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On Similarity and Entropy of Single Valued Neutrosophic Sets

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Abstract

Neutrosophic sets was defined by Smarandache, which is a generalized of fuzzy sets and intuitionistic fuzzy sets. A single valued neutrosophic set is an instance of neutrosophic sets defined by Wang et al. (2005). In this study, we introduce similarity measure on two single valued neutrosophic sets. We develop an entropy of single valued neutrosophic sets. Finally, we give an example which demonstrates the application of the similarity measure in the single valued neutrosophic multicriteria decision making.

Keywords: *Decision making, Entropy, Neutrosophic set, Single valued neutrosophic set, Similarity measure.*

1 Introduction

The concept of neutrosphy was introduced by Smarandache [12] in 1995. Neutrosophic set generalizes the concept of the classic set, fuzzy set [23], interval valued fuzzy set [16], intuitionistic fuzzy set [1], interval valued intuitionistic fuzzy sets [2], etc. A neutrosophic set consider truth-membership, indeterminancy-membership and falsity-membership which are independent and lies between the non-standard unit interval $]^{-}0, 1^{+}[$. Wang et al. [17] introduced single valued neutrosophic sets (SVNS) and provided the set-theoretic operators and various properties of SVNSs. The single valued neutrosophic set is a generalization of classic set, fuzzy set, intuitionistic fuzzy set etc. The single valued neutrosophic set theory is precious in modelling theory. Therefore it can be used in real scientific and engineering applications.

Entropy and similarity measures are two basic concepts in fuzzy set theory. Similarity measure is an important tool for determining the degree of similarity between two objects. Szmidt and Kacprzyk [13] extending the Hamming Distance, the Euclidean Distance, etc. to intuitionistic fuzzy set (IFS) environment. Hung and Yang [7] extended the Hausdorff distance to IFSs and proposed similarity measures. And then Hung and Yang [8] extended some similarity measures of fuzzy sets to intuitionistic fuzzy sets. Lie and Cheng [10] introduced a new similarity measure based on the membership degree and non membership degree for IFSs. Xu [19] introduced a series similarity measures for IFSs. Xu and Chen [20] given a series of distance and similarity measures which are generalized the weighted Hamming distance, the weighted Euclidean distance and the weighted Hausdorff distance. Broumi and Smarandache [3] gave some similarity measures of neutrosophic sets. Ye [22] introduced similarity measures based on the distances ofinterval neutrosophic sets (INSs). Entropy describes the degree of fuzziness in fuzzy set. Zadeh [24] mentioned entropy on fuzzy set for the first time. De Luca and Termini [5] introduced the noprobabilistic entropy on fuzzy sets. The entropy and similarity measure of intuitionistic fuzzy sets is given widely in decision making by Chen and Li [4]. Kaufmann [9] introduced a distance based measure of soft entropy. Yager [21] defined an entropy measure of a fuzzy set in terms of lack of distinction between the fuzzy set and its complement. Szmidt and Kacprzyk [14] introduced the fuzzy entropy of intuitionistic fuzzy sets. The fuzzy entropy for intuitionistic fuzzy sets is given by Huang [6]. Wei, et al. [18] defined entropy measure of interval valued intuitionistic fuzzy sets. Similarity measures and entropy of single valued neutrosophic sets was introduced by Majumdar and Samanta [11] for the first time.

2 Preliminaries

In this section, we give some basic definition related single valued neutrosophic sets (SVNS) from [17].

Definition 2.1: *Let X be a universal set, with generic element of X denoted by x. A single valued neutrosophic set A in X is characterized by a truth-membership function T_A , indeterminacy-membership function I_A and falsity-membership function F_A, with for each $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0,1]$.*

Note that for a SVNS A, the relation

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

holds. When the universal set X is continuous, a SVNS A can be written as

$$A = \int_X \langle T_A(x), I_A(x), F_A(x) \rangle / x \, , x \in X.$$

When the universal set X is discrete, a SVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle / x_i \, , x \in X.$$

Definition 2.2: *A SVNS A is contained in the other SVNS B, $A \subseteq B$, if and only if*

$$T_A(x) \leq T_B(x);\; I_A(x) \leq I_B(x);\; F_A(x) \geq F_B(x)$$

for all $x \in X$.

Definition 2.3: *The complement of a SVNS A is denoted by A^c and is defined by*

$$T_{A^c}(x) = F_A(x); I_{A^c}(x) = 1 - I_A(x); F_{A^c}(x) = T_A(x)$$

for all $x \in X$.

