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On Generalized (σ,τ)-n-Derivations in Prime Near–Rings

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Abstract

In this paper, we investigate prime near – rings with generalized (σ,τ)- n-derivations satisfying certain differential identities. Consequently, some well known results have been generalized.

Keywords: *Prime near-ring, (σ,τ)- n-derivations, Generalized (σ,τ)- n-derivations.*

1 Introduction

A right near – ring (resp. left near ring) is a set N together with two binary operations (+) and (.) such that (i) (N,+) is a group (not necessarily abelian). (ii) (N,.) is a semi group. (iii) For all a,b,c ϵ N ; we have (a + b).c = a.c + b.c (resp. a.(b + c) = a.b + b.c . Throughout this paper, N will be a zero symmetric left near – ring (i.e., a left near-ring N satisfying the property 0.x = 0 for all x ∈ N). We will denote the product of any two elements x and y in N, i.e.; x.y by xy. The symbol Z will denote the multiplicative centre of N, that is Z = {x ϵ N, xy = yx for all y ϵ N}. For any x, y ∈ N the symbol [x, y] = xy - yx and (x, y) = x + y – x - y stand for multiplicative commutator and additive commutator of x and y respectively. Let σ and τ be two endomorphisms of N. For any $x, y \in N$, set the symbol $[x, y]_{\sigma,\tau}$ will denote $x\sigma(y) - \tau(y)x$, while the symbol $(x \circ y)_{\sigma,\tau}$ will denote $x\sigma(y) + \tau(y)x$. N is called a prime near-ring if $xNy = \{0\}$ implies that either x = 0 or y = 0. For terminologies concerning near-rings, we refer to Pilz [9].

An additive endomorphism $d:N \longrightarrow N$ is called a derivation if $d(xy) = xd(y) + d(x)y$, (or equivalently $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$, as noted in [10, proposition 1]. The concept of derivation has been generalized in several ways by various authors. The notion of (σ,τ) derivation has been already introduced and studied by Ashraf [1]. An additive endomorphism $d:N \longrightarrow N$ is said to be a (σ,τ) derivation if $d(xy) = \sigma(x)d(y) + d(x)\tau(y)$, (or equivalently $d(xy) = d(x)\tau(y) + \sigma(x)d(y)$ for all $x, y \in N$, as noted in [1, Lemma 2.1].

The notions of symmetric bi-(σ,τ) derivation and permuting tri-(σ,τ) derivation have already been introduced and studied in near-rings by Ceven [6] and Öztürk [7], respectively.

Motivated by the concept of tri-derivation in rings, Park [8] introduced the notion of permuting n-derivation in rings. Further, the authors introduced and studied the notion of permuting n-derivation in near-rings (for reference see [2]). In [4] Ashraf introduced the notion of generalized n-derivation in near-ring N and investigate several identities involving generalized n-derivations of a prime near-ring N which force N to be a commutative ring.

Inspired by these concepts, Ashraf [3] introduced (σ,τ)-n-derivation in near-rings and studied its various properties.

Let n be a fixed positive integer. An n-additive (i.e.; additive in each argument) mapping $d:\underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ is called (σ,τ)-n-derivation of N if there exist outomorphisms $\sigma, \tau: N \longrightarrow N$ such that the equations

$$d(x_1x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \ldots, x_n)$$

$$d(x_1, x_2x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)\sigma(x_2') + \tau(x_2)d(x_1, x_2', \ldots, x_n)$$

$$\vdots$$

$$d(x_1, x_2, \ldots, x_nx_n') = d(x_1, x_2, \ldots, x_n)\sigma(x_n') + \tau(x_n)d(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in N$

Lemma 1.1 [5] *Let N be a prime near-ring. If there exists a non-zero element z of Z such that z + z ∈ Z, then (N, +) is abelian.*

Lemma 1.2 [1] *Let N be a prime near-ring and d be a nonzero (σ, τ)-derivation on N. Then xd(N) = {0} or d(N)x = {0}, implies x = 0.*

Lemma 1.3 [3] *Let N be a near-ring, then d is a (σ,τ)-n-derivation of N if and only if*

$d(x_1 x_1', x_2, \dots, x_n) = \tau(x_1)d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_1')$

$d(x_1, x_2 x_2', \dots, x_n) = \tau(x_2)d(x_1, x_2', \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_2')$

$\vdots$

$d(x_1, x_2, \dots, x_n x_n') = \tau(x_n)d(x_1, x_2, \dots, x_n') + d(x_1, x_2, \dots, x_n)\sigma(x_n')$

Lemma 1.4 [3] *Let N be a near-ring and d be a (σ,τ)-n-derivation of N, then*

$(d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n))y =$

$d(x_1, x_2, \dots, x_n)\sigma(x_1')y + \tau(x_1)d(x_1', x_2, \dots, x_n)y$

$(d(x_1, x_2, \dots, x_n)\sigma(x_2') + \tau(x_2)d(x_1, x_2', \dots, x_n))y =$

$d(x_1, x_2, \dots, x_n)\sigma(x_2')y + \tau(x_2)d(x_1, x_2', \dots, x_n)y$

$\vdots$

$(d(x_1, x_2, \dots, x_n)\sigma(x_n') + \tau(x_n)d(x_1, x_2, \dots, x_n'))y =$

$d(x_1, x_2, \dots, x_n)\sigma(x_n')y + \tau(x_n)d(x_1, x_2, \dots, x_n')y$

hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n', y \in N$.

Lemma 1.5 [3] *Let N be a near-ring and d be a (σ,τ)-n-derivation of N, then*

$(\tau(x_1)d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_1'))y =$

$\tau(x_1)d(x_1', x_2, \dots, x_n)y + d(x_1, x_2, \dots, x_n)\sigma(x_1')y$

$(\tau(x_2)d(x_1, x_2', \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_2'))y =$

$\tau(x_2)d(x_1, x_2', \dots, x_n)y + d(x_1, x_2, \dots, x_n)\sigma(x_2')y$

$\vdots$

$(\tau(x_n)d(x_1, x_2, \dots, x_n') + d(x_1, x_2, \dots, x_n)\sigma(x_n'))y =$

$\tau(x_n)d(x_1, x_2, \dots, x_n')y + d(x_1, x_2, \dots, x_n)\sigma(x_n')y$

hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n', y \in N$.

Lemma 1.6 [3] *Let N be a prime near-ring, d a nonzero (σ,τ)-n-derivation of N and x ϵ N.*

(i) If $d(N, N, \dots, N)x = \{0\}$, then $x = 0$.

(ii) If xd(N, N, . . ., N) ={0}, then x = 0.

In the present paper, we define generalized (σ,τ)-n-derivation in near-rings and study some properties involved there, which gives a generalization of (σ,τ)-n-derivation of near-rings.

2 Generalized (σ,τ)-n-Derivation on Prime Near-Rings

Definition 2.1 *Let N be a near-ring and d be (σ,τ)-n-derivation of N. An n-additive mapping* $f:\underbrace{N\times N\times\ldots\times N}_{n-times}\longrightarrow N$ *is called a right generalized (σ,τ)-n-derivation associated with (σ,τ)-n-derivation d if the relations*

$$f(x_1x_1', x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \ldots, x_n)$$

$$f(x_1, x_2x_2', \ldots, x_n) = f(x_1, x_2, \ldots, x_n)\sigma(x_2') + \tau(x_2)d(x_1, x_2', \ldots, x_n)$$

$$\vdots$$

$$f(x_1, x_2, \ldots, x_nx_n') = f(x_1, x_2, \ldots, x_n)\sigma(x_n') + \tau(x_n)d(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in N$.

