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Topologically α- Transitive Maps and Minimal Systems

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Abstract

In this paper, we define and introduce a new type of topological transitive map called topological α- transitive and investigate some of its properties. Further, we introduce the notions of α- minimal mapping. We have proved that every topologically α-transitive map is a topologically transitive map as every open set is α-open set but the converse not necessarily true, unless every α-open set is locally closed and that every α-minimal map is a minimal map as every open set is α-open set, but the converse not necessarily true, unless every α-open set is locally closed.

Keywords: Topologically α- transitive, α- minimal maps, α- continuous, α- dense.

1 Introduction

Recently there has been some interest in the notion of a locally closed subset of a topological space. According to Bourbaki [16] a subset S of a space (X,τ) is called locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [13] and [14] to define the concept of LC–continuity, i.e. a function $f:(X,\tau)\to(X,\sigma)$ is LC–continuous if the inverse with respect to f of any open set in Y is closed in X. The study of semi open sets and semi continuity in topological spaces was initiated by Levine [6]. Bhattacharya and Lahiri [8] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets which was introduced by Levine [5]. Throughout this paper, the word "space " will mean topological space The collections of semi-open, semi-closed sets and α–sets in (X,τ) will be denoted by $SO(X,\tau)$, $SC(X,\tau)$ and τ^{α} respectively. Ogata N. [7] has shown that τ^{α} is a topology on X with the following properties: $\tau\subseteq\tau^{\alpha}$, $(\tau^{\alpha})^{\alpha}=\tau^{\alpha}$ and $S\in\tau^{\alpha}$ if and only if $S=U\setminus N$ where $U\in\tau$ and N is nowhere dense $(i.e.\ Int(Cl(N))=\varphi)$ in (X,τ) . Hence $\tau=\tau^{\alpha}$ if and only if every nowhere dense (nwd) set in (X,τ) is closed Hence every transitive map implies α-transitive. Also if $LC(X,\tau)=LC(X,\tau^{\alpha})$ then every transitive map implies α-transitive; and this structure also occurs if $SO(X,\tau)\subseteq LC(X,\tau)$.Clearly every α–set is semi-open and every nwd set in (X,τ) is semi-closed. Andrijevic [1] has observed that $SO(X,\tau^{\alpha})=SO(X,\tau)$, and that $N\subseteq X$ is nwd in (X,τ^{α}) if and only if N is nwd in (X,τ) .

In this paper, we will define a new class of topological transitive maps called topological α- transitive and a new class of α- minimal maps. We will also study some of their properties.

2 Preliminaries and Definitions

In this section, we recall some of the basic definitions. Let X be a topological space and $A\subset X$. The interior (resp. closure) of A is denoted by Int(A) (resp. Cl(A).

Definition 2.1 *[6] A subset A of a topological space X will be termed semi- open (written S.O.) if and only if there exists an open set U such that* $U\subset A\subset Cl(U)$.

Definition 2.2 *[8] Let A be a subset of a space X then semi closure of A defined as the intersection of all semi – closed sets containing A is denoted by sClA.*

Definition 2.3 *[9] Let (X, τ) be a topological space and α an operator from τ to $\mathcal{P}(X)$ i.e $\alpha: \tau \to \mathcal{P}(X)$, where $\mathcal{P}(X)$ is a power set of X. We say that α is an operator associated with τ if $U \subset \alpha(U)$ for all $U \in \tau$*

Definition 2.4 *[10] Let (X,τ) be a topological space and α an operator associated with τ. A subset A of X is said to be α-open if for each $x \in X$ there exists an open set U containing x such that $\alpha(U) \subset A$. Let us denote the collection of all α-open, semi-open sets in the topological space (X,τ) by τ^{α}, $SO(\tau)$, respectively. We then have $\tau \subseteq \tau^{\alpha} \subseteq SO(\tau)$. A subset B of X is said to be α-closed [7] if its complement is α-open.*

Definition 2.5 *[9] Let (X,τ) be a space. An operator α is said to be regular if, for every open neighborhoods U and V of each $x \in X$, there exists a neighborhood W of x such that $\alpha(W) \subset \alpha(U) \subset \alpha(V)$.*

Note that the family τ^{α} of α –open sets in (X,τ) always forms a topology on X, when α is considered to be regular finer than τ.

