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Intuitionistic Fuzzy Almost π Generalized Semi Open Mappings in Topological Spaces

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy almost π generalized semi open mappings and intuitionistic fuzzy almost π generalized semi closed mappings in intuitionistic fuzzy topological space and we investigate some of its properties. Also we provide the relations between intuitionistic fuzzy almost π generalized semi closed mappings and other intuitionistic fuzzy closed mappings.

Keywords: *Intuitionistic fuzzy topology, intuitionistic fuzzy π generalized semi closed set, intuitionistic fuzzy almost π generalized semi closed mappings, intuitionistic fuzzy almost π generalized semi open mappings and intuitionistic fuzzy* $\pi T_{1/2}$ *(IF*$\pi T_{1/2}$*) space and intuitionistic fuzzy* $\pi gT_{1/2}$ *(IF*$\pi gT_{1/2}$*) space.*

1 Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [16] in1965. Using the concept of fuzzy sets, Chang [3] introduced the concept of fuzzy topological space. In [1], Atanassov introduced the notion of intuitionistic fuzzy sets in 1986. Using the notion of intuitionistic fuzzy sets, Coker [4] defined the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of the directions is related to the properties of intuitionistic fuzzy sets introduced by Gurcay [7] in 1997. Continuing the work done in the [10], [11],[12],[13],[14],[15] we define the notion of intuitionistic fuzzy almost π-generalized semi closed mappings and intuitionistic fuzzy almost π generalized semi open mappings. We discuss characterizations of intuitionistic fuzzy almost π generalized semi closed mappings and open mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic fuzzy closed mappings.

2 Preliminaries

Definition 2.1 [1] *An intuitionistic fuzzy set (IFS in short) A in X is an object having the form* $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$,*where the functions* $\mu_A(x): X \to [0, 1]$ *and* $\nu_A(x): X \to [0, 1]$ *denote the degree of membership (namely* $\mu_A(x)$*) and the degree of non-membership (namely* $\nu_A(x)$*) of each element* $x \in X$ *to the set A, respectively, and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for each* $x \in X$*. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.*

Definition 2.2 [1] *Let A and B be IFSs of the form*

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ *and* $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. *Then*

(a) $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$
(b) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3 [3] *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms.*

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ *for any family* $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 [3] *Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by*

int(A) = $\cup$ { G / G is an IFOS in X and G $\subseteq$ A },

cl(A) = $\cap$ { K / K is an IFCS in X and A $\subseteq$ K }.

Definition 2.5 [10] *A subset of A of a space (X, τ) is called:*

(i) regular open if A = int (cl(A)).

(ii) π open if A is the union of regular open sets.

Definition 2.6 [10] *An IFS A = { $\langle x, \mu_A, \nu_A \rangle$ } in an IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy semi open set (IFSOS in short) if A $\subseteq$ cl(int(A)),
(ii) intuitionistic fuzzy α-open set (IFαOS in short) if A $\subseteq$ int(cl(int(A))),
(iii) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)),
(iv) intuitionistic fuzzy pre open set (IFPOS in short) if A $\subseteq$ int(cl(A)).
(v) intuitionistic fuzzy semi-pre open set (IFSPOS) if there exists B $\in$ IFPO(X) such that B
$\subseteq$A$\subseteq$ Cl(B).

Definition 2.7 [10] *An IFS A = $\langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an*
(i) intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) $\subseteq$ A,
(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A)) $\subseteq$ A,
(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(A)),
(iv) intuitionistic fuzzy pre closed set (IFPCS in short) if cl(int(A)) $\subseteq$ A.

Definition 2.8 [10] *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π generalized semi closed set (IFπGSCS in short) if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFπOS in (X, τ). An IFS A is said to be an intuitionistic fuzzy π generalized semi open set (IFπGSOS in short) in X if the complement A^c is an IFπGSCS in X.*
The family of all IFπGSCSs of an IFTS (X, τ) is denoted by IFπGSC(X).

Result 2.9 [10] Every IFCS, IFGCS, IFRCS, IFαCS , IFαGCS, IFGSCS is an IFπGSCS but the converses may not be true in general.

Definition 2.10 [13] *Let A be an IFS in an IFTS (X, τ). Then π generalized Semi closure of A (πgscl(A) in short) and π generalized Semi interior of A (πgsint(A) in short) are defined by*

πgsint(A) = $\cup$ { G / G is an IFπGSOS in X and G $\subseteq$ A }

πgscl(A) = $\cap$ { K / K is an IFπGSCS in X and A $\subseteq$ K }.

Note that for any IFS A in (X, τ), we have $\pi gscl(A^c) = [\pi gsint(A)]^c$ and $\pi gsint(A^c) = [\pi gscl(A)]^c$.