Example 2.2 *Let n be a fixed positive integer and R be a commutative ring and S be zero symmetric left near-ring which is not a ring such that (S,+) is abelian, it can be easily verified that the set M = R × S is a zero symmetric left near-ring with respect to component wise addition and multiplication. Now suppose that*

$$N_1 = \left\{\begin{pmatrix}(0,0) & (x,x')\\(0,0) & (y,y')\end{pmatrix} \mid (x,x'),(y,y'),(0,0)\in M\right\}$$

It can be easily seen that N is a non commutative zero symmetric left near-ring with respect to matrix addition and matrix multiplication.

Define $d_1, f_1 : N_1\times N_1\times\cdots\times N_1\rightarrow N_1$ and $\sigma_1, \tau_1: N_1\rightarrow N_1$ such that

$$d_1\left(\begin{pmatrix}(0,0) & (x_1,x_1')\\(0,0) & (y_1,y_1')\end{pmatrix}, \begin{pmatrix}(0,0) & (x_2,x_2')\\(0,0) & (y_2,y_2')\end{pmatrix}, \ldots, \begin{pmatrix}(0,0) & (x_n,x_n')\\(0,0) & (y_n,y_n')\end{pmatrix}\right)$$

$$= \begin{pmatrix}(0,0) & (y_1\,y_2\,\ldots y_n,0)\\(0,0) & (0,0)\end{pmatrix}$$

$$f_1\left(\begin{pmatrix}(0,0) & (x_1,x_1')\\(0,0) & (y_1,y_1')\end{pmatrix}, \begin{pmatrix}(0,0) & (x_2,x_2')\\(0,0) & (y_2,y_2')\end{pmatrix}, \ldots, \begin{pmatrix}(0,0) & (x_n,x_n')\\(0,0) & (y_n,y_n')\end{pmatrix}\right)$$

$$= \begin{pmatrix}(0,0) & (0,0)\\(0,0) & (y_1\,y_2\,\ldots y_n,0)\end{pmatrix}$$

$$\sigma_1\left(\begin{pmatrix}(0,0) & (x,x')\\ (0,0) & (y,y')\end{pmatrix}\right)=\begin{pmatrix}(0,0) & (-x,-x')\\ (0,0) & (y,y')\end{pmatrix},$$

$$\tau_1\left(\begin{pmatrix}(0,0) & (x,x')\\ (0,0) & (y,y')\end{pmatrix}\right)=\begin{pmatrix}(0,0) & (x,-x')\\ (0,0) & (y,y')\end{pmatrix}$$

It can be easily verified that d_1 is a (σ_1,τ_1)-n-derivation of N_1 and f_1 is a right (but not left) generalized(σ_1,τ_1)-n-derivation associated with d_1, where σ_1 and τ_1 are automorphisms.

Definition 2.3 *Let N be a near-ring and d be (σ,τ)-n-derivation of N. An n-additive mapping* $f: \underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ *is called a left generalized (σ,τ)-n-derivation associated with(σ,τ)-n-derivation d if the relations*

$$f(x_1x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \ldots, x_n)$$

$$f(x_1, x_2x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)\sigma(x_2') + \tau(x_2)f(x_1, x_2', \ldots, x_n)$$

$$\vdots$$

$$f(x_1, x_2, \ldots, x_nx_n') = d(x_1, x_2, \ldots, x_n)\sigma(x_n') + \tau(x_n)f(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in N$.

Example 2.4 *Let M be a zero symmetric left near-ring as defined in Example 2.2. Now suppose that*

$$N_2=\left\{\begin{pmatrix}(x,x') & (y,y')\\ (0,0) & (0,0)\end{pmatrix} \mid (x,x'),(y,y'),(0,0)\in M\right\}$$

It can be easily seen that N_2 is a non commutative zero symmetric left near-ring with respect to matrix addition and matrix multiplication.

Define $d_2, f_2 : N_2\times N_2\times\cdots\times N_2\longrightarrow N_2$ and $\sigma_2, \tau_2: N_2\longrightarrow N_2$ such that

$$d_2\left(\begin{pmatrix}(x_1,x_1') & (y_1,y_1')\\ (0,0) & (0,0)\end{pmatrix},\begin{pmatrix}(x_2,x_2') & (y_2,y_2')\\ (0,0) & (0,0)\end{pmatrix},\ldots,\begin{pmatrix}(x_n,x_n') & (y_n,y_n')\\ (0,0) & (0,0)\end{pmatrix}\right)$$

$$=\begin{pmatrix}(0,0) & (x_1\, x_2\, \ldots x_n, 0)\\ (0,0) & (0,0)\end{pmatrix}$$

$$f_2\left(\begin{pmatrix}(x_1,x_1') & (y_1,y_1')\\ (0,0) & (0,0)\end{pmatrix},\begin{pmatrix}(x_2,x_2') & (y_2,y_2')\\ (0,0) & (0,0)\end{pmatrix},\ldots,\begin{pmatrix}(x_n,x_n') & (y_n,y_n')\\ (0,0) & (0,0)\end{pmatrix}\right)$$

$$=\begin{pmatrix}(x_1\, x_2\, \ldots x_n, 0) & (0,0)\\ (0,0) & (0,0)\end{pmatrix}$$

$$\sigma_2\left(\begin{pmatrix}(x,x') & (y,y')\\(0,0) & (0,0)\end{pmatrix}\right)=\begin{pmatrix}(x,x') & (y,-y')\\(0,0) & (0,0)\end{pmatrix},$$

$$\tau_2\left(\begin{pmatrix}(x,x') & (y,y')\\(0,0) & (0,0)\end{pmatrix}\right)=\begin{pmatrix}(x,x') & (-y,-y')\\(0,0) & (0,0)\end{pmatrix}$$

It can be easily verified that d_2 is a (σ_2,τ_2)-n-derivation of N_2 and f_2 is a left (but not right) generalized(σ_2,τ_2)-n-derivation associated with d_2, where σ_2 and τ_2 are automorphisms.

Definition 2.5 *Let N be a near-ring and d be (σ,τ)-n-derivation of N. An n-additive mapping* $f:\underbrace{N\times N\times\ldots\times N}_{n-times}\longrightarrow N$ *is called a generalized (σ,τ)-n-derivation associated with(σ,τ)-n-derivation d if it is both a right generalized (σ,τ)-n-derivation associated with(σ,τ)-n-derivation d as well as a left generalized (σ,τ)-n-derivation associated with(σ,τ)-n-derivation d.*

Example 2.6 *Let M be a zero symmetric left near-ring as defined in Example 2.2. Now suppose that*

$$N_3=\left\{\begin{pmatrix}(0,0) & (x,x') & (y,y')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z,z')\end{pmatrix},(x,x'),(y,y'),(z,z'),(0,0)\in M\right\}$$

It can be easily seen that N_3 is a non commutative zero symmetric left near-ring with respect to matrix addition and matrix multiplication.