Definition 2.4: *Two SVNSs A and B are equal if and only if $A \subseteq B$and $B \subseteq A$.*

Definition 2.5: *The union of two SVNSs A and B is a SVNS C, written as $C = A \cup B$, is defined as follow*

$$T_C(x) = max\{T_A(x), T_B(x)\}\,; I_C(x) = max\{I_A(x), I_B(x)\}\,;$$
$$F_C(x) = min\{F_A(x), F_B(x)\}$$

for all $x \in X$.

Definition 2.6: *The intersection of two SVNSs A and B, written as $C = A \cap B$, is defined as follow*

$$T_C(x) = min\{T_A(x), T_B(x)\}\,; I_C(x) = min\{I_A(x), I_B(x)\}\,; F_C(x) = max\{F_A(x), F_B(x)\}$$

for all $x \in X$.

3 Similarity of Single Valued Neutrosophic Sets

The concept of similarity for SVNSs is defined by Majumdar and Samanta [11] as follow.

Definition 3.1: *Let $N(X)$ be all SVNSs on X and $A, B \in N(X)$. A similarity measure between two SVNSs is a function $S: N(X) \times N(X) \to [0,1]$ which is satisfies the following conditions:*

i. $0 \leq S(A, B) \leq 1$
ii. $S(A, B) = 1$ *iff* $A = B$
iii. $S(A, B) = S(B, A)$
iv. *If* $A \subseteq B \subseteq C$ *then* $S(A, C) \leq S(A, B)$ *and* $S(A, C) \leq S(B, C)$ *for all* $A, B, C \in N(X)$.

Definition 3.2: *Let A, B be two neutrosophic sets in X. The similarity measure between the neutrosophic sets A and B can be evaluated by the function*

$$S(A, B) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3}\right]$$

for all $x_i \in X$.

We shall prove this similarity measure satisfies the properties of the Definition 3.1.

Proof: We show that the $S(A, B)$ satisfies the all properties 1-4 as above.It is obvious, the properties 1-3 is satisfied of definition 3.1. In the following we only prove 4.

Let $A \subseteq B \subseteq C$, the we have $T_A(x_i) \leq T_B(x_i) \leq T_C(x_i)$, $I_A(x_i) \leq I_B(x_i) \leq I(x_i)$ and $F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$.

It follows that $|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|$, $|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|$ and $|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|$. Then

$$\sum_{i=1}^{n}\left[1 - \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3}\right] \\ \geq \sum_{i=1}^{n}\left[1 - \frac{|T_A(x_i) - T_C(x_i)| + |I_A(x_i) - I_C(x_i)| + |F_A(x_i) - F_C(x_i)|}{3}\right]$$

It means that $S(A, C) \leq S(A, B)$.

Similarly, it seems that $S(A, C) \leq S(B, C)$.

The proof is completed.

4 Entropy of a Single Valued Neutrosophic Set

Herewe transform the entropy formula for intuitionistic fuzzy sets in [4] to be entropy formula for a single valued neutrosophic set. Entropy of SVNSs is defined by Majumdar and Samanta [11].

Definition 4.1: *Let $N(X)$ be all SVNSs on X and $A \in N(X)$. An entropy on SVNSs is a function $E_N\colon N(X) \to [0,1]$ which is satisfies the following axioms:*

i. $E_N(A) = 0$ *if A is crisp set*
ii. $E_N(A) = 1$ *if* $\left(T_A(x), I_A(X), F_A(X)\right) = (0.5,0.5,0.5)$ *for all $x \in X$*
iii. $E_N(A) \geq E_N(B)$ *if* $A \subset B$, *i.e.,* $T_A(x) \leq T_B(x)$, $F_A(x) \geq F_B(x)$ *and* $I_A(x) \leq I_B(x)$ *for all $x \in X$*
iv. $E_N(A) = E_N(A^c)$ *for all $A \in N(X)$.*

Definition 4.2: *The entropy of SVNS set A is,*

$$E_N(A) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{1}{b-a}\int_a^b |T_A(x_i) - F_A(x_i)||I_A(x_i) - I_{A^c}(x_i)|\, dx\right)$$

for all $x \in X$.