Definition 2.11 [7] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.*

Definition 2.12 [12] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous) if $f^{-1}(B) \in IFGCS(X)$ for every IFCS B in Y.*

Definition 2.13 [14] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy almost π generalized semi continuous mappings (IFAπGS continuous) if $f^{-1}(B) \in IFGCS(X)$ for every IFRCS B in Y.*

Definition 2.14 [15] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy α generalized continuous mappings (IFαG continuous) if $f^{-1}(B) \in IF\alpha GCS(X)$ for every IFRCS B in Y.*

Definition 2.15 [15] *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized semi closed mappings (IFGSCM) if $f^{-1}(B) \in IFGSCS(X)$ for every IFRCS B in Y.*

Definition 2.16 *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy almost closed mappings (IFACM) if $f^{-1}(B) \in IFC(Y)$ for every IFRCS B in X.*

Definition 2.17 *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy almost α generalized closed mappings (IFAαGCM) if $f^{-1}(B) \in IF\alpha GC(Y)$ for every IFRCS B in X.*

Definition 2.18 [5] *The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta}\rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP) in X.*

Note that an IFP c(α, β) is said to belong to an IFS $A = \langle x, \mu_A, \nu_A\rangle$ of X denoted by $c(\alpha, \beta) \in A$ if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.19 [5] *Let c(α, β) be an IFP of an IFTS (X, τ). An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN) of c(α, β) if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.*

Definition 2.20 [7] *An IFS A is said to be an intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFTS (X, τ), if cl(A) = B.*

Definition 2.21 [11] *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\pi}T_{1/2}$ (IF$_{\pi}T_{1/2}$ in short) space if every IFπGSCS in X is an IFCS in X.*

Definition 2.22 [11] *An IFTS (X, τ) is said to be an intuitionistic fuzzy $_{\pi g}T_{1/2}$ (IF $_{\pi g}T_{1/2}$ in short) space if every IFπGSCS in X is an IFGCS in X.*

Result 2.23 [9] (i) Every IFπOS is an IFOS in (X, τ).
(ii) Every IFπCS is an IFCS in (X, τ)

3 Intuitionistic Fuzzy almost π Generalized Semi Open Mappings

In this section we introduce intuitionistic fuzzy almost π generalized semi open mappings, intuitionistic fuzzy almost π generalized semi closed mappings and studied some of its properties.

Definition 3.1 *A mapping f: X → Y is called an intuitionistic fuzzy almost π generalized semi open mappings (IFAπGSOM for short) if f(A) is an IFπGSOS in Y for each IFROS A in X.*

Definition 3.2 *A mapping f: (X, τ) → (Y,σ) is called an intuitionistic fuzzy almost π generalized semi closed mappings (IFAπGSCM) if f (B) is an IFπGSCS in (Y, σ) for every IFRCS B of (X, τ).*

Example 3.3 Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.2_a, 0.2_b), (0.6_a, 0.7_b) \rangle$, $G_2 = \langle y, (0.4_u, 0.2_v), (0.6_u, 0.7_v) \rangle$. Then, τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFAπGSCM.

Theorem 3.4 (i) *Every IFCM is an IFAπGSCM but not conversely.*
(ii) Every IFαGCM is an IFAπGSCM but not conversely.
(iii) Every IFACM is an IFAπGSCM but not conversely.
(iv) Every IFAαGCM is an IFAπGSCM but not conversely.

Proof (i) Assume that f : (X, τ) → (Y, σ) is an IFCM. Let A be an IFRCS in X. This implies A is an IFCS in X. Since f is an IFCM, f (A) is an IFCS in Y. Every IFCS is an IFπGSCS, f (A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Proof (ii) Let f: (X, τ) → (Y,σ) be an IFαGCM. Let A be an IFRCS in X. This implies A is an IFCS in X. Then by hypothesis f (A) is an IFαGCS in Y. Since every IFαGCS is an IFGSCS and every IFGSCS is an IFπGSCS, f(A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Proof (iii) Let f: (X, τ) → (Y, σ) be an IFACM. Let A be an IFRCS in X. Since f is IFACM, f(A) is an IFCS in Y. Since every IFCS is an IFπGSCS, f(A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Proof (iv) Let f: (X, τ) → (Y,σ) be an IFAαGCM. Let A be an IFRCS in X. Since f is IFACM. Then by hypothesis f(A) is an IFαGCS in Y. Since every IFαGCS is an IFGSCS and every IFGSCS is an IFπGSCS, f(A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Example (i) Let X = {a, b}, Y = { u, v } and G_1 = ⟨ x, (0.4_a, 0.2_b), (0.5_a, 0.4_b) ⟩,G_2 = ⟨ y, (0.3_u, 0.2_v), (0.6_u, 0.7_v) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then, f is an IFAπGSCM. But f is not an IFCM since G_1^c = ⟨ x, (0.5_a, 0.4_b), (0.4_a, 0.2_b) ⟩ is an IFCS in X but f (G_1^c) = ⟨ y, (0.5_u, 0.4_v), (0.4_u, 0.2_v) ⟩ is not an IFCS in Y.

Example (ii) Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.3_a, 0.4_b), (0.4_a, 0.5_b) ⟩,G_2 = ⟨ y, (0.7_u, 0.6_v), (0.3_u, 0.4_v) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then, f is an IFAπGSCM but not an IFαGCM since G1c = ⟨ x, (0.4_a, 0.5_b), (0.3_a, 0.4_b) ⟩ is an IFCS in X but f (G_1^c) = ⟨ y, (0.4_u, 0.2_v), (0.3_u, 0.4_v) ⟩ is not an IFαGCS in Y.

