*

## 1 Introduction

Levine[7] offered a new and useful notion in General Topology, that is the notion of a generalized closed set. This notion has been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and intersting concepts, e.g. new covering properties and new separation axioms weaker than $T_1$. Some of these separation axioms have been found to be useful in computer science and digital topology. After the introduction of generalized closed sets there are many research papers which deal with different types of generalized closed sets. Devi et al.[2,1] have introduced semi-generalised closed sets (briefly sg-closed), generalised semi-closed sets (briefly gs-closed), generalised $\alpha$-closed (briefly g$\alpha$-closed) sets and $\alpha$-generalised closed (briefly $\alpha$g-closed) sets respectively. Jafari et al.[3] have introduced $\widetilde{g}_\alpha$-closed sets and studied their properties using $^{\#}$gs-open sets[10]. In this paper we have introduced $\widetilde{g}_\alpha$-closed maps. Using these new types of functions, several chracterizations and its properties have been obtained. Also the relationship between $\widetilde{g}_\alpha$-closed maps and strongly $\widetilde{g}_\alpha$-closed maps have been established. We have introduced $\widetilde{g}_\alpha$-regular space and studied its properties intrems of $\widetilde{g}_\alpha$-closed maps.

## 2 Preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset $A$ of $(X,\tau)$ are denoted by $Int(A)$ and $Cl(A)$, respectively. Throughout the present paper $(X,\tau)$ and $(Y,\sigma)$(or $X$ and $Y$) represent non-empty topological spaces on which no separation axiom is defined, unless otherwise mentioned.

**Definition 2.1** *A subset $A$ of a topological space $(X,\tau)$ is called*

*(i) a generalized closed set (briefly g-closed) [7] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X,\tau)$),*

*(ii) an $\omega$-closed set [8] ($= \widehat{g} -$ closed ) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X,\tau)$,*

*(iii) a *g-closed set [9] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\omega$-open in $(X,\tau)$,*

*(iv) a $^{\#}$g-semi-closed set(briefly $^{\#}$gs-closed)[10] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is *g-open in $(X,\tau)$ and*

*(v) a $\widetilde{g}_\alpha$-closed[3] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $^{\#}$gs-open in X.*

*The complement of g-closed(resp *g-closed,$\omega$-closed,$^{\#}$gs-closed and $\widetilde{g}_\alpha$-closed)set is said to be g-open(resp *g-open,$\omega$-open,$^{\#}$gs-open and $\widetilde{g}_\alpha$-open)respectively.*

**Definition 2.2** *A function $f : (X,\tau) \to (Y,\sigma)$ is called*

*(i) g-continuous [7] if $f^{-1}(V)$ is a g-closed set of $(X,\tau)$ for each closed set $V$ of $(Y,\sigma)$,*

*(ii) $\omega$-continuous [8] if $f^{-1}(V)$ is an $\omega$-closed subset of $(X,\tau)$ for every closed set $V$ of $(Y,\sigma)$,*

*(iii)$\widetilde{g}_\alpha$-continuous[4] if $f^{-1}(V)$ is a $\widetilde{g}_\alpha$-closed subset of $(X,\tau)$ for every closed set $V$ of $(Y,\sigma)$ and*

*(iv) strongly $\widetilde{g}_\alpha$-continuous[5] if $f^{-1}(V)$ is a closed subset of $(X,\tau)$ for every $\widetilde{g}_\alpha$-closed set $V$ of $(Y,\sigma)$*

**Definition 2.3** *Let $f:(X,\tau)\to(Y,\sigma)$ be a map. Then f is said to be*

*(i) $^{\#}$g-semi-irresolute [10] (briefly $^{\#}$gs-irresolute) if $f^{-1}(V)$ is $^{\#}$gs-closed in $(X,\tau)$ for each $^{\#}$gs-closed set $V$ of $(Y,\sigma)$*

*(ii) $\alpha$-irrsolute[1] if $f^{-1}(V)$ is $\alpha$-open in $(X,\tau)$ for each $\alpha$-open set $V$ of $(Y,\sigma)$*

*(iii)$\widetilde{g}_\alpha$-irresolute[5] if $f^{-1}(V)$ is $\widetilde{g}_\alpha$-closed in $(X,\tau)$ for each $\widetilde{g}_\alpha$-closed set $V$ of $(Y,\sigma)$*

**Definition 2.4** *Let $f:(X,\tau)\to(Y,\sigma)$ be a map then f is said to be pre-$\alpha$-closed[1] if the image of every $\alpha$-closed set in $(X,\tau)$ is $\alpha$-closed in $(Y,\sigma)$.*

**Definition 2.5** *A space $(X,\tau)$ is called a*

*(i)$^{\#}T_{\widetilde{g}_\alpha}$-space[3] if every $\widetilde{g}_\alpha$-closed set in it is a closed set,*

*(ii)$T_{1/2}$-space [7] if every g-closed set in it is a closed set and*

*(iii)$T_\omega$-space[8] if every $\omega$-closed set in it is a closed set.*

**Definition 2.6** *For a subset A of $(X,\tau)$ the $\widetilde{g}_\alpha$-closure of A is defined to be the intersection of all $\widetilde{g}_\alpha$-closed sets containing A and the $\widetilde{g}_\alpha$-interior is defined to be the union of all $\widetilde{g}_\alpha$-open sets contained in A. The class of all $\widetilde{g}_\alpha$-open sets is denoted by $\widetilde{G}_\alpha O(X)$ and the class of all $\widetilde{g}_\alpha$-closed sets is denoted by $\widetilde{G}_\alpha C(X)$.*

# 3 $\widetilde{g}_\alpha$-closed Maps

**Definition 3.1** *A map $f:(X,\tau)\to(Y,\sigma)$ is said to be $\widetilde{g}_\alpha$-closed if the image of every closed set in $(X,\tau)$ is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$.*