Theorem:*The SVN entropy of $E_N(A)$ is an entropy measure for SVN sets.*

Proof: We show that the $E_N(A)$ satisfies the all properties given in Definition 4.1.

i. When A is a crisp set, i.e., $T_A(x_i) = 0$, $I_A(x_i) = 0$, $F_A(x_i) = 1$ or $T_A(x_i) = 1$, $I_A(x_i) = 0$, $F_A(x_i) = 0$, for all $x_i \in X$. It is clear that $E_N(A) = 0$.

ii. Let $\left(T_A(x), I_A(X), F_A(X)\right) = (0.5,0.5,0.5)$ for all $x \in X$. Then
$$E_N(A) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{1}{b-a}\int_a^b |0.5 - 0.5||0.5 - 0.5|\, dx\right) = 1$$

iii. If $A \subset B$, then $T_A(x) \leq T_B(x)$, $F_A(x) \geq F_B(x)$ and $I_A(x) \leq I_B(x)$ for all $x \in X$. So $T_A(x_i) - F_A(x_i) \leq T_B(x_i) - F_B(x_i)$ and $I_A(x) - I_{A^c}(x_i) \leq I_B(x) - I_{B^c}(x_i)$. Therefore $E_N(A) \geq E_N(B)$.

iv. Since $T_{A^c}(x) = F_{A^c}(x)$, $I_{A^c}(x) = 1 - I_A(x)$ and $F_{A^c}(x) = F_A(x)$, it is clear that $E_N(A) = E_N(A^c)$.

The proof is completed.

5 Example

In multicriteria decision-making environments, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives. Hence, we can define an ideal criterion value $a_j^* = \left(t_j^*, i_j^*, f_j^*\right) = (1,0,0)(j = 1,2, \ldots, n)$ in the ideal alternative A^*.

In order to demonstrate the application of the proposed approach, a multicriteria decision making problem adapted from Tan and Chen [15] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A = \{A_1, A_2, A_3, A_4\}$ whose core competencies are evaluated by means of the following four criteria (C_1, C_2, C_3, C_4) : (1) the level of technology innovation(C_1), (2) the control ability of flow (C_2), (3) the ability of management (C_3), (4) the level of service (C_4). Then, the weight vector for the four criteria is $w = (0.25, 0.30, 0.20, 0.25)$.

The proposed decision making method is applied to solve this problem for selecting suppliers. For the evaluation of an alternative A_i $(i = 1,2,3,4)$ with respect to a criterion C_j $(j = 1,2,3,4)$, it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. For the neutrosophic notation, it can be expressed as $a_{11} = (0.5, 0.1, 0.3)$. Thus, when the four possible alternatives with respect to the above four criteria are evaluated by the similar method from the expert, we can obtain the following single valued neutrosophic decision matrix E:

$$E = \begin{pmatrix} \{0.4, 0.2, 0.3\} & \{0.5, 0.1, 0.4\} & \{0.7, 0.1, 0.2\} & \{0.3, 0.2, 0.1\} \\ \{0.4, 0.2, 0.3\} & \{0.3, 0.2, 0.4\} & \{0.9, 0.0, 0.1\} & \{0.5, 0.3, 0.2\} \\ \{0.4, 0.3, 0.1\} & \{0.5, 0.1, 0.3\} & \{0.5, 0.0, 0.4\} & \{0.6, 0.2, 0.2\} \\ \{0.6, 0.1, 0.2\} & \{0.2, 0.2, 0.5\} & \{0.4, 0.3, 0.2\} & \{0.7, 0.2, 0.1\} \end{pmatrix}$$

By applying Definition 3.2, the similarity measure between an alternative A_i $(i = 1,2,3,4)$ and the ideal alternative A^* are as follows:

$$S(A^*, A_1) = 0.171, S(A^*, A_2) = 0.170, S(A^*, A_3) = 0.175, S(A^*, A_4) = 0.167.$$

According to the similarity measures, thus the ranking order of the four suppliers is $A_3 \succ A_1 \succ A_2 \succ A_4$. Hence, the best supplier is A_3.

From the example, we can see that the proposed single valued neutrosophic multicriteria decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. The technique proposed in this paper extends existing fuzzy decision-making methods and provides a new way for decision-makers.

To represent uncertainty, imprecise, incomplete, and inconsistent information that exist in real world, Smarandache [12] gave the concept of a neutrosophic set from philosophical point of view. The neutrosophic set is a powerful general formal framework that generalizes the concept of the classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval valued intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set. In the neutrosophic set, truth membership, indeterminacy membership, and falsity membership are represented independently. Since the neutrosophic set generalizes the above-mentioned sets from the philosophical point of view, the main advantage of the proposed decision-making approach based on similarity measure (or entropy) is due to the fact that our model not only accommodate the single valued neutrosophic environment but also automatically take into account the indetermine information provided by decision makers than the existing decision-making methods.

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