Define d_3, f_3:$N_3\times N_3\times\cdots\times N_3\longrightarrow N_3$ and σ_3, τ_3: $N_3\longrightarrow N_3$ such that

$$d_3\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (y_1,y_1')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_1,z_1')\end{pmatrix},\begin{pmatrix}(0,0) & (x_2,x_2') & (y_2,y_2')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_2,z_2')\end{pmatrix},\ldots,\begin{pmatrix}(0,0) & (x_n,x_n') & (y_{n1},y_n')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_n,z_n')\end{pmatrix}\right)$$

$$=\begin{pmatrix}(0,0) & (x_1x_2\ldots x_n,0) & (0,0)\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (0,0)\end{pmatrix}$$

$$f_3\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (y_1,y_1')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_1,z_1')\end{pmatrix},\begin{pmatrix}(0,0) & (x_2,x_2') & (y_2,y_2')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_2,z_2')\end{pmatrix},\ldots,\begin{pmatrix}(0,0) & (x_n,x_n') & (y_{n1},y_n')\\(0,0) & (0,0) & (0,0)\\(0,0) & (0,0) & (z_n,z_n')\end{pmatrix}\right)$$

$$=\begin{pmatrix}(0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (0,0)\end{pmatrix}$$

$$\sigma_3\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (y_1,y_1')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (z_1,z_1')\end{pmatrix}\right)=\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (-y_1,-y_1')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (z_1,z_1')\end{pmatrix}\right),$$

$$\tau_3\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (y_1,y_1')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (z_1,z_1')\end{pmatrix}\right)=\left(\begin{pmatrix}(0,0) & (x_1,x_1') & (y_1,y_1')\\ (0,0) & (0,0) & (0,0)\\ (0,0) & (0,0) & (-z_1,-z_1')\end{pmatrix}\right)$$

It can be easily seen that d_3 is (σ_3, τ_3)-n-derivations of N and f_3 is a nonzero generalized (σ_3, τ_3)-n-derivations associated with d_3, where σ_3 and τ_3 are automorphisms of near-rings N_3.

If f = d then generalized (σ,τ)-n-derivation is just (σ,τ)-n-derivation. If σ = τ = 1, the identity map on N, then generalized (σ,τ)-n-derivation is simply a generalized n-derivation. If σ = τ = 1 and d = f, then generalized (σ,τ)-n-derivation is an n-derivation. Hence the class of generalized (σ,τ)-n-derivations includes those of n-derivations, generalized n-derivations and (σ,τ)-n-derivation.

Lemma 2.7 *Let N be a near-ring, then*

(i) f is a right generalized (σ,τ)-n-derivation of N associated with (σ,τ)-n-derivation d if and only if

$$f(x_1x_1', x_2, \dots, x_n) = \tau(x_1)d(x_1', x_2, \dots, x_n) + f(x_1, x_2, \dots, x_n)\sigma(x_1')$$

$$f(x_1, x_2x_2', \dots, x_n) = \tau(x_2)d(x_1, x_2', \dots, x_n)+f(x_1, x_2, \dots, x_n)\sigma(x_2')$$

$$\vdots$$

$$f(x_1, x_2, \dots, x_nx_n') = \tau(x_n)d(x_1, x_2, \dots, x_n')+f(x_1, x_2, \dots, x_n)\sigma(x_n')$$

for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in N$.

(ii) f is a left generalized (σ,τ)-n-derivation of N if and only if

$$f(x_1x_1', x_2, \dots, x_n) = \tau(x_1)f(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_1')$$

$$f(x_1, x_2x_2', \dots, x_n) = \tau(x_2)f(x_1, x_2', \dots, x_n)+d(x_1, x_2, \dots, x_n)\sigma(x_2')$$

$$\vdots$$

$$f(x_1, x_2, \dots, x_nx_n') = \tau(x_n)f(x_1, x_2, \dots, x_n')+d(x_1, x_2, \dots, x_n)\sigma(x_n')$$

for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in N$.

Proof:

(i) By hypothesis, we get for all $x_1, x_1', x_2, \dots, x_n \in N$.

$f(x_1(x_1' + x_1'), x_2, \dots, x_n)$

$= f(x_1, x_2, \dots, x_n)\sigma(x_1' + x_1') + \tau(x_1)d(x_1' + x_1', x_2, \dots, x_n)$

$$= f(x_1, x_2, \dots, x_n)\sigma(x_1') + f(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n) + \tau(x_1)d(x_1', x_2, \dots, x_n) \quad (1)$$

And

$f(x_1(x_1' + x_1'), x_2, \dots, x_n)$

$= f(x_1x_1' + x_1x_1', x_2, \dots, x_n)$

$= f(x_1x_1', x_2, \dots, x_n) + f(x_1x_1', x_2, \dots, x_n)$

$$= f(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n) + f(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n) \quad (2)$$

Comparing the two equations (1) and (2), we conclude that

$$f(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n) = \tau(x_1)d(x_1', x_2, \dots, x_n) + f(x_1, x_2, \dots, x_n)\sigma(x_1')$$

for all $x_1, x_1', x_2, \dots, x_n \in N$.

Similarly we can prove the remaining $(n-1)$ relations. Converse can be proved in a similar manner.

(ii) Use same arguments as used in the proof of (i).

Lemma 2.8 *Let N be a near-ring admitting a right generalized (σ,τ)-n-derivation f associated with (σ,τ)-n-derivation d of N, then*

$$(f(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \dots, x_n))y = f(x_1, x_2, \dots, x_n)\sigma(x_1')y + \tau(x_1)d(x_1', x_2, \dots, x_n)y$$

$(f(x_1,x_2,\dots,x_n)\sigma(x_2{}') + \tau(x_2)d(x_1,x_2{}',\dots,x_n))y =$

$$f(x_1,x_2,\dots,x_n)\sigma(x_2{}')y + \tau(x_2)d(x_1,x_2{}',\dots,x_n)y$$

$$\vdots$$

$(f(x_1,x_2,\dots,x_n)\sigma(x_n{}') + \tau(x_n)d(x_1,x_2,\dots,x_n{}'))y =$

$$f(x_1,x_2,\dots,x_n)\sigma(x_n{}')y + \tau(x_n)d(x_1,x_2,\dots,x_n{}')y$$

hold for all $x_1,x_1{}',x_2,x_2{}',\dots,x_n,x_n{}',y \in N$.