**Example 3.2** *Let $X=Y=\{a,b,c\},\tau=\{\phi,X,\{a\}\},\sigma=\{\phi,Y,\{a,c\}\}$ $\widetilde{G}_\alpha C(Y)=\{\phi,X,\{b\},\{a,b\},\{b,c\}\}$, The function f is defined as $f(a)=a,f(b)=c,f(c)=b$. The function f is a $\widetilde{g}_\alpha$-closed map.*

**Proposition 3.3** *A map $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-closed if and only if $\widetilde{g}_\alpha$-$Cl(f(A))\subseteq f(Cl(A))$ for every subset A of $(X,\tau)$.*

**Proof.** If f is a $\widetilde{g}_\alpha$-closed map and $A\subseteq X$.Then f(Cl(A)) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$. $f(A)\subseteq f(Cl(A))$.
$\widetilde{g}_\alpha$-$Clf(A)\subseteq\widetilde{g}_\alpha$-$Cl(f(Cl(A)))=f(Cl(A))$.Conversely let A be any closed set in $(X,\tau)$.Then $A=Cl(A)$ and $f(A)=f(Cl(A))$ and by hypothesis $\widetilde{g}_\alpha$-$Cl(f(A))\subseteq f(Cl(A))=f(A)$,also $f(A)\subseteq\widetilde{g}_\alpha$-$Cl(f(A))$ and hence $f(A)=\widetilde{g}_\alpha$-$Cl(f(A))$ Therefore f(A) is $\widetilde{g}_\alpha$-closed.

**Theorem 3.4** *A map $f : (X,\tau) \to (Y,\sigma)$ is $\widetilde{g}_\alpha$-closed if and only if for each subset S of $(Y,\sigma)$ and for each open set U containing $f^{-1}(S)$ there is a $\widetilde{g}_\alpha$-open set V of $(Y,\sigma)$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.*

**Proof.**
Suppose that f is a $\widetilde{g}_\alpha$-closed map. Let $S \subseteq Y$ and U be an open subset of $(X,\tau)$ such that $f^{-1}(S) \subseteq U$. Then $V = (f(U^c))^c$ is a $\widetilde{g}_\alpha$-open set containing S such that $f^{-1}(V) \subseteq U$. Conversely, let S be a closed set of $(X,\tau)$. Then $f^{-1}((f(S))^c \subseteq S^c$ and $S^c$ is open. By assumption, there exists a $\widetilde{g}_\alpha$-open set V of $(Y,\sigma)$ such that $(f(S))^c \subseteq V$ and $f^{-1}(V) \subseteq S^c$ and so $S \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(S) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(S) = V^c$. Since $V^c$ is $\widetilde{g}_\alpha$-closed, f(S) is $\widetilde{g}_\alpha$-closed and therefore f is $\widetilde{g}_\alpha$-closed.

**Proposition 3.5** *Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping such that $\widetilde{g}_\alpha$-$Cl(f(A)) \subseteq f(Cl(A))$ for every subset A of $(X,\tau)$.Then the image f(A) of a closed set A in $(X,\tau)$ is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$.*

**Proof.** Let A be a closed set in $(X,\tau)$ By hypothesis $\widetilde{g}_\alpha$-$Cl(f(A)) \subseteq f(Cl(A)) = f(A)$ and $f(A) \subseteq \widetilde{g}_\alpha$-$Cl(f(A))$ so $\widetilde{g}_\alpha$-$Cl(f(A)) = f(A)$. Therefore f(A) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$.

**Proposition 3.6** *If $f : (X,\tau) \to (Y,\sigma)$ is $^{\#}$gs-irresolute and pre-$\alpha$-closed and A is a $\widetilde{g}_\alpha$-closed subset of $(X,\tau)$ then f(A) is $\widetilde{g}_\alpha$-closed.*

**Proof.** Let U be any $^{\#}gs$-open set in $(Y,\sigma)$ such that $f(A) \subseteq U$ and since f is $^{\#}$gs-irresolute, $f^{-1}(U)$ is $^{\#}$gs-open in $(X,\tau)$. Hence $\alpha Cl(A) \subseteq f^{-1}(U)$ as A is $\widetilde{g}_\alpha$-closed in $(X,\tau)$. Since f is pre-$\alpha$-closed $f(\alpha Cl(A))$ is a $\alpha$-closed set contained in the $^{\#}$gs-open set U which gives $\alpha Cl(f(\alpha Cl(A))) \subseteq U$ and we have $A \subseteq \alpha Cl(A)$. Therefore $f(A) \subseteq f(\alpha Cl(A))$, $\alpha Cl(f(A)) \subseteq \alpha Cl f(\alpha Cl(A) \subseteq U$. Therefore f(A) is $\widetilde{g}_\alpha$-closed.

**Remark 3.7** *The composition of two $\widetilde{g}_\alpha$-closed functions need not be $\widetilde{g}_\alpha$-closed.*

**Example 3.8** *Let $X = Y = Z = \{a,b,c\}, \tau = \{\phi, X, \{a\}\}, \sigma = \{\phi, Y, \{a,b\}\}$
$\eta = \{\phi, Z, \{b\}\}$,
$\widetilde{G}_\alpha C(X) = \{\phi, X, \{b\}, \{c\}, \{b,c\}\}$,
$\widetilde{G}_\alpha C(Y) = \{\phi, Y, \{c\}, \{a,c\}, \{b,c\}\}$,
$\widetilde{G}_\alpha C(Z) = \{\phi, Z, \{a\}, \{c\}, \{a,c\}\}$,. The function f and g are defined as $f(a) = a, f(b) = c, f(c) = b$,$g(c) = a, g(b) = b, g(a) = c$,f and g are $\widetilde{g}_\alpha$-closed. But $(gof)(\{b,c\}) = g(f(\{b,c\}) = \{a,b\}$ which is not $\widetilde{g}_\alpha$-closed in Z.*

**Corollary 3.9** *If $f : (X,\tau) \to (Y,\sigma)$ is a $\widetilde{g}_\alpha$-closed map and $g : (Y,\sigma) \to (Z,\eta)$ is a pre-$\alpha$-closed and $^{\#}$gs-irresolute map then their composition $gof : (X,\tau) \to (Z,\eta)$ is $\widetilde{g}_\alpha$-closed.*

**Proof.** Let A be a closed set in $(X,\tau)$. Then f(A) is $\widetilde{g}_\alpha$-closed set in $(Y,\sigma)$. Since g is both $^{\#}gs$-irresolute and pre$\alpha$-closed,then by Proposition 3.6 $g(f(A)) = (gof)(A)$ is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$ and therefore gof is $\widetilde{g}_\alpha$-closed.

