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Effect Boundary Roughness on Rayleigh-Taylor Instability of a Couple-Stress Fluid

Krishna B. Chavaraddi[1], Vishwanath B. Awati[2] and Priya M. Gouder[3]

[1] Department of Mathematics
Government First Grade College
Yellapur (U.K) - 581359, Karnataka, India
E-mail: ckrishna2002@yahoo.com

[2] Department of Mathematics
Rani Channamma University
Belgaum, Karnataka, India
E-mail: await_vb@yahoo.com

[3] Department of Mathematics
K.L.E. Society's Dr. M. S. Sheshagiri College of
Engineering & Technology, Belgaum - 590008, Karnataka, India
E-mail: priyagouder@gmail.com

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Abstract

The effect of boundary roughness on Rayleigh-Taylor instability (RTI) of a couple–stress fluid layer bounded above a clear fluid and below by a rigid surface with roughness boundary is studied using linear stability analysis. Because of the growing importance of non-Newtonian fluids (Couple-stress fluid) in modern technology and industries as well as various practical applications investigations on such fluids are desirable. An expression for the growth rate of RTI is derived using suitable boundary and surface conditions in addition to couple-stress boundary conditions. From this it is clear that the effects of couple-

stress parameter, roughness parameter and bond number play a significant role in maintaining the stability on the two fluid system.

Keywords: *Rayleigh-Taylor instability (RTI), couple-stress fluid, surface roughness.*

1 Introduction

The Rayleigh-Taylor Instability (RTI) occurs when a heavy fluid is supported by a lighter one in a gravitational or equivalently, when a heavy fluid is accelerated by a lighter one. Similar to pouring of water into a oil, the heavier fluid, once perturbed, streams to the bottom, pushing the light fluid aside. This notion for a fluid in a gravitational fluid was first discovered by Lord Rayleigh [1] and later applied to all accelerated fluids by Sir Geoffrey Taylor [2]. He RTI has been addressed in several studies owing to its importance in science, engineering and technology. RTI in hydrodynamics and magnetohydrodynamics has been extensively investigated (see Chandrashekar [3]). Bhatia [4] has studied the stability of a plane interface separating two incompressible superposed conducting fluids of uniform density, when the whole system is under the influence of a uniform magnetic field. He has carried out the stability analysis of two highly viscous fluids of equal kinematic viscosity and different uniform densities. RTI of two viscoelastic (Oldroyd) superposed fluds have been studied by Sharma and Sharma [5].

Nevertheless, much attention has not been given in the literature to the study of RTI in a poorly conducting non-Newtonian fluid like Couple stress fluid with the effect of surface roughness that in spite of frequently occurring in many engineering and physical situations namely, inertial fusion energy (IFE), geophysics and supernova, the consideration of such fluids is desirable. The couple-stress effects are considered as result of the action of one part of a deforming body on its neighbourhood. Stokes [6] has formulated the theory of a couple-stress fluid. The theory of Stokes [6] allows for the polar effects such as the presence of couple-stresses and body couples and has been applied to the study of some simple lubrication problems(see Sinha et al. [7], Bujurke and Jayaraman [8]). According to Stokes [6], couple-stresses appear in fluids with very high molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [9] modeled synovial fluid as couple-stress fluid in human joints. The presence of small amounts of additives in a lubricant can improve the bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. This additive in a lubricant also reduces the coefficient of friction and increases the temperature range in which bearing can operate. Later Rudraiah et al., [10] have studied the RTI in a non-Newtonian (power-law) fluid layer. The RTI of two superposed infinitely conducting couple-stress fluids of uniform densities in a porous medium in the presence of a uniform magnetic field by Sunil et al., [11]. Recently, Rudraiah et al., [12] have studied the electro hydrodynamic RTI in a couple-stress fluid layer bounded above by porous layer. It is clear notice that the profound effect of surface roughness with couple-stress fluid in region -1 with proper choice of couple-stress parameter and roughness parameter in reducing the asymmetry of the two fluid composite systems at the interface.