Proof: for all $x_1,x_1{}',x_1{}'',x_2,\dots,x_n \in N$, we have

$f\big((x_1x_1{}')x_1{}'',x_2,\dots,x_n\big)$

$= f(x_1x_1{}',x_2,\dots,x_n)\sigma(x_1{}'') + \tau(x_1x_1{}')d(x_1{}'',x_2,\dots,x_n)$

$= (f(x_1,x_2,\dots,x_n)\sigma(x_1{}') + \tau(x_1)d(x_1{}',x_2,\dots,x_n))\sigma(x_1{}'') +$

$$\tau(x_1)\tau(x_1{}')d(x_1{}'',x_2,\dots,x_n). \qquad (3)$$

Also

$f(x_1(x_1{}'x_1{}''),x_2,\dots,x_n)$

$= f(x_1,x_2,\dots,x_n)\sigma(x_1{}'x_1{}'') + \tau(x_1)d(x_1{}'x_1{}'',x_2,\dots,x_n)$

$= f(x_1,x_2,\dots,x_n)\sigma(x_1{}')\sigma(x_1{}'') + \tau(x_1)d(x_1{}',x_2,\dots,x_n)\sigma(x_1{}'') +$

$$\tau(x_1)\tau(x_1{}')d(x_1{}'',x_2,\dots,x_n). \qquad (4)$$

Combining relations (3) and (4), we get

$(f(x_1,x_2,\dots,x_n)\sigma(x_1{}') + \tau(x_1)d(x_1{}',x_2,\dots,x_n))\sigma(x_1{}'') =$
$f(x_1,x_2,\dots,x_n)\sigma(x_1{}')\sigma(x_1{}'') + \tau(x_1)d(x_1{}',x_2,\dots,x_n)\sigma(x_1{}'')$

for all $x_1,x_1{}',x_1{}'',x_2,\dots,x_n \in N$. Since σ is an automorphism, putting y in place of $\sigma(x_1{}'')$, we find that

$(f(x_1,x_2,\dots,x_n)\sigma(x_1{}')+ \tau(x_1)d(x_1{}',x_2,\dots,x_n))y =$

$$f(x_1,x_2,\dots,x_n)\sigma(x_1{}')y + \tau(x_1)d(x_1{}',x_2,\dots,x_n)y.$$

for all $x_1,x_1{}',x_2,\dots,x_n,y \in N$.

Similarly other (n−1) relations can be proved.

Lemma 2.9 *Let N be a near-ring admitting a generalized (σ,τ)-n-derivation f associated with (σ,τ)-n-derivation d of N, then*

$$(d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n))y =$$

$$d(x_1, x_2, \dots, x_n)\sigma(x_1')y + \tau(x_1)f(x_1', x_2, \dots, x_n)y,$$

$$(d(x_1, x_2, \dots, x_n)\sigma(x_2') + \tau(x_2)f(x_1, x_2', \dots, x_n))y =$$

$$d(x_1, x_2, \dots, x_n)\sigma(x_2')y + \tau(x_2)f(x_1, x_2', \dots, x_n)y$$

$$\vdots$$

$$(d(x_1, x_2, \dots, x_n)\sigma(x_n') + \tau(x_n)f(x_1, x_2, \dots, x_n'))y =$$

$$d(x_1, x_2, \dots, x_n)\sigma(x_n')y + \tau(x_n)f(x_1, x_2, \dots, x_n')y$$

hold for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n', y \in N$.

Proof: for all $x_1, x_1', x_1'', x_2, \dots, x_n \in N$, we have

$$f((x_1x_1')x_1'', x_2, \dots, x_n)$$

$$= f(x_1x_1', x_2, \dots, x_n)\sigma(x_1'') + \tau(x_1x_1')d(x_1'', x_2, \dots, x_n)$$

$$= (d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n))\sigma(x_1'') +$$

$$\tau(x_1)\tau(x_1')d(x_1'', x_2, \dots, x_n). \qquad (5)$$

Also

$$f(x_1(x_1'x_1''), x_2, \dots, x_n)$$

$$= d(x_1, x_2, \dots, x_n)\sigma(x_1'x_1'') + \tau(x_1)f(x_1'x_1'', x_2, \dots, x_n)$$

$$= d(x_1, x_2, \dots, x_n)\sigma(x_1')\sigma(x_1'') + \tau(x_1)f(x_1', x_2, \dots, x_n)\sigma(x_1'') +$$
$$\tau(x_1)\tau(x_1')d(x_1'', x_2, \dots, x_n). \qquad (6)$$

Combining relations (5) and (6), we get

$$(d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n))\sigma(x_1'') =$$

$$d(x_1, x_2, \dots, x_n)\sigma(x_1')\sigma(x_1'') + \tau(x_1)f(x_1', x_2, \dots, x_n)\sigma(x_1'')$$

for all $x_1, x_1', x_1'', x_2, \dots, x_n \in N$.

Since σ is an automorphism, putting y in place of $\sigma(x_1^{''})$, in previous equation we find that

$$(d(x_1, x_2, \dots, x_n)\sigma(x_1^{'}) + \tau(x_1)f(x_1^{'}, x_2, \dots, x_n))y =$$

$$d(x_1, x_2, \dots, x_n)\sigma(x_1^{'})y + \tau(x_1)f(x_1^{'}, x_2, \dots, x_n)y$$

for all $x_1, x_1^{'}, x_1^{''}, x_2, \dots, x_n \in N$.

Similarly other (n−1) relations can be proved.

Lemma 2.10 *Let N be a prime near-ring admitting a generalized (σ,τ)-n-derivation f with associated nonzero (σ,τ)-n-derivation d of N and x∊ N.*

(i) If f(N, N, . . ., N)x = {0}, then x = 0.
(ii) If xf(N, N, . . ., N) = {0}, then x = 0

Proof:

(i) By our hypothesis we have

$$f(x_1, x_2, \dots, x_n)x = 0, \text{ for all } x_1, x_2, \dots, x_n \in N \quad (7)$$

Putting x_1x_1' in place of x_1, where $x_1' \in N$, in equation (7) and using Lemma 2.9 we get

$$d(x_1, x_2, \dots, x_n)\sigma(x_1')x + \tau(x_1)f(x_1', x_2, \dots, x_n)x = 0 \text{ for all } x_1, x_1', x_2, \dots, x_n \in N.$$

Using (7) again we get $d(x_1, x_2, \dots, x_n)\sigma(x_1')x = 0$ for all $x_1, x_1', x_2, \dots, x_n \in N$.

Since σ is an automorphism, then we have $d(x_1, x_2, \dots, x_n)Nx = \{0\}$ for all $x_1, x_2, \dots, x_n \in N$. Since $d \neq 0$, primeness of N implies that x = 0.

(ii) It can be proved in a similar way.

3 Commutativity Results for Prime Near-Rings with Generalized (σ,τ)-n-Derivation

In [3, Theorem 3.1] M. Ashraf and M. A. Siddeeque proved that if a prime near-ring N admits a nonzero (σ,τ)-n-derivation d such that $d(N,N,\dots,N) \subseteq Z$, then N is a commutative ring. We have extended this result in the setting of generalized (σ,τ)-n-derivation f of N.

Theorem 3.1 *Let N be a prime near-ring admitting a nonzero generalized (σ,τ)-n-derivation f with associated (σ,τ)-n-derivation d of N. If* $f(N,N,\dots,N) \subseteq Z$, *then N is a commutative ring.*

Proof: Since $f(N,N,...,N) \subseteq Z$ and f is a nonzero generalized (σ,τ)-n-derivation, there exist nonzero elements $x_1, x_2, . . ., x_n \in N$, such that $f(x_1, x_2, . . ., x_n) \in Z\backslash\{0\}$. We have $f(x_1+ x_1, x_2, . . ., x_n) = f(x_1, x_2, . . ., x_n) + f(x_1, x_2, . . ., x_n) \in Z$. By Lemma 1.1 we obtain that (N, +) is abelain. By hypothesis we get

$f(y_1, y_2, . . ., y_n)y = yf(y_1, y_2, . . ., y_n)$ for all $y,y_1, y_2, . . ., y_n \in N$.