**Proposition 3.10** *If $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ are $\widetilde{g}_\alpha$-closed maps and $(Y,\sigma)$ be a $^{\#}T_{\widetilde{g}_\alpha}$-space then their composition $gof : (X,\tau) \to (Z,\eta)$ is $\widetilde{g}_\alpha$-closed.*

**Proof.** Let A be a closed subset of $(X,\tau)$. Then by hypothesis f(A) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$. Since $(Y,\sigma)$ is a $^{\#}T_{\widetilde{g}_\alpha}$-space f(A) is closed in $(Y,\sigma)$. Again by hypothesis g(f(A)) is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$. Thus $g(f(A)) = (gof)(A)$ is $\widetilde{g}_\alpha$-closed.

**Proposition 3.11** *If $f : (X,\tau) \to (Y,\sigma)$ is a closed map and $g : (Y,\sigma) \to (Z,\eta)$ is a $\widetilde{g}_\alpha$-closed map then their composition $gof : (X,\tau) \to (Z,\eta)$ is $\widetilde{g}_\alpha$-closed.*

**Proof.** Let A be a closed set in $(X,\tau)$ then f(A) is closed in $(Y,\sigma)$.Then $(gof)(A) = g(f(A))$ is $\widetilde{g}_\alpha$-closed.

**Remark 3.12** *If $f : (X,\tau) \to (Y,\sigma)$ is a $\widetilde{g}_\alpha$-closed and $g : (Y,\sigma) \to (Z,\eta)$ is a closed map then their composition $gof : (X,\tau) \to (Z,\eta)$ need not be $\widetilde{g}_\alpha$-closed.*

**Example 3.13** *Let $X = Y = Z = \{a,b,c\}, \tau = \{\phi, X, \{b\}\}, \sigma = \{\phi, Y, \{b,c\}\}$ $\eta = \{\phi, Z, \{a\}, \{a,b\}\}, \widetilde{G}_\alpha C(Y) = \{\phi, Y, \{a\}, \{a,b\}, \{a,c\}\}, \widetilde{G}_\alpha C(Z) = \{\phi, Z, \{b\}, \{c\}, \{b,c\}\},$. The function f and g are defined as $f(a) = c, f(b) = b, f(c) = a$ and $g(a) = c, g(c) = a, g(b) = b$. $(gof)(\{b,c\}) = g(f(\{a,c\}) = \{a,c\}$ which is not $\widetilde{g}_\alpha$-closed in Z.*

**Theorem 3.14** *Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ are two mappings such that their composition $gof : (X,\tau) \to (Z,\eta)$ be a $\widetilde{g}_\alpha$-closed mapping.Then the following are true.*

*(i)If f is continuous and surjective then g is $\widetilde{g}_\alpha$-closed.*

*(ii)If g is $\widetilde{g}_\alpha$-irresolute and injective then f is $\widetilde{g}_\alpha$-closed.*

*(iii)If f is g-continuous ,surjective and $(X,\tau)$ is a $T_{1/2}$-space then g is $\widetilde{g}_\alpha$-closed.*

*(iv)If f is $\omega$-continuous,surjective and $(X,\tau)$ is a $T_\omega$-space then g is $\widetilde{g}_\alpha$-closed.*

*(v)If g is strongly $\widetilde{g}_\alpha$-continuous and injective then f is closed .*

**Proof.**

(i) Let A be a closed set in $(Y,\sigma)$.Since f is continuous, $f^{-1}(A)$ is closed in $(X,\tau)$ and $(gof)f^{-1}(A)$ is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$i.e $g(A)$ is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$.(since f is surjective). Therefore $g$ is a $\widetilde{g}_\alpha$-closed map.

(ii)Let B be a closed set of $(X,\tau)$. Since $(gof)$ is $\widetilde{g}_\alpha$-closed,(gof)(B) is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$. Since $g$ is $\widetilde{g}_\alpha$-irresolute ,$g^{-1}(gof)(B)$ is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$.i.e f(B) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$(since g is injective). Thus f is a $\widetilde{g}_\alpha$-closed map.

(iii) Let C be a closed set of $(Y,\sigma)$. Since f is $g$-continuous $f^{-1}(C)$ is g-closed in $(X,\tau)$. Since $(X,\tau)$ is a $T_{1/2}$-space $f^{-1}(A)$ is closed in $(X,\tau)$ and so $(gof)f^{-1}(A)$ is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$(since f is surjective). Hence $g$ is $\widetilde{g}_\alpha$-closed.

(iv)Let C be a closed in $(Y,\sigma)$. Since f is $\omega$-continuous, $f^{-1}(C)$ is $\omega$ closed in $(X,\tau)$. Since $(X,\tau)$ is a $T_\omega$ space, $f^{-1}(C)$ is closed in $(X,\tau)$. Then $(gof)f^{-1}(C) = g(C)$ is a $\widetilde{g}_\alpha$-closed map.