Definition 2.6 *[4]: Let A be a subset of a space X. A point x is said to be an α - limit point of A if for each α - open U containing x, $U \cap (A \setminus x) \neq \phi$. The set of all α - limit points of A is called the α -derived set of A and is denoted by $D_{\alpha}(A)$.*

Definition 2.7 *[4] For subsets A and B of a space X, the following statements hold true:*

1) $D_{\alpha}(A) \subset D(A)$ where D(A) is the derived set of A

2)if $A \subset B$ then $D_{\alpha}(A) \subset D_{\alpha}(B)$

3) $D_{\alpha}(A) \cup D_{\alpha}(B) \subset D_{\alpha}(A \cup B)$

4) $D_{\alpha}(A \cup D_{\alpha}(A)) \subset A \cup D_{\alpha}(A)$.

Definition 2.8 *[10]: The point $x \in X$ is in the α –closure of a set $A \subset X$ if $\alpha(U) \cap A \neq \varphi$, for each open set U containing x. The α - closure of a set A is the intersection of all α - closed sets containing A and is denoted by $Cl_{\alpha}(A)$.*

Remark 2.9: *For any subset A of the space X, $A \subset Cl(A) \subset Cl_{\alpha}(A)$*

Definition 2.10 *[10] Let (X,τ) be a topological space. We say that a subset A of X is α - compact if for every α -open covering Π of A there exists a finite sub-collection $\{C_1, C_2, ..., C_n\}$ of Π such that $A \subset \bigcup_{i=1}^{n} C_i$. Properties of α –compact spaces have been* *investigated by Rosa, E etc. and Kasahara, S [9,10].The following results were given by Rosas, E etc.*

Theorem 2.11 *[9] Let (X,τ) be a topological space and α an operator associated with τ. A ⊂ X and K ⊂ A. If A is α –compact and K is α –closed then K is α –compact.*

Theorem 2.12 *[9] Let (X,τ) be a topological space and α be a regular operator on τ. If X is α -T_2 (see Rosa, E etc. and Kasahara, S) [9,10] and K⊂X is α –compact then K is α –closed.*

Definition 2.13 *[10] The intersection of all α –closed sets containing A is called the α –closure of A, denoted by $Cl_\alpha(A)$.*

Lemma 2.14 *For subsets A and A_i (i ϵ I) of a space (X,τ), the following hold:*

1) $A \subset Cl_\alpha(A)$

2) $Cl_\alpha(A)$ is closed; $Cl_\alpha(Cl_\alpha(A)) = Cl_\alpha(A)$

3) If $A \subset B$ then $Cl_\alpha(A) \subset Cl_\alpha(B)$

4) $Cl_\alpha(\cap(A_i : i \in I)) \subset \cap(Cl_\alpha(A) : i \in I)$

5) $Cl_\alpha(\cup(A_i : i \in I)) = \cup(Cl_\alpha(A) : i \in I)$

Lemma 2.15 *[9] The collection of α –compact subsets of X is closed under finite unions. If α is a regular operator and X is an α-T_2 space then it is closed under arbitrary intersection.*

Definition 2.16 *Let (X,τ) be a topological any space, A subset of X, The $\operatorname{int}_\alpha(A) = \cup\{U : U$ is α–open and $U \subset A\}$.*

Remark 2.17 *A subset A is α –open if and only if $\operatorname{int}_\alpha(A) = A$.*

Proof: The proof is obvious from the definition.

Definition 2.18 *Let (X, τ) and (Y, σ) be two topological spaces, a map $f : X \to Y$ is said to be α –continuous if for each open set H of Y, $f^{-1}(H)$ is α –open in X.*

Theorem 2.19 *[4]: For any subset A of a space X, $Cl_\alpha(A) = A \cup Cl_\alpha(A)$.*

Theorem 2.20 *[4]: For subsets A, B of a space X, the following statements are true:*

1) $\text{int}_\alpha(A)$ *is the largest α –open contained in A*

2) $\text{int}_\alpha(\text{int}_\alpha(A)) = \text{int}_\alpha(A)$

3) If $A \subset B$ *then* $\text{int}_\alpha(A) \subset \text{int}_\alpha(B)$

4) $\text{int}_\alpha(A) \cup \text{int}_\alpha(B) \subset \text{int}_\alpha(A \cup B)$

5) $\text{int}_\alpha(A) \cap \text{int}_\alpha(B) \supset \text{int}_\alpha(A \cap B)$

Lemma 2.21 *[7] For any α–open set A and any α-closed set C, we have*

1) $Cl_\alpha(A) = Cl(A)$

2) $\text{int}_\alpha(C) = \text{int}(C)$

3) $\text{int}_\alpha(Cl_\alpha(A)) = \text{int}(Cl(A))$

Remark 2.22 *[4]: It is not always true that every α-open set is an open set, as shown in the following example:*

Example 2.23 Let X={a, b, c, d} with topology τ ={ ϕ ,{c, d}, X}.Hence α(τ)={ ϕ ,{c, d},{b, c, d}, {a, c, d},X}
So {b, c, d} is α-open but not open.