Now replacing y_1 by y_1y_1', where $y_1' \in N$, in previous equation we have

$$(d(y_1, y_2, . . ., y_n)\sigma(y_1') + \tau(y_1)f(y_1', y_2, . . ., u_n))\, y =$$

$$y(d(y_1, y_2, . . ., y_n)\sigma(y_1') + \tau(y_1)f(y_1', y_2, . . ., u_n))$$

for all $y, y_1, y_1', y_2, . . ., y_n \in N$. (8)

Putting $\tau(y_1)$ for y in (8) and using Lemma 2.9 we get

$$d(y_1, y_2, . . ., y_n)\sigma(y_1')\tau(y_1) + \tau(y_1)f(y_1', y_2, . . ., y_n)\tau(y_1)$$

$$= \tau(y_1)d(y_1, y_2, . . ., y_n)\sigma(y_1') + \tau(y_1)\tau(y_1)f(y_1', y_2, . . ., y_n)$$

for all $y, y_1, y_1', y_2, . . ., y_n \in N$.

By using hypothesis again the preceding equation reduces to

$$d(y_1, y_2, . . ., y_n)\sigma(y_1')\tau(y_1) = \tau(y_1)d(y_1, y_2, . . ., y_n)\sigma(y_1')$$

for all $y_1,y_1',y_2, . . ., y_n \in N$.

Replacing y_1' by $y_1'x$, where $x \in N$, in previous equation and using it again we get $d(y_1, y_2, . . ., y_n)\sigma(y_1')[\tau(y_1), \sigma(x)] = 0$ for all $x,y_1,y_1',y_2, . . .,y_n \in N$. Since σ is an auotomorphism we conclude that

$d(y_1, y_2, . . ., y_n)N[\tau(y_1), \sigma(x)] = \{0\}$ for all $x,y_1,y_2, . . ., y_n \in N$. Primeness of N implies that for each $y_1 \in N$ either $[\tau(y_1), \sigma(x)] = 0$ for all $x \in N$ or $d(y_1, y_2, . . ., y_n) = 0$ for all $y_2, . . ., y_n \in N$.

If $d(y_1, y_2, . . ., y_n) = 0$ for all $y_2, . . .,y_n \in N$, then equation (8) takes the form $f(y_1', y_2, . . ., y_n)N[y, \tau(y_1)] = \{0\}$. Since $f \neq 0$, primeness of N implies that $[y, \tau(y_1)] = \{0\}$ for all $y \in N$. But τ is an automorphism, we conclude that $y_1 \in Z$. On the other hand if $[\tau(y_1), \sigma(x)] = 0$ for all $x \in N$, then again $y_1 \in Z$, hence we find that N = Z, and N is a commutative ring.

Corollary 3.1 [4, Theorem 3.1] *Let N be a prime near-ring and f a nonzero generalized n-derivation with associated n-derivation d of N. If* $f(N,N,...,N) \subseteq Z$*, then N is a commutative ring.*

Corollary 3.2 [3, Theorem 3.1] *Let N be a prime near-ring and d a nonzero* (σ,τ)*-n-derivation d of N. If* $d(N,N,...,N) \subseteq Z$*, then N is a commutative ring.*

Theorem 3.2 *Let N be a prime near-ring and* f_1 *and* f_2 *be any two generalized* (σ,τ)*-n-derivations with associated nonzero* (σ,τ)*-n-derivations* d_1, d_2 *respectively. If* $[f_1(N, N, . . ., N) , f_2(N, N, . . ., N)] = \{0\}$*, then (N,+) is abelian.*

Proof: Assume that $[f_1(N, N, . . ., N) , f_2(N, N, . . ., N)] = \{0\}$. If both z and z + z commute element wise with $f_2(N, N, . . ., N)$, then for all $x_1, x_2, . . ., x_n \in N$ we have

$$zf_2(x_1, x_2, . . ., x_n) = f_2(x_1, x_2, . . ., x_n)z \qquad (9)$$

and

$$(z + z)f_2(x_1, x_2, . . ., x_n) = f_2(x_1, x_2, . . ., x_n)(z + z) \qquad (10)$$

Substituting $x_1 + x_1'$ instead of x_1 in (10) we get

$(z + z)f_2(x_1 + x_1', x_2, . . ., x_n) = f_2(x_1 + x_1', x_2, . . ., x_n)(z + z)$ for all $x_1 ,x_2,\ldots,x_n \in N$.

From (9) and (10) the previous equation can be reduced to

$zf_2(x_1 + x_1'- x_1- x_1', x_2, . . ., x_n) = 0$ for all $x_1, x_1' ,x_2,\ldots,x_n \in N$.

Which means that

$zf_2((x_1 , x_1'), x_2, . . ., x_n) = 0$ for all $x_1 , x_1',x_2,\ldots,x_n \in N$.

Putting $z = f_1(y_1, y_2, . . ., y_n)$, in previous equation, we get

$f_1(y_1, y_2, . . ., y_n)f_2((x_1 , x_1'), x_2, . . ., x_n) = 0$

for all $x_1, x_1' ,x_2,\ldots,x_n , y_1, y_2, . . ., y_n \in N$.

By Lemma 2.10 (i) we conclude that

$$f_2((x_1 , x_1'), x_2, . . ., x_n) = 0 \text{ for all } x_1, x_1' ,x_2,\ldots,x_n \in N. \qquad (11)$$

Since we know that for each $w \in N$, $w(x_1 , x_1') = w(x_1 + x_1'- x_1- x_1') = wx_1 + wx_1'- wx_1- wx_1' = (wx_1 , wx_1')$ which is again an additive commutator, putting $w(x_1 , x_1')$ instead of (x_1 , x_1') in (11) we get

$f_2(w(x_1 , x_1'), x_2, . . ., x_n) = 0$ for all $x_1, x_1' ,x_2,\ldots,x_n,w \in N$.

Therefore

$d_2(w, x_2, . . ., x_n)\sigma(x_1 , x_1') + \tau(w)f_2((x_1 , x_1'), x_2, . . ., x_n) = 0$ for all $x_1, x_1' ,x_2,\ldots,x_n,w \in N$.

Using (11) in previous equation yields

$d_2(w, x_2, . . ., x_n)\sigma(x_1 , x_1') = 0$ for all $x_1, x_1' ,x_2,\ldots,x_n,w \in N$. Since σ is an automorphism, using Lemma 1.6 (i) we conclude that $(x_1 , x_1') = 0$. Hence (N, +) is abelain.