(v) Let D be a closed set of $(X,\tau)$. Since (gof) is $\widetilde{g}_\alpha$-closed (gof)(D) is $\widetilde{g}_\alpha$-closed in $(Z,\eta)$. Since g is strongly $\widetilde{g}_\alpha$-continuous $g^{-1}(gof)(D)$ is closed in $(Y,\sigma)$.i.e f(D) is closed in $(Y,\sigma)$.(since $g$ is injective). Therefore f is a closed map.

**Theorem 3.15** *Let $(X,\tau)$ and $(Y,\sigma)$ be any two topological spaces.*

*(i)If $f:(X,\tau)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-closed and $A$ is a closed subset of $(X,\tau)$ then $f_A:(A,\tau_A)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-closed.*

*(ii)If $f:(X,\tau)\to(Y,\sigma)$ is a surjective $\widetilde{g}_\alpha$-closed (resp closed) and $A = f^{-1}(B)$ for some closed (resp $\widetilde{g}_\alpha$-closed) set $B$ of $(Y,\sigma)$ then $f_A:(A,\tau_A)\to(Y,\sigma)$ is $\widetilde{g}_\alpha$-closed.*

**Proof.**

(i)Let B be a closed set of A.Then $B = (A\cap F)$ for some closed set F of $(X,\tau)$ then B is closed in $(X,\tau)$. By hypothesis f(B) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$. But $f(B) = f_A(B)$ and hence $f_A$ is a $\widetilde{g}_\alpha$-closed map.

(ii)Let D be a closed set of A then $D = A\cap H$ for some closed set H of $(X,\tau)$. $f_A(D) = f(D) = f(A\cap H) = f(f^{-1}(B)\cap H) = B\cap f(H)$. Since f is $\widetilde{g}_\alpha$-closed, f(H) is $\widetilde{g}_\alpha$-closed and so $B\cap f(H)$ is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$. Therefore $f_A$ is a $\widetilde{g}_\alpha$-closed map.

# 4 $\widetilde{g}_\alpha$-Open Maps

**Definition 4.1** *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be a $\widetilde{g}_\alpha$-open map if f(A) is $\widetilde{g}_\alpha$-open for each open set A in $(X\tau)$.*

**Theorem 4.2** *A map $f : (X, \tau) \to (Y\sigma)$ is $\widetilde{g}_\alpha$-open if and only if for each subset S of $(Y, \sigma)$ and for each closed set U containing $f^{-1}(S)$ there is a $\widetilde{g}_\alpha$-closed set V of $(Y, \sigma)$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.*

**Proof.** Suppose that f is a $\widetilde{g}_\alpha$-open map. Let $S \subseteq Y$ and U be a closed subset of $(X, \tau)$ such that $f^{-1}(S) \subseteq U$. Then $V = (f(U^c))^c$ is a $\widetilde{g}_\alpha$-closed set containing S such that $f^{-1}(V) \subseteq U$. Conversely, let S be an open set of $(X, \tau)$. Then $f^{-1}((f(S))^c \subseteq S^c$ and $S^c$ is closed. By assumption, there exists a $\widetilde{g}_\alpha$-closed set V of $(Y, \sigma)$ such that $(f(S))^c \subseteq V$ and $f^{-1}(V) \subseteq S^c$ and so $S \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(S) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(S) = V^c$. Since $V^c$ is $\widetilde{g}_\alpha$-open, f(S) is $\widetilde{g}_\alpha$-open and therefore f is $\widetilde{g}_\alpha$-open.

**Corollary 4.3** *A function $f : (X, \tau) \to (Y\sigma)$ is $\widetilde{g}_\alpha$-open if and only $f^{-1}(\widetilde{g}_\alpha$-$Cl(B) \subseteq Cl(f^{-1}(B))$ for every subset B of $(Y, \sigma)$.*

**Proof.** Let f be $\widetilde{g}_\alpha$-open.For any subset B of Y,$f^{-1}(B) \subseteq Cl(f^{-1}(B)$.By the theorem 4.2 there exists a $\widetilde{g}_\alpha$-closed set A of Y such that $B \subseteq A$ and $f^{-1}(A) \subseteq Cl(f^{-1}(B))$. Therefore $f^{-1}(\widetilde{g}_\alpha\text{-}Cl(B)) \subseteq f^{-1}(A) \subseteq Cl(f^{-1}(B))$,since A is a $\widetilde{g}_\alpha$-closed set in $(Y, \sigma)$.
Conversely let S be any subset of $(Y, \sigma)$ and F be any closed set containing $f^{-1}(S)$.Take $A = \widetilde{g}_\alpha\text{-}Cl(S)$.Then A is a $\widetilde{g}_\alpha$-closed set and $S \subseteq A$. By assumption $f^{-1}(A) = f^{-1}(\widetilde{g}_\alpha\text{-}Cl(S)) \subseteq Cl(f^{-1}(S)) \subseteq F$ and therefore by the theorem 4.2 f is $\widetilde{g}_\alpha$-open

**Proposition 4.4** *For any bijection $f : (X, \tau) \to (Y, \sigma)$ the following are equivalent.*

*(i)$f^{-1} : (Y, \sigma) \to (X\tau)$ is $\widetilde{g}_\alpha$-continuous.*

*(ii)f is a $\widetilde{g}_\alpha$-open map.*

*(iii)is a $\widetilde{g}_\alpha$-closed map.*

**Proof.**

$(i) \Rightarrow (ii)$ Let U be an open set of $(X, \tau)$. By assumption $(f^{-1})^{-1}(U) = f(U)$ is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$ and so f is $\widetilde{g}_\alpha$-open.

$(ii) \Rightarrow (iii)$ Let F be a closed set of $(X, \tau)$ then $F^c$ is open in $(X, \tau)$. By assumption $f(F^c)$ is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$.i.e $f(F^c) = (f(F))^c$ is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$ and therefore f(F) is $\widetilde{g}_\alpha$-closed in $(Y, \sigma)$. Hence f is $\widetilde{g}_\alpha$-closed.