3 Transitivity and Minimal Systems

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we mean a topological space X together with a continuous map $f : X \to X$. The space X is sometimes called the phase space of the system. A set $A \subseteq X$ is called $f-inveriant$ if $f(A) \subseteq A$.

A dynamical system (X, f) is called *minimal* if X does not contain any non-empty, proper, closed $f-inveriant$ subset. In such a case we also say that the map f itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them.

Given a point x in a system (X, f), $O_f(x) = \{x, f(x), f^2(x), ...\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω -limit set, i.e. the set of limit points of the sequence $x, f(x), f^2(x), ...$. The following conditions are equivalent:

- (X, f) is minimal,
- every orbit is dense in X ,
- $\omega_f(x) = X$ for every x ∈ X .

A minimal map f is necessarily surjective if X is assumed to be Hausdorff and compact.
Now, we will study the Existence of minimal sets. Given a dynamical system (X, f), a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f).
The basic fact discovered by G. D. Birkhoff is that in any compact system (X, f) there are minimal sets. This follows immediately from the Zorn's lemma. Since any orbit closure is invariant, we get that *any compact orbit closure contains a minimal set*. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) either are disjoint or coincide. A minimal set A is strongly $f-inveriant$, i.e. $f(A) = A$. Provided it is compact Hausdorff

Definition 3.1 *Let (X,τ) be a topological space, and $f : X \to X$ a continuous map, then f is said to be a topologically transitive map if for every pair of open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$*

3.1 Topologically α-Transitive Maps

In this subsection, we define a new class of topologically transitive maps that are called α-transitive maps on a space (X, τ), and we study some of their properties and prove some results associated with these new definitions. We will also define and introduce a new class of α-minimal maps.

Definition 3.1.1 *Let (X, τ) be a topological space. A subset A of X is called α-dense in X if $Cl_\alpha(A) = X$.*

Remark 3.1.2 *Any α-dense subset in X intersects any α-open set in X.*

Proof: Let A be an α-dense subset in X, then by definition, $Cl_\alpha(A) = X$, and let U be a non-empty α-open set in X. Suppose that A∩U=ϕ. Therefore $B = U^c$ is α-closed and $A \subset U^c$=B. So $Cl_\alpha(A) \subset Cl_\alpha(B)$, i.e. $Cl_\alpha(A) \subset$B, but $Cl_\alpha(A) = X$, so X⊂B, this contradicts that U $\neq \varphi$

Definition 3.1.3 *[12] A map $f : X \to Y$ is called α-irresolute if for every α-open set H of Y, $f^{-1}(H)$ is α-open in X.*

Example 3.1.4 Let (X, τ) be a topological space such that X={a, b, c, d} and τ ={ϕ, X ,{a, b}, {b}}. We have the set of all α-open sets is α(X, τ)={ϕ, X, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}}and the set of all α-closed sets is αC(X, τ)={ϕ, X, {c, d, {a, c, d}, {a, d}, {}a, c}, {d}, {c}}. Then define the map f : X→X as follows f(a)= a, f(b)= b, f(c)= d, f(d)= c, we have f is α-irresolute because {b} is α-open and f^{-1}({b})={b} is α-open; {a, b} is α-open and f^{-1}({a, b})={a, b} is α-open; {b, c} is α-open and f^{-1}({b, c})={b, d} is α-open; {a, b, c} is α-open and f^{-1}({a, b, c})={a, b, d} is α-open; {a, b, d} is α-open and f^{-1}({a, b, d})={a, b, c} is α-open so f is α-irresolute.

Definition 3.1.5 *A subset A of a topological space (X, τ) is said to be nowhere α-dense, if its α-closure has an empty α-interior, that is,* $\operatorname{int}_{\alpha}(Cl_{\alpha}(A)) = \phi$.