Keeping in mind the importance of non-Newtonian(couple-stress) fluids in modern technology and industries as well as various applications mentioned above, the RTI in a poorly conducting couple-stress fluid layer bounded above by a clear fluid with the effect of boundary roughness(surface roughness condition at the boundary formulated by Miksis and Devis [13] in this paper. The plan of this paper is as follows. The mathematical formulation subjected to the boundary and surface conditions is given in Section 2. The expression for the dispersion relation is derived using the basic equations with boundary and surface conditions in section 3. The cutoff and maximum wave numbers and the corresponding maximum growth rate are also obtained in section 4 and some important conclusions are drawn in final section of this paper.

2 Mathematical Formulation

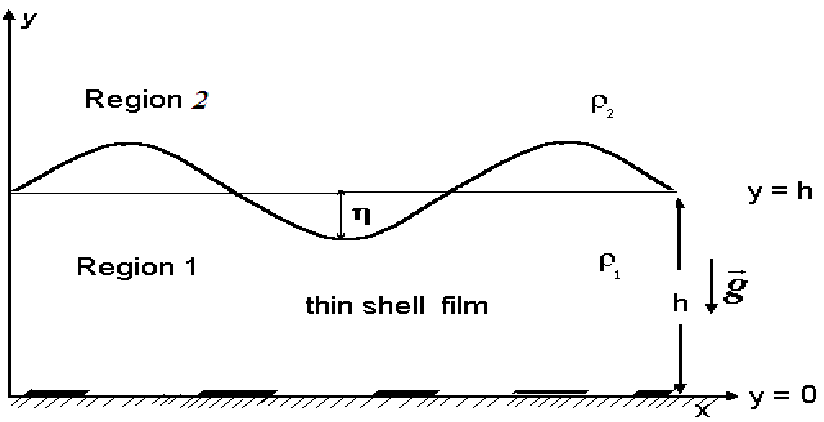

Fig. 1: Physical configuration

The physical configuration is shown in Fig.1. It consists of a thin target shell in the form of a thin film of unperturbed thickness *h* (region 1) filled with an incompressible, viscous, poorly electrically conducting light couple-stress fluid of density ρ_1 bounded below by a rough rigid surface at *y=0* and above by dense incompressible, viscous poorly conducting clear fluid of density ρ_2 of large extent compared to the shell thickness *h*. The fluid in the thin film is set in motion by acceleration normal to the interface whereas in the clear fluid it is assumed to be static and small perturbations are amplified when acceleration is directed from the lighter fluid in the thin film to the heavy clear fluid above the interface. This instability at the interface by definition of Rayleigh-Taylor instability (ERTI). To investigate this RTI, we consider a rectangular coordinate system (*x*, *y*) with the xaxis parallel to the film and y-axis normal to it. The interface between the clear fluid and thin film (couple stress fluid) is described by $\eta(x,t)$ as the perturbed interface between two fluids in regions -1 and 2, where region-2 is a region of dense liquid and region-1 is a region of light couple stress liquid.

To investigate the problems posed in the paper the following combined lubrication and Stokes approximations are used.

(i) The clear dense liquid is homogeneous and isotropic.
(ii) The film thickness h is much smaller than the thickness H of the porous layer bounded above the film. That is, $h << H$.
(iii) The Strouhal number S is assumed to be negligibly small.
(iv) The surface elevation η is assumed to be small compared to film thickness h. That is, $\eta << h$.
(v) Nonuniform polarization and electric charge injection are negligible.
(vi) The fluid viscosity and thermal conductivity are assumed to be constants.

Following these assumptions and approximations, the basic equations are

$$\nabla\cdot\vec{q}=0 \tag{2.1}$$

$$\rho\left(\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right)=-\nabla p+\mu\nabla^{2}\vec{q}-\lambda\nabla^{4}\vec{q} \tag{2.2}$$

$$\sigma=\sigma_0\left[1+\alpha_h(C-C_0)\right] \tag{2.3}$$

where $\vec{q}=(u,v)$ the fluid velocity, λ the couple-stress parameter, ε_e the dielectric constant, p the pressure, C the concentration, σ_0 the electrical conductivity at the reference concentration C_0, α_h is the volumetric expansion coefficient of σ, μ the fluid viscosity and ρ the fluid density.