$(iii) \Rightarrow (i)$ Let F be a closed set in $(X, \tau)$. By assumption f(F) is $\widetilde{g}_\alpha$-closed in $(Y, \sigma)$ and $f(F) = (f^{-1})^{-1}(F)$ and therefore $f^{-1}$ is $\widetilde{g}_\alpha$-continuous.

**Definition 4.5** *Let $x$ be a point of $(X, \tau)$ and $V$ be a subset of $X$. Then $V$ is called a $\widetilde{g}_\alpha$-neighbourhood of $x$ in $(X, \tau)$ if there exist a $\widetilde{g}_\alpha$-open set $U$ of $(X, \tau)$ such that $x \in U \subseteq V$.*

**Theorem 4.6** *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following are equivalent if arbitrary union of $\widetilde{g}_\alpha$-closed sets is also $\widetilde{g}_\alpha$-closed in $Y$.*

*(i)f is a $\widetilde{g}_\alpha$-open mapping.*

*(ii)For a subset $A$ of $(x, \tau), f(Int(A)) \subseteq \widetilde{g}_\alpha - Int(f(A))$.*

*(iii)For each $x \in X$ and for each neighbourhood $U$ of $x$ in $(X, \tau)$ there exist a $\widetilde{g}_\alpha$-neighbourhood $W$ of f(x) in $(Y, \sigma)$ such that $W \subseteq f(U)$.*

**Proof.**

$(i) \Rightarrow (ii)$Suppose f is $\widetilde{g}_\alpha$-open. Let $A \subseteq X$. Then Int(A) is open in $(X, \tau)$ and so f(Int(A)) is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$. We have $f(Int(A)) \subseteq f(A)$. Therefore $f(Int(A) \subseteq \widetilde{g}_\alpha - Int(f(A))$.

$(ii) \Rightarrow (iii)$Suppose (ii) holds. Let $x \in X$ and U be a neighbourhood of x. Then there exist an open set G such that $x \in G \subseteq U$. By assumption $f(G) = f(Int(G)) \subseteq \widetilde{g}_\alpha\text{-}Int(f(G))$. This implies that $f(G) = \widetilde{g}_\alpha\text{-}(Int(f(G))$. Hence f(G) is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$. Further $f(x) \in f(G) \subseteq f(U)$ and so (iii) holds where $W = f(G)$.

$(iii) \Rightarrow (i)$Suppose (iii) holds. Let U be an open set in $(X, \tau)$, $x \in U, f(x) = y \in f(U)$. For each $y \in f(U)$ there exist a $\widetilde{g}_\alpha$-neighbourhood $W_y$ of y in $(Y, \sigma)$ such that $W_y \subseteq f(U)$(by hypothesis). Since $W_y$ is a $\widetilde{g}_\alpha$-neighbourhood of y there exist a $\widetilde{g}_\alpha$-open set $V_y \in (Y, \sigma)$ such that $y \in V_y \subseteq W_y$. Therefore $f(U) = \bigcup(\{V_y : y \in f(U)\})$ is a $\widetilde{g}_\alpha$-open set in $(Y, \sigma)$. Thus f is a $\widetilde{g}_\alpha$-open map. Since arbitrary union of $\widetilde{g}_\alpha$-open sets is $\widetilde{g}_\alpha$-open.

# 5 Strongly $\widetilde{g}_\alpha$-closed and Strongly $\widetilde{g}_\alpha$-open Maps

**Definition 5.1** *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be a strongly $\widetilde{g}_\alpha$-closed map if the image f(A) is $\widetilde{g}_\alpha$-closed in $(Y, \sigma)$ for each $\widetilde{g}_\alpha$-closed set $A$ in $(X, \tau)$.*

**Proposition 5.2** *Every strongly $\widetilde{g}_\alpha$-closed map is $\widetilde{g}_\alpha$-closed.*

**Proof.** Since every closed set is $\widetilde{g}_\alpha$-closed the result follows.

**Example 5.3** *Let* $X = Y = \{a,b,c\}, \tau = \{\phi, X, \{a,b\}\}, \sigma = \{\phi, Y, \{a\}\}$ $\widetilde{G}_\alpha C(X) = \{\phi, Y, \{c\}, \{a,c\}, \{b,c\}\}, \widetilde{G}_\alpha C(Y) = \{\phi, Y, \{b\}, \{c\}, \{b,c\}\},$. *The function f is the identity function. The function f is* $\widetilde{g}_\alpha$*-closed but not strongly* $\widetilde{g}_\alpha$*-closed .Since for the* $\widetilde{g}_\alpha$*-closed set* $\{a,c\}$, $f(\{a,c\}) = \{a,c\}$ *which is not* $\widetilde{g}_\alpha$*-closed in* $(Y,\sigma)$.

**Proposition 5.4** *A map* $f : (X,\tau) \to (Y,\sigma)$ *is strongly* $\widetilde{g}_\alpha$*-closed if and only if* $\widetilde{g}_\alpha$-$Cl(f(A)) \subseteq f(\widetilde{g}_\alpha$-$Cl(A))$ *for every subset A of* $(X,\tau)$.

**Proof.** Suppose that f is strongly $\widetilde{g}_\alpha$-closed and $A \subseteq X$.Then $f(\widetilde{g}_\alpha$-$Cl(A)$ is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$.We have $f(A) \subseteq f(\widetilde{g}_\alpha$-$Cl(A))$, $\widetilde{g}_\alpha$-$Cl(f(A)) \subseteq \widetilde{g}_\alpha$-$Cl(f(\widetilde{g}_\alpha$-$Cl(A)) = f(\widetilde{g}_\alpha$-$Cl(A))$for every subset A of $(X,\tau)$.Conversely let A be a $\widetilde{g}_\alpha$-closed subset of X.Then $A = \widetilde{g}_\alpha$-$Cl(A)$ and so $f(A) = f(\widetilde{g}_\alpha$-$Cl(A)) \supseteq \widetilde{g}_\alpha$-$Cl(f(A))$.By definition $f(A) \subseteq \widetilde{g}_\alpha$-$Cl(f(A)$.Therefore $f(A) = \widetilde{g}_\alpha$-$Cl(f(A))$.i.e f(A) is $\widetilde{g}_\alpha$-closed and hence f is strongly $\widetilde{g}_\alpha$-closed .