Definition 3.1.6a *Let (X, τ) be a topological space ,* $f : X \to X$ *be α-irresolute map then f is said to be topological α-transitive if every pair of non-empty α-open sets U and V in X there is a positive integer n such that* $f^{n}(U) \cap V \neq \phi$*.In the forgoing example 3.1.4: we have f is α-transitive because b belongs to any non-empty α-open set V and also belongs to f(U) for any α-open set it means that f(U)∩V≠* ϕ *so f is . α - transitive.*

Example 3.1.7 Let (X, τ) be a topological space such that X ={a, b, c}and τ={ϕ, {a}, X}.Then the set of all α-open sets is τ^{α}={ϕ, {a}, {a, b}, {a, c}, X}. Define f : X→X as follows f(a)=b, f(b)=b, f(c)=c. Clearly f is continuous because {a} is open and f({a})=ϕ is open. Note that f is transitive because f({a})={b} implies that f({a})∩{b}≠ϕ. But f is not α-transitive because for each n in N , f^n({a})∩{a, c}=ϕ; since f^n({a})={b} for every n ε N, and {b}∩{a, c}=ϕ. So we have f is not α-transitive, so we show that transitivity not implies α-transitivity.

Definition 3.1.6b *The function* $f : X \to X$ *is said to be topologically transitive if there exists a point x* **ε** *X such that its orbit* $\{f^{n}(x) : n \geq o\}$ *is dense in X.*
Note the two definitions of topological transitive are clearly not equivalent:

Example 3.1.7: We consider the continuous function $f : X \to X$ where
$X = \{0\} \cup \{\frac{1}{n} : n \in N\}$ equipped with the metric $d = |x - y|$ for each x, y in X the function is defined by f(0)=0 and f(1/n)= $1/(n+1)$ for n=1,2,3,…Then if we choose U={1/2} and v={0} the f does not satisfy the definition 3.1.6a.
Now we can observe that the point x=1 has a dense orbit in X so the definition 3.1.6b is satisfied and so its not equivalent with definition 3.1.6a.

Associated with this new definition 3.1.6a we can prove the following new theorem.

Theorem 3.1.8: *Let (X, τ) be a topological space and $f: X \to X$ be α -irresolute map. Then the following statements are equivalent:*

(1) f is topological α-transitive map

(2) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^n(U)$ *is α-dense in X*

(3) For every nonempty α-open set U in X, $\bigcup_{n=0}^{\infty} f^{-n}(U)$ *is α-dense in X*

(4) If $B \subset X$ *is α-closed and B is f- invariant i.e.* $f(B) \subset B$*. then B=X or B is nowhere α-dense*

(5) If U is α-open and $f^{-1}(U) \subset U$

then U=ϕ or U is α-dense in X. We have to prove this theorem:

Proof:

$(1) \Rightarrow (2)$

Assume that $\bigcup_{n=0}^{\infty} f^n(U)$ is not α-dense.

Then there exists a non-empty α-open set V such that $\bigcup_{n=0}^{\infty} f^n(U) \cap V = \phi$.

This implies that $f^n(U) \cap V = \phi$ for all n ϵ N .This is a contradiction to the α-transitivity of f. Hence $\bigcup_{n=0}^{\infty} f^n(U)$ is α-dense in X.

$(2) \Rightarrow (1)$

Let U and V be two nonempty α-open sets in X, and let $\bigcup_{n=0}^{\infty} f^n(U)$ be α-dense in X, this implies that $\bigcup_{n=0}^{\infty} f^n(U) \cap V \neq \phi$ by Remark 3.1.2.This implies that there exists m ϵ N such that $f^n(U) \cap V \neq \phi$. Hence f is topologically α-transitive map.

$(1) \Rightarrow (3)$

It is obvious that $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is α-open and since f is α-transitive, it has to meet every α-open set in X, and hence is α-dense by Remark 3.1.2..

$(3) \Rightarrow (1)$

Let V and W be two α-open subsets in X. Then $\bigcup_{n=0}^{\infty} f^{-n}(W)$ is α-dense, this implies that $\bigcup_{n=0}^{\infty} f^{-n}(W) \cap V \neq \phi$, by Remark 3.1.2 .This implies that there exists m ϵ N such that $f^{-m}(W) \cap V \neq \phi$..So $f^m(f^{-m}(W) \cap V) = W \cap f^m(V) \neq \phi$. Therefore f is α-transitive.