Let us non-dimensionalize the equations using

$$x^*=\frac{x}{h},\, y^*=\frac{y}{h},\, u^*=\frac{u}{\delta h^2/\mu},\, v^*=\frac{v}{\delta h^2/\mu},\, p^*=\frac{p}{\delta h} \tag{2.4}$$

Following the assumptions and approximations as stated above (i.e., Stokes and lubrication approximations), assuming that the heavy fluid in the porous layer is almost static because of creeping flow approximation and substituting Eq.(2.4) into Eqs.(2.1) and (2.2), we obtain (after neglecting the asterisks for simplicity)

$$0=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} \tag{2.5}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M_0^2 \frac{\partial^4 u}{\partial y^4} \tag{2.6}$$

$$0 = -\frac{\partial p}{\partial y} \tag{2.7}$$

where $M_0 = \sqrt{\lambda / \mu h^2}$ is the couple-stress parameter.

3 Dispersion Relation

To find the dispersion relation, first we have to find the velocity distribution from Eq. (2.6) using the following boundary and surface conditions in addition to couple-stress boundary conditions.

(i) Roughness condition

$$-\beta \frac{\partial u}{\partial y} = u \quad at \quad y = 0 \tag{3.1}$$

(ii) no-shear condition:

$$\frac{\partial u}{\partial y} = 0 \quad at \quad y = 1 \tag{3.2}$$

(iii) Couple-stress conditions:

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad at \quad y = 0 \; \& \; 1 \tag{3.3}$$

(iv) Kinematic condition: $v = \frac{\partial \eta}{\partial t}$ at $y = 1$ (3.4)

(v) Dynamic condition: $p = -\eta - \frac{1}{B}\frac{\partial^2 \eta}{\partial x^2}$ at $y = 1$. (3.5)

Where $B = \delta h^2 / \gamma$ is the Bond number, β is the roughness parameter and $\eta = \eta(x, y, t)$ is the elevation of the interface.

The solution of (2.6) subject to the above conditions is

$$u = C_1 + C_2 y + C_3 cosh(\frac{y}{M_0}) + C_4 sinh(\frac{y}{M_0}) + \frac{P}{2} M_0 y^2 \tag{3.6}$$

Where

$$a_0 = \frac{M_0^3(-1+\cosh(1/M_0))}{\sinh(1/M_0)}$$

$$a_1 = M_0^2 \sinh(1/M_0) - \frac{a_0}{M_0}\cosh(1/M_0) - M_0$$

$$a_2 = -\beta a_1 + M_0^3 - \frac{\beta a_0}{M_0}, \quad P = \frac{\partial p}{\partial x}$$

$$C_1 = a_2 P, \quad C_2 = a_1 P, \quad C_3 = -PM_0^3, \quad C_4 = a_0 P.$$

After integrating Eq. (2.5) with respect to y between $y = 0$ and 1 and using Eq.(3.6), we get

$$v(1) = v_1 = -\int_0^1 \frac{\partial u}{\partial x} dy = \frac{\partial^2 p}{\partial x^2} N \tag{3.7}$$

Where

$$N = \frac{2M_0^4 \sinh(1/M_0) + a_0 M_0 - a_0 M_0 \cosh(1/M_0) - a_2 - (a_1/2) - (M_0/6)}{M_0^2}$$

Then Eq. (3.4), using Eqs. (3.5) and (3.7), becomes

$$\frac{\partial \eta}{\partial t} = -\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B}\frac{\partial^4 \eta}{\partial x^4}\right] N. \tag{3.8}$$

To investigate the growth rate, n, of the periodic perturbation of the interface, we look for the solution of Eq. (3.8) in the form

$$\eta = \eta(y)\exp\{i\ell x + nt\} \tag{3.9}$$

where ℓ is the wave number and $\eta(y)$ is the amplitude of perturbation of the interface.