**Definition 5.5** *A map* $f : (X,\tau) \to (Y,\sigma)$ *is said to be strongly a* $\widetilde{g}_\alpha$*-open map if the image f(A) is* $\widetilde{g}_\alpha$*-open in* $(Y,\sigma)$ *for each* $\widetilde{g}_\alpha$*-open set A in* $(X,\tau)$.

**Proposition 5.6** *For any bijection* $f : (X,\tau) \to (Y,\sigma)$ *the following are equivalent.*

*(i)*$f^{-1} : (Y,\sigma) \to (X\tau)$ *is* $\widetilde{g}_\alpha$*-irresolute.*

*(ii)f is a strongly* $\widetilde{g}_\alpha$*-open map.*

*(iii)f is a strongly* $\widetilde{g}_\alpha$*-closed map.*

**Proof.**

$(i) \Rightarrow (ii)$ Let U be a $\widetilde{g}_\alpha$-open set of $(X,\tau)$. By assumption $(f^{-1})^{-1}(U) = f(U)$ is $\widetilde{g}_\alpha$-open in $(Y,\sigma)$ and so f is strongly $\widetilde{g}_\alpha$-open.

$(ii) \Rightarrow (iii)$ Let F be a $\widetilde{g}_\alpha$-closed set of $(X,\tau)$ then $F^c$ is $\widetilde{g}_\alpha$-open in $(X,\tau)$. By assumption $f(F^c)$ is $\widetilde{g}_\alpha$-open in $(Y,\sigma)$.i.e $f(F^c) = (f(F))^c$ is $\widetilde{g}_\alpha$-open in $(Y,\sigma)$ and therefore f(F) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$. Hence f is strongly $\widetilde{g}_\alpha$-closed.

$(iii) \Rightarrow (i)$ Let F be a $\widetilde{g}_\alpha$-closed set in $(X,\tau)$. By assumption f(F) is $\widetilde{g}_\alpha$-closed in $(Y,\sigma)$ and $f(F) = (f^{-1})^{-1}(F)$ and therefore $f^{-1}(F)$ is $\widetilde{g}_\alpha$-irresolute.

**Theorem 5.7** *If* $f : (X,\tau) \to (Y,\sigma)$ *is* $^{\#}$*gs-irresolute,pre-*$\alpha$*-closed and* $\widetilde{g}_\alpha$*-closed then it is a strongly* $\widetilde{g}_\alpha$*-closed map.*

**Proof.** It follows from Proposition 3.6

### 5.0.1 Applications

In this section we introduce the definition of $\widetilde{g}_\alpha$-regular spaces and prove some preservation theorems of Normality and Regularity.

**Definition 5.8** *A space $(X,\tau)$ is said to be $\widetilde{g}_\alpha$-regular if for each closed set $F$ of $X$ and each point $x \notin F$ there exist disjoint $\widetilde{g}_\alpha$-open sets $U$ and $V$ such that $F \subseteq U$ and $x \in V$.*

**Theorem 5.9** *In a topological space $(X,\tau)$, the following statements are equivalent.*

*(i) $(X,\tau)$ is $\widetilde{g}_\alpha$-regular.*

*(ii) for every point $x$ of $(X,\tau)$ and every open set $V$ containing $x$ there exists an $\widetilde{g}_\alpha$-open set $A$ such that $x \in A \subseteq \alpha Cl(A) \subseteq V$.*

**Proof.**

$(i) \Rightarrow (ii)$. Let $x \in X$ and $V$ be an open set containing $x$. Then $X - V$ is closed and $x \notin X - V$. By $(i)$ there exist disjoint $\widetilde{g}_\alpha$-open sets $A$ and $B$ such that $x \in A$ and $X - V \subseteq B$. That is $X - B \subseteq V$. Since every open set is $^{\#}$gs-open, $V$ is $^{\#}$gs-open. $X - B$ is $\widetilde{g}_\alpha$-closed. Therefore $\alpha Cl(X - B) \subseteq V$. Since $A \cap B = \phi, A \subseteq X - B$. Therefore $x \in A \subseteq \alpha Cl(A) \subseteq \alpha Cl(X - B) \subseteq V$. Hence $x \in A \subseteq \alpha Cl(A) \subseteq V$.

$(ii) \Rightarrow (i)$. Let $F$ be a closed set and $x \notin F$. This implies that $X - F$ is an open set containing $x$. By $(ii)$, there exists a $\widetilde{g}_\alpha$-open set $A$ such that $x \in A \subseteq \alpha Cl(A) \subseteq X - F$. That is $X - \alpha Cl(A) \supseteq F$. Since every $\alpha$-closed set is $\widetilde{g}_\alpha$-closed, $\alpha Cl(A)$ is $\widetilde{g}_\alpha$-closed and $X - \alpha Cl(A)$ is $\widetilde{g}_\alpha$-open. Therefore $A$ and $X - \alpha Cl(A)$ are the required $\widetilde{g}_\alpha$-open sets containing x and F respectively.