$(1) \Rightarrow (4)$

Suppose f is α-transitive map, $E \subset X$ is α-closed and $f(E) \subset E$. Assume that E $\neq \phi$ and E has a nonempty α-interior (i.e. $\mathrm{int}_\alpha(E) \neq \phi$). If we define V=X\E so V is α-open because V is the compliment of α-closed. Let $W \subset E$ be α-open since $\text{int}_\alpha(E) \neq \phi$. We have $f^n(W) \subset E$ since E is invariant. Therefore $f^n(W) \cap V = \phi$, for all n є N. This is a contradiction to topological α-transitive. Hence E=X or E is nowhere α-dense .

(4) $\Rightarrow$(1)

Let V be a nonempty α-open set in X. Suppose f is not a topological α-transitive map, from (3) of this theorem $\bigcup_{n=0}^{\infty} f^{-n}(V)$ is not α-dense, but α-open. Define $E = X \setminus \bigcup_{n=0}^{\infty} f^{-n}(V)$ which is α-closed, because it is the complement of α-open, and E≠X. Clearly $f(E) \subset E$. Since $\bigcup_{n=0}^{\infty} f^{-n}(V)$ is not α-dense so by Remark3.1.2, there exists a non-empty α-open W in X such that $\bigcup_{n=0}^{\infty} f^{-n}(V) \cap W = \phi$. This implies that W$\subset$E. This is contradiction to the fact that E is nowhere α-dense. Hence f is a topological α-transitive map..

(1) $\Rightarrow$(5)

Suppose that f is α-transitive, $U \subset X$ is α-open and $f^{-1}(U) \subset U$. Assume that $U \neq \phi$ and U is not α-dense in X(i.e. $Cl_\alpha(U) \neq X$). Then there exists a non-empty α-open V in X such that $U \cap V = \phi$. Further $f^{-n}(U) \cap V = \phi$ for all n є N. This implies $U \cap f^n(V) = \phi$ for all n є N, a contradiction to f being a topological α-transitive map. Therefore $U = \phi$ or U is α-dense in X.

4 α-Minimal Functions

In this section, we introduce a new definition on α-minimal maps and we study some new theorems associated with this new definition.

Given a topological space X, we ask whether there exists α-continuous map on X such that the set $\{f^n(x) : n \geq 0\}$, called the orbit of x and denoted by $O_f(x)$, is α-dense in X for each x є X.. A partial answer will be given in this section. Let us begin with a new definition.

Definition 4.1 *(α-minimal) Let X be a topological space and f an α-continuous map on X with α-regular operator associated with the topology on X. Then (X,f) is called α-minimal system (or f is called α-minimal map on X) if one of the three equivalent conditions hold:*

1) The orbit of each point of X is α-dense in X.

2) $Cl_\alpha(O_f(x)) = X$ for each x є X .

3) Given x ϵ X and a nonempty α-open U in X, there exists n ϵ N such that $f^n(x) \in U$.

Theorem 4.2 *For* (X, f) *the following statements are equivalent:*
(1) f *is an α-minimal map.*
(2) If E is an α-closed subset of X with $f(E) \subset E$, *we say E is invariant. Then E=*ϕ *or E=X.*
(3) If U is a nonempty α-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

Proof: (1) $\Rightarrow$(2): If A ≠ ϕ, let x ϵ A. Since A is invariant and α-closed, $Cl_\alpha(O_f(x)) \subset A$. On other hand $Cl_\alpha(O_f(x)) = X$. So A=X

(2)$\Rightarrow$(3) Let A=X\ $\bigcup_{n=0}^{\infty} f^{-n}(U)$. Since U is nonempty, A ≠ X. Since U is α-open and f is α-continuous, A is α-closed. Also $f(A) \subset A$, so A must be an empty set.
(3) $\Rightarrow$(1): Let x ϵ X and U be a nonempty α-open subset of X. Since x ϵ X $= \bigcup_{n=0}^{\infty} f^{-n}(U)$ .Therefore x ϵ $f^{-n}(U)$ for some n>0. So $f^n(x) \in U$

5. Conclusion

There are the main results of the paper.

Proposition 5.1 *Every topologically α-transitive map is a topologically transitive map as every open set is* α*-open set, but the converse not necessarily true, unless every α-open set is locally closed.*

Proposition 5.2 *Every α-minimal map is a minimal map as every open set is* α*-open set, but the converse not necessarily true, unless every α-open set is locally closed.*

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