Corollary 3.3 [4, Theorem 3.16] *Let N be a prime near-ring and f_1 and f_2 be any two generalized n-derivations with associated nonzero n-derivations d_1 , d_2 respectively such that $[f_1(N, N, . . ., N) , f_2(N, N, . . ., N)] = \{0\}$. Then (N,+) is abelian.*

Corollary 3.4 [3, Theorem 3.2] *Let N be a prime near-ring and d_1 and d_2 be any two nonzero (σ,τ)-n-derivations. If $[d_1(N, N, . . ., N) , d_2(N, N, . . ., N)] = \{0\}$ then (N,+) is abelian.*

Theorem 3.3 *Let N be a prime near-ring and f_1 and f_2 be any two generalized (σ,τ)-n-derivations with associated nonzero (σ,τ)-n-derivations d_1, d_2 respectively. If $f_1(x_1, x_2, . . ., x_n)f_2(y_1, y_2, . . ., y_n) + f_2(x_1, x_2, . . ., x_n)f_1(y_1, y_2, . . ., y_n) = 0$ for all $x_1,x_2,\ldots,x_n,y_1,y_2,\ldots,y_n \in N$, then (N,+) is abelian.*

Proof: By our hypothesis we have,

$$f_1(x_1, x_2, . . ., x_n)f_2(y_1 , y_2, . . ., y_n) + f_2(x_1, x_2, . . ., x_n)f_1(y_1 , y_2, . . ., y_n) = 0$$

for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. (12)

Substituting $y_1 + y_1'$ instead of y_1 in (12) we get

$$f_1(x_1, x_2, . . ., x_n)f_2(y_1 + y_1' , y_2, . . ., y_n) +f_2(x_1, x_2, . . ., x_n)f_1(y_1 + y_1', y_2, . . ., y_n) = 0$$

for all $x_1 ,x_2,\ldots,x_n, y_1, y_1' ,y_2,\ldots,y_n \in N$. So we get

$$f_1(x_1, x_2, . . ., x_n)f_2(y_1, y_2, . . ., y_n) + f_1(x_1, x_2, . . ., x_n)f_2(y_1' , y_2, . . ., y_n) +$$
$$f_2(x_1, x_2, . . ., x_n)f_1(y_1, y_2, . . ., y_n) + f_2(x_1, x_2, . . ., x_n)f_1(y_1', y_2, . . ., y_n) = 0$$

for all $x_1 ,x_2,\ldots,x_n, y_1,y_1', y_2,\ldots,y_n \in N$.

Using (12) again in last equation we get

$f_1(x_1,x_2, . . ., x_n)f_2(y_1, y_2, . . ., y_n) + f_1(x_1, x_2, . . ., x_n)f_2(y_1' , y_2, . . ., y_n)$

$+ f_1(x_1, x_2, . . ., x_n)f_2(-y_1, y_2, . . ., y_n) + f_1(x_1, x_2, . . ., x_n)f_2(-y_1', y_2, . . ., y_n) = 0$

for all $x_1 ,x_2,\ldots,x_n, y_1 , y_1', y_2,\ldots,y_n \in N$. Thus, we get

$f_1(x_1, x_2, . . ., x_n)f_2((y_1, y_1'), y_2, . . ., y_n) = 0$ for all $x_1 ,x_2,\ldots,x_n,y_1,y_1', y_2,\ldots,y_n \in N$.

By Lemma 2.10 (i) we obtain

$f_2((y_1, y_1'), y_2, . . ., y_n) = 0$ for all $y_1 , y_1', y_2,\ldots,y_n \in N$.

Now pitting $w(y_1, y_1')$ instead of (y_1,y_1'), where $w \in N$, in previous equation and using it again we get

$d_2(w, y_2, . . ., y_n)\sigma(y_1, y_1') = 0$ for all $w, x_1 ,x_2,\ldots,x_n, y_1 , y_1', y_2,\ldots,y_n \in N$, using Lemma 1.6 (i); as used in the Theorem 3.2, we conclude that (N, +) is abelain.

Corollary 3.5 *Let N be a prime near-ring and f_1 and f_2 be any two generalized n-derivations with associated nonzero n-derivations d_1 , d_2 respectively.*

If $f_1(x_1, x_2, . . ., x_n)f_2(y_1 , y_2, . . ., y_n) + f_2(x_1, x_2, . . ., x_n)f_1(y_1 , y_2, . . ., y_n) = 0$

for all $x_1 ,x_2,\ldots,x_n , y_1 ,y_2,\ldots,y_n \in N$, then (N,+) is abelian.

Corollary 3.6 [3, Theorem 3.3] *Let N be a prime near-ring and d_1 and d_2 be any two nonzero (σ,τ)-n-derivations.*

If $d_1(x_1, x_2, . . ., x_n)d_2(y_1 , y_2, . . ., y_n) + d_2(x_1, x_2, . . ., x_n)d_1(y_1 , y_2, . . ., y_n) = 0$

for all $x_1 ,x_2,\ldots,x_n , y_1 ,y_2,\ldots,y_n \in N$, then (N,+) is abelian.

Theorem 3.4 *Let N be a prime near-ring, let f_1 and f_2 be any two generalized (σ,τ)-n-derivations with associated nonzero (σ,τ)-n-derivations d_1 , d_2 respectively.*

If $f_1(x_1, x_2, . . ., x_n)\sigma f_2(y_1 , y_2, . . ., y_n) + \tau f_2(x_1, x_2, . . ., x_n)f_1(y_1 , y_2, . . ., y_n) = 0$

for all $x_1,x_2,\ldots,x_n,y_1,y_2,\ldots,y_n \in N$, then (N,+) is abelian.

Proof: By our hypothesis we have,

$f_1(x_1, x_2, . . ., x_n)\sigma(f_2(y_1 , y_2, . . ., y_n)) + \tau(f_2(x_1, x_2, . . ., x_n))f_1(y_1 , y_2, . . ., y_n) = 0$

for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. (13)

Substituting $y_1 + y_1'$, where $y_1' \in N$, for y_1 in (13) we get

$f_1(x_1, x_2, . . ., x_n)\sigma(f_2(y_1 + y_1', y_2, . . ., y_n)) +$

$\tau(f_2(x_1, x_2, . . ., x_n))f_1(y_1 + y_1', y_2, . . ., y_n) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \in N$. Thus, we get

$f_1(x_1,x_2,. . .,x_n)\sigma(f_2(y_1,y_2,. . .,y_n)) + f_1(x_1,x_2,. . .,x_n)\sigma(f_2(y_1',y_2, . . .,y_n)) +$

$\tau(f_2(x_1,x_2,. . .,x_n))f_1(y_1,y_2, . . .,y_n) + \tau(f_2(x_1,x_2, . . .,x_n))f_1(y_1',y_2, . . .,y_n) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \in N$.

Using (13) in previous equation implies

$f_1(x_1,x_2, . . .,x_n)\sigma(f_2(y_1,y_2, . . .,y_n)) + f_1(x_1,x_2,. . .,x_n)\sigma(f_2(y_1',y_2, . . .,y_n)) +$

$f_1(x_1,x_2, . . ., x_n)\sigma(f_2(-y_1,y_2,. . .,y_n)) + f_1(x_1,x_2,. . .,x_n)\sigma(f_2(-y_1',y_2, . . .,y_n)) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \in N$.

which means that

$f_1(x_1, x_2, . . ., x_n)\sigma(f_2((y_1, y_1'), y_2, . . ., y_n)) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \in N$.

Now using Lemma 2.10, in previous equation, we conclude that

$\sigma(f_2((y_1, y_1'), y_2, . . ., y_n)) = 0$ for all $y_1, y_1', y_2, \ldots, y_n \in N$.Since σ is an automorphism of N, we conclude that $f_2((y_1, y_1'), y_2, . . ., y_n) = 0$ for all $y_1, y_1', y_2, \ldots, y_n \in N$. Now putting $w(y_1, y_1')$ instead of (y_1, y_1'), where $w \in N$ in last equation and using it again, we get

$d_2(w, y_2, . . ., y_n)\sigma(y_1, y_1') = 0$ for all $y_1, y_1', y_2, \ldots, y_n, w \in N$. Using Lemma 12.6 (i) as used in the Theorem 3.2, we conclude that (N, +) is abelain.