**Theorem 5.10** *If $f : (X,\tau) \to (Y,\sigma)$ is a continuous $\alpha$-open and $\widetilde{g}_\alpha$-closed surjective map and $(X,\tau)$ is a regular space then $(Y,\sigma)$ is $\widetilde{g}_\alpha$-regular.*

**Proof.** Let $y \in Y$ and $V$ be an open set containing $y$ of $(Y,\sigma)$. Let $x$ be a point of $(X,\tau)$ such that $y = f(x)$. Since $f$ is continuous, $f^{-1}(V)$ is open in $(X,\tau)$. Since $(X,\tau)$ is regular, there exists an open set $U$ such that $x \in U \subseteq Cl(U) \subseteq f^{-1}(V)$. Hence $y = f(x) \in f(U) \subseteq f(Cl(U)) \subseteq V$. Since $f$ is a $\widetilde{g}_\alpha$-closed map, $f(Cl(U))$ is a $\widetilde{g}_\alpha$-closed set contained in the open set $V$ which is $^{\#}$gs-open. Hence, we have $\alpha Cl(f(Cl(U))) \subseteq V$. Therefore $y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq \alpha Cl(f(Cl(U))) \subseteq V$. This implies $y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq V$. Since $f$ is an $\alpha$-open map and $U$ is open in $X$, $f(U)$ is $\alpha$-open in $(Y,\sigma)$. Every $\alpha$-open set is $\widetilde{g}_\alpha$-open and hence $f(U)$ is $\widetilde{g}_\alpha$-open in $(Y,\sigma)$. Thus for every

point $y$ of $(Y, \sigma)$ and every open set $V$
containing $y$ there exists an $\widetilde{g}_\alpha$-open set $f(U)$ such that
$y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq V$. Hence by Theorem 5.9, $(Y, \sigma)$ is $\widetilde{g}_\alpha$-regular.

**Theorem 5.11** *If $f : (X, \tau) \to (Y, \sigma)$ is continuous and strongly $\widetilde{g}_\alpha$-closed bijective map and if $(X, \tau)$ is a $\widetilde{g}_\alpha$-regular space then $(Y, \sigma)$ is $\widetilde{g}_\alpha$-regular.*

**Proof.** Let $y \in (Y, \sigma)$ and $V$ be an open set containing $y$. Let $x$ be a point of $(X, \tau)$ such that $y = f(x)$. Since $f$ is continuous, $f^{-1}(V)$ is open in $(X, \tau)$ By assumptions and Theorem 5.9, there exists an $\widetilde{g}_\alpha$-open set $U$ such that $x \in U \subset \alpha Cl(U) \subseteq f^{-1}(V)$. Then $y \in f(U) \subseteq f(\alpha Cl(U)) \subseteq V$. We know that $\alpha Cl(U)$ is $\alpha$-closed and hence $\widetilde{g}_\alpha$-closed. Since $f$ is strongly $\widetilde{g}_\alpha$-closed , $f(\alpha Cl(U))$ is $\widetilde{g}_\alpha$-closed. Every open set is $^{\#}$gs-open and hence $V$ is $^{\#}$gs-open. Therefore, we have $\alpha Cl(f(\alpha Cl(U)) \subseteq V$. This implies
$y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq \alpha Cl(f(\alpha Cl(U)) \subseteq V$. That is
$y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq V$. Now $U$ is $\widetilde{g}_\alpha$-open implies $X - U$ is $\widetilde{g}_\alpha$-closed in $(X, \tau)$. Since $f$ is a strongly $\widetilde{g}_\alpha$-closed map, $f(X - U)$ is $\widetilde{g}_\alpha$-closed in $(Y, \sigma)$. That is, $Y - f(U)$ is $\widetilde{g}_\alpha$-closed in $(Y, \sigma)$. This implies $f(U)$ is $\widetilde{g}_\alpha$-open in $(Y, \sigma)$.Thus, for every point $y$ of $(Y, \sigma)$ and every open set $V$ containing $y$ there exists an $\widetilde{g}_\alpha$-open set $f(U)$ such that $y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq V$. Hence by Theorem 6.2, $(Y, \sigma)$ is $\widetilde{g}_\alpha$-regular.

**Theorem 5.12** *If $f : (X, \tau) \to (Y, \sigma)$ is continuous, strongly $\widetilde{g}_\alpha$-open bijective map and if $X$ is a normal space, then $Y$ is normal.*

**Proof.** Let $A$ and $B$ be disjoint closed sets of $Y$. Since $f$ is continuous bijective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets of $X$. Since $X$ is normal, there exist disjoint open sets $G$ and $H$ of $X$ such that $G \supset f^{-1}(A)$ and $H \supset f^{-1}(B)$. Every open set is $\widetilde{g}_\alpha$-open and hence $G$ and $H$ are disjoint $\widetilde{g}_\alpha$-open sets of $X$. Since $f$ is strongly $\widetilde{g}_\alpha$-open bijective, $f(G)$ and $f(H)$ are disjoint $\widetilde{g}_\alpha$-open sets of $Y$ containing $A$ and $B$ respectively. Since every closed set is $^{\#}g$s-closed, $A$ and $B$ are $^{\#}g$s-closed sets in $Y$. Then we have $\alpha Int(f(G)) \supset A$ and $\alpha Int(f(H)) \supset B$ and $\alpha Int(f(G)) \cap \alpha Int(f(H)) \subset f(G) \cap f(H) = \phi$. Therefore, there exist disjoint $\alpha$-open sets $\alpha Int(f(G))$ say $U$ and $\alpha Int(f(H))$ say $V$ of $Y$ containing $A$ and $B$ respectively. $U$ and $V$ are $\alpha$-open sets imply $U \subseteq Int(Cl(Int(U)))$ and $V \subseteq Int(Cl(Int(V)))$. Since $Int(Cl(Int(U))) \cap Int(Cl(Int(V))) = \phi$,
$A \subseteq \alpha Int(f(G)) = U \subseteq Int(Cl(Int(U)))$ and $B \subseteq \alpha Int(f(H)) = V \subseteq Int(Cl(Int(V)))$. Hence, $Y$ is normal.