Substituting Eq. (3.9) into Eq.(3.8), we obtain the dispersion relation in the form

$$n = \ell^2 \left(1 - \frac{\ell^2}{B}\right) N. \tag{3.10}$$

In the absence of couple-stress parameters, that is $M_0 \to 0$, the growth rate given by Eq.(3.10) reduces to n_b. Now the dispersion formula can be expressed in the form

$$n = n_b - \ell \beta v_a \tag{3.11}$$

Where

$$n_b = \frac{\ell^2}{3}\left(1-\frac{\ell^2}{B}\right), \quad v_a = N\ell\left\{1-\frac{\ell^2}{B}\right\}, \qquad \beta = \frac{\left(\frac{1-3N}{3N}\right)\left(1-\frac{\ell^2}{B}\right)}{\left(1--\frac{\ell^2}{B}\right)}.$$

Setting $n = 0$ in Eq.(3.10), we obtain the cut-off wavenumber, ℓ_{ct} in the form

$$\ell_{ct} = \sqrt{B}\,. \tag{3.12}$$

The maximum wave number, ℓ_m obtained from Eq.(3.10) by setting $\partial n / \partial \ell = 0$ is

$$\ell_m = \sqrt{\frac{B}{2}} = \frac{\ell_{ct}}{\sqrt{2}}. \tag{3.13}$$

The corresponding maximum growth rate, n_m for applied voltage opposing gravity is

$$n_m = \frac{B}{4}N\,. \tag{3.14}$$

Similarly, using $\ell_m = \sqrt{B/2}$, we obtain

$$n_{bm} = \frac{B}{12}\,. \tag{3.15}$$

Therefore,

$$G_m = \frac{n_m}{n_{bm}} = 3N\,. \tag{3.16}$$

The growth rate given by Equation (3.10) is computed numerically for different values of parameters and the results are presented graphically in **Figures 2-4**.

4 Results and Discussion

In this study we have shown the effect of boundary roughness on RTI in couple-stress fluid above by a clear dense fluid and below by rigid rough surface. Numerical calculations were performed to determine the growth rate at different wave numbers for various fluid properties like couple stress parameter M_0, Bond number B and roughness parameter β. We have plotted the dimensionless growth rate of the perturbation against the dimensionless wavenumber for some of the cases only.

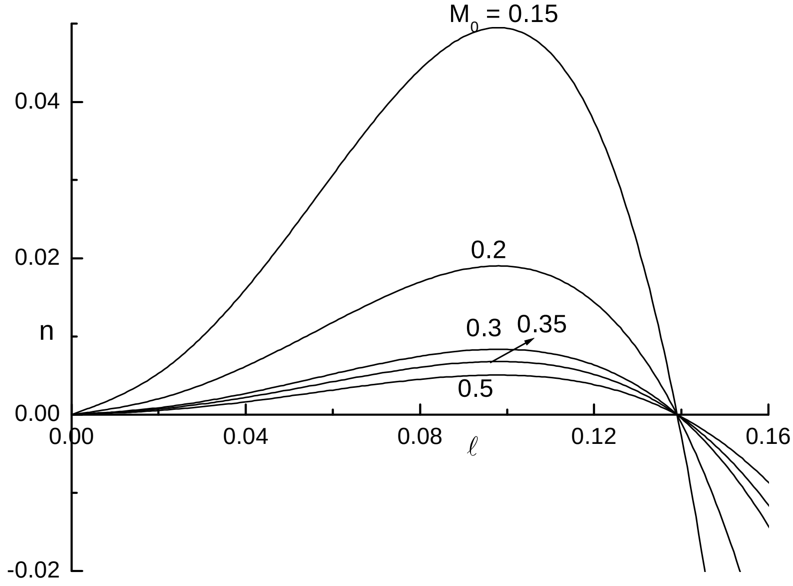

Figure 2: Growth rate, *n* versus the wavenumber, ℓ for different values of couple stress parameter, M_0 when B = *0.02* and $\beta = 3.3 \times 10^{-3}$