Corollary 3.7 *Let N be a prime near-ring, let f_1 and f_2 be any two generalized n-derivations with associated nonzero n-derivations d_1 , d_2 respectively.*

If $f_1(x_1, x_2, . . ., x_n)f_2(y_1, y_2, . . ., y_n) + f_2(x_1, x_2, . . ., x_n)f_1(y_1, y_2, . . ., y_n) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$, then (N,+) is abelian.

Corollary 3.8 [3, Theorem 3.4] *Let N be a prime near-ring, let d_1 and d_2 be any two nonzero (σ,τ)-n-derivations.*

If $d_1(x_1, x_2, . . ., x_n)\sigma(d_2(y_1, y_2, . . ., y_n)) + \tau(d_2(x_1, x_2, . . ., x_n))d_1(y_1, y_2, . . ., y_n) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$, then (N,+) is abelian.

Theorem 3.5 *Let N be a prime near-ring, let f_1 be a generalized (σ,τ)-n-derivation with associated nonzero (σ,τ)-n-derivation d_1 and f_2 be a generalized n-derivation with associated nonzero n-derivation d_2.*

(i) If $f_1(x_1, x_2, \ldots, x_n)\sigma(f_2(y_1, y_2, \ldots, y_n))+\tau(f_2(x_1, x_2, \ldots, x_n))f_1(y_1, y_2, \ldots, y_n)= 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$, then (N,+) is abelian.

(ii) If $f_2(x_1, x_2, \ldots, x_n)\sigma(f_1(y_1, y_2, \ldots, y_n))+\tau(f_1(x_1, x_2, \ldots, x_n)f_2(y_1, y_2, \ldots, y_n) = 0$

for all $x_1,x_2,\ldots,x_n,y_1,y_2,\ldots,y_n \in N$, then (N,+) is abelian.

Proof: (i) By our hypothesis we have,

$f_1(x_1, x_2, \ldots, x_n)\sigma(f_2(y_1, y_2, \ldots, y_n)) + \tau(f_2(x_1, x_2, \ldots, x_n))f_1(y_1, y_2, \ldots, y_n) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in N$. (14)

Substituting $y_1 + y_1'$, where $y_1' \in N$, for y_1 in (14) we get

$$f_1(x_1, x_2, \ldots, x_n)\sigma(f_2(y_1 + y_1', y_2, \ldots, y_n)) + \tau(f_2(x_1, x_2, \ldots, x_n))f_1(y_1 + y_1', y_2, \ldots, y_n) = 0$$

for all $x_1,x_2,\ldots,x_n,y_1,y_1',y_2,\ldots,y_n \in N$.

So we have

$$f_1(x_1,x_2,\ldots,x_n)\sigma(f_2(y_1,y_2,\ldots, y_n)) + f_1(x_1,x_2,\ldots,x_n)\sigma(f_2(y_1', y_2, \ldots,y_n)) + \tau(f_2(x_1, x_2,\ldots, x_n))f_1(y_1,y_2,\ldots,y_n) + \tau(f_2(x_1,x_2,\ldots, x_n))f_1(y_1',y_2,\ldots,y_n) = 0$$

for all $x_1,x_2,\ldots,x_n,y_1,y_1',y_2,\ldots,y_n \in N$.

Using (14) in previous equation implies

$$f_1(x_1, x_2,\ldots, x_n)\sigma(f_2(y_1,y_2, \ldots, y_n)) + f_1(x_1,x_2, \ldots, x_n)\sigma(f_2(y_1',y_2, \ldots,y_n)) + f_1(x_1,x_2,\ldots,x_n)\sigma(f_2(-y_1 ,y_2, \ldots,y_n)) + f_1(x_1,x_2,\ldots,x_n)\sigma(f_2(-y_1',y_2,\ldots,y_n)) = 0$$

for all $x_1,x_2,\ldots,x_n,y_1 ,y_1',y_2,\ldots,y_n \in N$. Thus, we get

$f_1(x_1, x_2, \ldots, x_n)\sigma(f_2((y_1 , y_1'), y_2, \ldots, y_n)) = 0$

for all $x_1, x_2, \ldots, x_n, y_1, y_1', y_2, \ldots, y_n \in N$.

Now, using Lemma 2.10 we conclude that $\sigma(f_2((y_1,y_1'),y_2,\ldots,y_n)) = 0$ for all $y_1,y_1',y_2,\ldots,y_n \in N$.

Since σ is an automorphism of N, we conclude that

$f_2((y_1, y_1'), y_2, \ldots, y_n) = 0$ for all $y_1, y_1', y_2,\ldots,y_n \in N$. Treating f_2 as generalized (I,I)-n-derivation of N and d_2 as (I,I)-n-derivation, where I is the identity automorphism of N and arguing on similar lines as in case of Theorem 3.2; we conclude that (N, +) is an abelain group.

(ii) Use same arguments as used in the proof of (i).

Corollary 3.9 *Let N be a prime near-ring, let d_1 be a nonzero (σ,τ)-n-derivation and d_2 be a nonzero n-derivation.*

(i) If $d_1(x_1, x_2, \ldots, x_n)\sigma(d_2(y_1, y_2, \ldots, y_n)) + \tau(d_2(x_1, x_2, \ldots, x_n))d_1(y_1, y_2, \ldots, y_n) = 0$ for all $x_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N$, then (N,+) is abelian.

(ii) If $d_2(x_1, x_2, \ldots, x_n)\sigma(d_1(y_1, y_2, \ldots, y_n) + \tau(d_1(x_1, x_2, \ldots, x_n))d_2(y_1, y_2, \ldots, y_n) = 0$ for all $x_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N$, then (N,+) is abelian.

Theorem 3.6 *Let N be a semiprime near-ring. Let f be a generalized (σ,τ)-n-derivation associated with the (σ,τ)-n-derivation d,*

If $\tau(x_1)f(y_1, y_2, \ldots, y_n) = f(x_1, x_2, \ldots, x_n)\sigma(y_1)$ for all $x_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N$, then d = 0.

Proof: By our hypothesis we have,

$$\tau(x_1)f(y_1, y_2, \ldots, y_n) = f(x_1, x_2, \ldots, x_n)\sigma(y_1) \text{ for all } x_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N. \quad (13)$$

Substituting z_1x_1 for x_1 in (13), where $z_1 \in N$, and using Lemma 2.11 we get

$$\begin{aligned}\tau(z_1x_1)f(y_1, y_2, \ldots, y_n) &= f(z_1x_1, x_2, \ldots, x_n)\sigma(y_1)\\ &= d(z_1, x_2, \ldots, x_n)\sigma(x_1)\sigma(y_1) + \tau(z_1)f(x_1, x_2, \ldots, x_n)\sigma(y_1)\end{aligned}$$

for all $x_1, z_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N$.