**Theorem 5.13** *The following are equivalent for a space $(X, \tau)$.*

*(i) $(X, \tau)$ is normal.*

*(ii) for any disjoint closed sets A and B, there exist disjoint $\widetilde{g}_\alpha$-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.*

*(iii). for any closed set A and any open set V containing A, there exists an $\widetilde{g}_\alpha$-open set U of X such that $A \subseteq U \subset \alpha Cl(U) \subseteq V$.*

**Proof.**

$(i) \Rightarrow (ii)$. This is obvious since every open set is $\widetilde{g}_\alpha$-open.

$(ii) \Rightarrow (iii)$. Let $A$ be a closed set and $V$ be an open set containing $A$. Then $A$ and $X - V$ are disjoint closed sets. By $(ii)$, there exist disjoint $\widetilde{g}_\alpha$-open sets $U$ and $W$ such that $A \subseteq U$ and $X - V \subseteq W$. Since every closed set is $^{\#}g$s-closed, $X - V$ is $^{\#}g$s-closed. $W$ is $\widetilde{g}_\alpha$-open implies $X - V \subseteq \alpha Int(W)$ and $U \cap \alpha Int(W) = \phi$. Therefore, we obtain $\alpha Cl(U) \cap \alpha Int(W) = \phi$(By theorem 4.19[3]) and hence $A \subset U \subseteq \alpha Cl(U) \subseteq X - \alpha Int(W) \subseteq V$.

$(iii) \Rightarrow (i)$. Let $A$ and $B$ be disjoint closed sets of $X$. Then $A \subseteq X - B$ and $X - B$ is open. By $(iii)$, there exists a $\widetilde{g}_\alpha$-open set $O$ of $X$ such that $A \subseteq O \subset \alpha Cl(O) \subseteq X - B$. Since $A$ is closed, it is $^{\#}g$s-closed. $A \subseteq O$ and $O$ is $\widetilde{g}_\alpha$-open implies $A \subseteq \alpha Int(O)$. $\alpha Cl(O) \subseteq X - B$ implies $B \subseteq X - \alpha Cl(O)$. Put $U = Int(Cl(Int(\alpha Int(O))))$ and $V = Int(Cl(Int(X - \alpha Cl(O))))$. Then $U$ and $V$ are disjoint open sets of $X$ such that $A \subseteq U$ and $B \subseteq V$. Hence $X$ is normal.

**Definition 5.14** *A spacec $(X, \tau)$ is said to be $\alpha$-regular if for every closed set F and a point $x \notin X$,there exist disjoint $\alpha$-open sets A and B such that $x \in A$ and $F \subseteq B$[1]*

**Theorem 5.15** *In a topological space $(X, \tau)$ the following are equivalent.*

*(i) $(X, \tau)$ is $\alpha$-regular*

*(ii)for every point $x \in X$ and every open set V containing X there exist an $\alpha$-open set U such that $x \in U \subseteq \alpha Cl(U) \subseteq V$[1]*

**Theorem 5.16** *If $f : (X, \tau) \to (Y, \sigma)$ is a continuous $\alpha$-open and strongly $\widetilde{g}_\alpha$-closed surjective map and if $(X, \tau)$ is a regular space then $(Y, \sigma)$ is $\alpha$-regular.*

**Proof.** Let $y \in Y$ and V be an open set containing y in Y.Let x be a point of $(X, \tau)$ such that $y = f(x)$.Since $(X, \tau)$ is regular and f is continuous there is an $^{\#}$gs-open set U such that $x \in U \subseteq Cl(U) \subseteq f^(-1(V))$.Hence $f(x) = y \in f(U) \subseteq f(Cl(U)) \subseteq V$.Since f is a $\widetilde{g_\alpha}$-closed map $f(C(U))$ is a $\widetilde{g_\alpha}$-closed set contained in the $^{\#}$gs-open set V (since any open set is $\widetilde{g}_\alpha$-open).Hence $\alpha Cl(f\alpha Cl(U)) \subseteq V$.Therefore $y \in f(U) \subseteq \alpha Cl(f(U)) \subseteq \alpha Cl(f(\alpha Cl(U)) \subseteq V$ and f(U) is $\alpha$-open. By the theorem 5.15 $(Y, \sigma)$ is $\alpha$-regular.

# References

[1] R. Devi, K. Balachandran and H. Maki, Generalized $\alpha$-closed maps and $\alpha$ generalized closed maps, *Indian J. Pure. Appl. Math*, 29(1998), 37-49.

[2] R. Devi, H. Maki and K. Balachandran, Semi-generalzed closed maps and generalized semi-closed maps, *Mem. Fac. Sci. Kochi Univ. Ser. App. Math.*, 14(1993), 41-54.

[3] S. Jafari, M.L. Thivagar and N.R. Paul, Remarks on $\widetilde{g}_\alpha$-closed sets, *International Mathematical Forum*, 5(2010), 1167-1178.

[4] M.T. Lellis and N.R. Paul, On continuous function via $\widetilde{g_\alpha}$-closed sets, *Communicated.*

[5] M.T. Lellis and N.R. Paul, Remarks on $\widetilde{g_\alpha}$-irresolute maps, *Bol. Soc. Paran. Mat.*, 29(2011), 49-60.

[6] M.T. Lellis and N.R. Paul, On topological $\widetilde{g_\alpha}$-quotient mappings, *Journal of Advanced Studies in Topology*, 1(2010), 9-16.

[7] N. Levine, Generalized closed sets in topology, *Rend. Circlo. Mat. Palermo*, 19(2) (1970), 89-96.

[8] P. Sundram, On $\omega$-closed sets in topology, *Acta Ciencia Indica*, 4(2000), 389-392.

[9] M.K.R.S. Veera Kumar, Between $g^*$-closed and g-closed sets, *Mem. Fac. Sci. Kochi Univ. Ser. Appl. Math.*, 21(2000), 1-19.

[10] M.K.R.S. Veera Kumar, $^\#$g-semi-closed sets in topological spaces, *Antarctica J. Math*, 2(2005), 201-222.