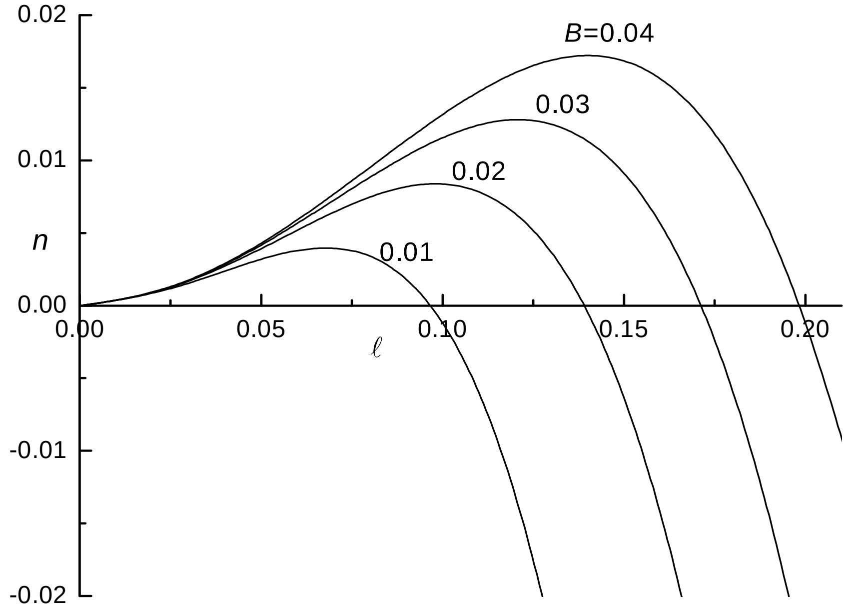

Figure 3: Growth rate, *n* versus the wavenumber, ℓ for different values of Bond number *B* when $M_0 = 0.3$ and $\beta = 3.3 \times 10^{-3}$

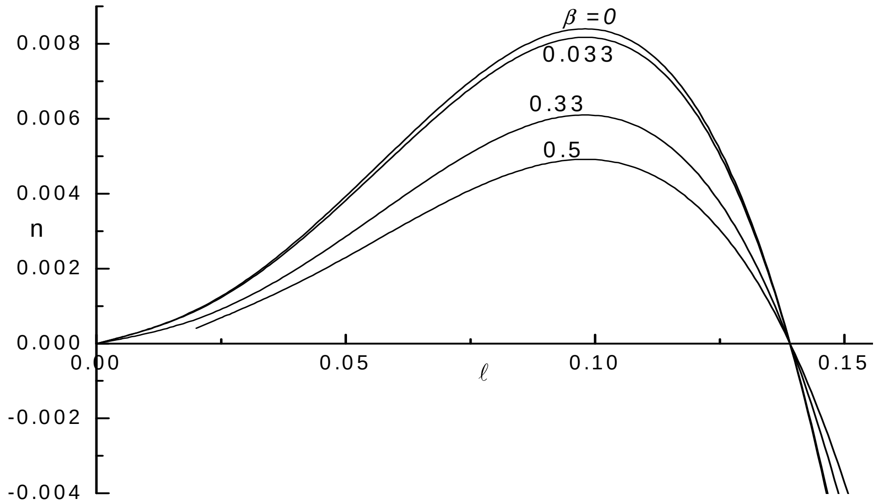

Figure 4: Growth rate, *n* versus the wavenumber, ℓ for different values of Roughness parameter, β when B = *0.02* and M_0=0.3

When we fix all the input parameters except the ratio of the Hartmann number *M*, we find that the higher the couple –stress parameter the more stable the interface is. In Figure 2, we have plotted the growth rate against the wavenumber in the case where B = *0.02* and $\beta = 3.3 \times 10^{-3}$ for different values of the couple-stress parameter M_0. Increasing the couple-stress ratio results in slightly increasing the critical wavenumber and decreasing the maximum growth rate this is because of the action of the body couples on the system. Thus it has a stabilizing effect for the selected values of input parameters due to the increased in couple-stress parameter.

In addition, we have investigated the effect of the surface tension of the fluid on the instability of the interface. In our sample calculations, we have taken $M_0 = 0.3$ and $\beta = 3.3 \times 10^{-3}$ and varied the Bond number *B*. For this input parameters, the critical wavenumber and maximum growth rate decreased as the ratio of the Bond number *B* decreased from 0.4 to 0.1 as observed in Figure 3. The Bond number is reciprocal of surface tension and thus showing that an increase in surface tension decreases the growth rate and hence make the interface more stable.

However, in order to understand the effect of surface roughness properties on the instability, we now fix values of other parameters B = *0.02* and M_0=0.3 and vary the ratios of the roughness parameter β. We note that an increase in surface roughness parameter decreases the growth rate of the interface; this is because the resistance offered by the surface roughness should be overcome, in that process a part of kinetic energy is converted into potential energy. Hence the effect of surface roughness is to reduce the growth rate of the interface and hence to make the system stable.

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