Using (13) in previous equation we get

$$\begin{aligned}\tau(z_1x_1)f(y_1, y_2, \ldots, y_n) &= d(z_1, x_2, \ldots, x_n)\sigma(x_1)\sigma(y_1) + \tau(z_1)\tau(x_1)f(y_1, y_2, \ldots, y_n)\\ &= d(z_1, x_2, \ldots, x_n)\sigma(x_1)\sigma(y_1) + \tau(z_1x_1)f(y_1, y_2, \ldots, y_n)\end{aligned}$$

for all $x_1, z_1, x_2,\ldots,x_n, y_1, y_2,\ldots,y_n \in N$. This yields that

$d(z_1, x_2, \ldots, x_n)\sigma(x_1)\sigma(y_1) = 0$ for all $x_1,z_1,x_2,\ldots,x_n,y_1,y_2,\ldots,y_n \in N$. Since σ is an automorphism of N, we get

$d(z_1, x_2, \ldots, x_n)uv = 0$ for all $z_1,x_2,\ldots,x_n,u,v \in N$. Now replacing v by $d(z_1, x_2, \ldots, x_n)$ in previous equation we get

$d(z_1, x_2, \ldots, x_n)Nd(z_1, x_2, \ldots, x_n) = \{0\}$ for all $z_1,x_2,\ldots,x_n \in N$.

Semiprimeness of N implies that d = 0.

Corollary 3.10 *Let N be a semiprime near-ring, let f be a generalized n-derivation associated with the n-derivation d, If* $x_1f(y_1, y_2, \ldots, y_n) = f(x_1, x_2, \ldots, x_n)\, y_1$ *for all* $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. *Then d = 0.*

Corollary 3.11 [3, Theorem 3.8] *Let N be a semiprime near-ring, let d be a (σ,τ)-n-derivation,*

If $\tau(x_1)d(y_1, y_2, \ldots, y_n) = d(x_1, x_2, \ldots, x_n)\sigma(y_1)$ for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. Then d = 0.

Example 3.1 *Let S be a zero-symmetric left near-ring. Let us define*

$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, x, y \in S \right\}$ is zero symmetric near-ring with regard to matrix addition and matrix multiplication .

Define $f_1, f_2, d_1, d_2 : \underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ such that

$$f_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1x_2\ldots x_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_1\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & y_1y_2\ldots y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$f_2\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 & y_1y_2\ldots y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d_2\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1x_2\ldots x_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now we define $\sigma, \tau : N \rightarrow N$ by

$$\tau\begin{pmatrix}0 & x & y\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} = \begin{pmatrix}0 & y & x\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

It can be easily seen τ are automorphism of near-rings N which is not semiprime, having d_1, d_2 as a nonzero(σ, τ)-n-derivations and f_1 and f_2 are nonzero generalized (σ, τ)-n-derivations associated with the (σ, τ)-n-derivations d_1 , d_2 respectively where σ = I, the identity outomorphism of N. We also have

(i) $f_1(N,N,...,N) \subseteq Z$

(ii) $[f_1(N, N, . . ., N) , f_2(N, N, . . ., N)] = \{0\}$,

(iii) $f_1(x_1, x_2, . . ., x_n)f_2(y_1 , y_2, . . ., y_n) = - f_2(x_1, x_2, . . ., x_n)f_1(y_1 , y_2, . . ., y_n)$ for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$,

(iv) $f_1(x_1, x_2, . . ., x_n)\sigma(f_2(y_1 , y_2, . . ., y_n)) + \tau(f_2(x_1, x_2, . . ., x_n))f_1(y_1 , y_2, . . ., y_n) = 0$ for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. However (N, +) is non abelain.

(v) $\tau(x_1)f_1(y_1, y_2, . . ., y_n) = f_1(x_1, x_2, . . ., x_n)\sigma(y_1)$ for all $x_1 ,x_2,\ldots,x_n, y_1 ,y_2,\ldots,y_n \in N$. However $d \neq 0$.

Theorem 3.7 *Let N be a prime near-ring, let f be a generalized (σ,τ)-n-derivation associated with the (σ,τ)-n-derivation d,*

If $K = \{ a \in N \mid [f(N, N, . . ., N), \tau(a)] = \{0\}$, then $a \in K$ implies either $d(a, x_2, . . ., x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $a \in Z$**.**

Proof: Assume that $a \in K$, we have

$f(x_1, x_2, . . ., x_n)\tau(a) = \tau(a)f(x_1, x_2, . . ., x_n)$ for all $x_1 ,x_2,\ldots,x_n \in N$. (14)

Putting ax_1 in place of x_1 in (14) and using Lemma 2.9 we get

$$d(a, x_2, . . ., x_n)\sigma(x_1)\tau(a) + \tau(a)f(x_1, x_2, . . ., x_n)\tau(a) = \tau(a)d(a, x_2, . . ., x_n)\sigma(x_1) + \tau(a)\tau(a)f(x_1, x_2, . . ., x_n)$$

for all $x_1 ,x_2,\ldots,x_n \in N$. Using (14) in previous equation we get

$d(a, x_2, . . ., x_n)\sigma(x_1)\tau(a) = \tau(a)d(a, x_2, . . ., x_n)\sigma(x_1)$ for all $x_1 ,x_2,\ldots,x_n \in N$ (15)

Putting x_1y_1, where $y_1 \in N$, for x_1 in (15) and using it again

$d(a, x_2, . . ., x_n)\sigma(x_1)[\sigma(y_1) , \tau(a)] = 0$. Since σ is an automorphism, we get

$d(a, x_2, \ldots, x_n)N[\sigma(y_1),\tau(a)] = \{0\}$ for all $x_1, x_2,\ldots,x_n \in N$. Since σ and τ are automorphisms, primeness of N yields either $d(a, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $a \in Z$.

Theorem 3.8 *Let N be a prime near-ring, let f be a generalized (σ,τ)-n-derivation associated with the (σ,τ)-n-derivation d.*

If $[f(N, N, \ldots, N), a]_{\sigma,\tau} = \{0\}$, then either $d(a, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $a \in Z$.

Proof: For all $x_1, x_2,\ldots, x_n \in N$ we have

$$f(x_1, x_2, \ldots, x_n)\sigma(a) = \tau(a)f(x_1, x_2, \ldots, x_n). \qquad (16)$$

Putting ax_1 in place of x_1 in (16) and using Lemma 2.9 we get

$$d(a, x_2, \ldots, x_n)\sigma(x_1)\sigma(a) + \tau(a)f(x_1, x_2, \ldots, x_n)\sigma(a) =$$

$$\tau(a)d(a, x_2, \ldots, x_n)\sigma(x_1) + \tau(a)\tau(a)f(x_1, x_2, \ldots, x_n)$$

for all $x_1, x_2,\ldots, x_n \in N$. Using (16) in previous equation we get

$$d(a, x_2, \ldots, x_n)\sigma(x_1)\sigma(a) = \tau(a)d(a, x_2, \ldots, x_n)\sigma(x_1) \qquad (17)$$

Putting x_1y_1, where $y_1 \in N$, for x_1 in (17) and using it again

$d(a, x_2, \ldots, x_n)\sigma(x_1)[\sigma(y_1), \sigma(a)] = 0$. Since σ is an automorphism, we have

$d(a, x_2, \ldots, x_n)N [\sigma(y_1), \sigma(a)] = \{0\}$.

Since σ is an automorphisms, Primeness of N yields either $d(a, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $a \in Z$.

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