Example (iii) In example (i), f is an IFAπGSCM but f is not an IFACM since G_1^c = ⟨ x, (0.5_a, 0.4_b), (0.4_a, 0.2_b) ⟩ is an IFRCS in X but f (G_1^c) = ⟨ y, (0.5_u, 0.4_v), (0.4_u, 0.2_v) ⟩ is not an IFCS in Y.

Example (iv) In example (ii), f is an IFAπGSCM. But f is not an IFAαGCM since G1c = ⟨ x, (0.4_a, 0.5_b), (0.3_a, 0.4_b) ⟩ is an IFRCS in Y but f (G_1^c) = ⟨ y, (0.4_u, 0.2_v), (0.3_u, 0.4_v) ⟩ is not an IFαGCS in Y.

Theorem 3.5 A bijective mapping f : X → Y is an IFAπGS closed mapping if and only if the image of each IFROS in X is an IFπGSOS in Y.

Proof Necessity: Let A be an IFROS in X. This implies A^c is IFRCS in X. Since f is an IFAπGS closed mapping, f (A^c) is an IFπGSCS in Y. Since f (A^c) = $(f(A))^c$, f(A) is an IFπGSOS in Y.

Sufficiency: Let A be an IFRCS in X. This implies A^c is an IFROS in X. By hypothesis, f(A^c) is an IFπGSOS in Y. Since f(A^c) = $(f(A))^c$, f(A) is an IFπGSCS in Y. Hence f is an IFAπGS closed mapping.

Theorem 3.6 *Let f :(X, τ) → (Y, σ) be an IFAπGS closed mapping. Then f is an IFA closed mapping if Y is an IF*$_\pi T_{1/2}$ *space.*

Proof Let A be an IFRCS in X. Then f(A) is an IFπGSCS in Y, by hypothesis. Since Y is an IF$\pi T_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IFA closed mapping.

Theorem 3.7 *Let f: X $\to$ Y be a bijective mapping. Then the following are equivalent.*

(i) *f is an IFAπGSOM*
(ii) *f is an IFAπGSCM*

Proof Straightforward

Theorem 3.8 *Let f: X $\to$ Y be a mapping where Y is an IF$\pi T_{1/2}$ space. Then the following are equivalent.*

(i) *f is an IFAπGSCM*
(ii) *scl(f (A)) $\subseteq$ f (cl(A)) for every IFSPOS A in X*
(iii) *scl(f (A)) $\subseteq$ f (cl(A)) for every IFSOS A in X*

Proof (i) $\Rightarrow$ (ii) Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFπGSCS in Y. Since Y is an IF$\pi T_{1/2}$ space. This implies scl(f(cl(A))) =f (cl(A)). Now scl(f(A)) $\subseteq$ scl(f(cl(A))) = f(cl(A)). Thus scl(f(A)) $\subseteq$ f(cl(A)). (ii) $\Rightarrow$ (iii) Since every IFSOS is an IFSPOS, the proof directly follows. (iii) $\Rightarrow$ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, scl(f(A)) $\subseteq$ f(cl(A)) = f(A) $\subseteq$ scl(f(A)). Hence f(A) is an IFSCS and hence is an IFπGSCS in Y. Thus f is an IFAπGSCM.

Theorem 3.9 Let *f: X $\to$ Y be a mapping where Y is an IF$\pi T_{1/2}$ space. Then the following are equivalent.*

(i) *f is an IFAπGSCM*
(ii) *f (A) $\subseteq$ sint(f (int(cl(A)))) for every IFPOS A in X*

Proof (i) $\Rightarrow$ (ii) Let A be an IFPOS in X. Then A $\subseteq$ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFπGSOS in Y. Since Y is an IF$\pi T_{1/2}$ space, f(int(cl(A))) is an IFSOS in Y. Therefore f(A) $\subseteq$ f(int(cl(A))) $\subseteq$ sint(f(int(cl(A)))). (ii) $\Rightarrow$ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) $\subseteq$ sint(f(int(cl(A)))) = sint(f(A)) $\subseteq$ f(A). This implies f(A) is an IFSOS in Y and hence is an IFπGSOS in Y. Therefore f is an IFAπGSCM, by Theorem 3.6.

Theorem 3.10 *The following are equivalent for a mapping f: X $\to$ Y, where Y is an IF$\pi T_{1/2}$ space.*

(i) *f is an IFAπGSCM*
(ii) *scl(f(A)) $\subseteq$ f(αcl(A)) for every IFSPOS A in X*
(iii) *scl(f(A)) $\subseteq$ f(αcl(A)) for every IFSOS A in X*
(iv) *f(A) $\subseteq$ sint(f(scl(A))) for every IFPOS A in X*

Proof (i) ⇒ (ii) Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFπGSCS in Y and hence is an IFSCS in Y, since Y is an IFπ$T_{1/2}$ space. This implies scl(f(cl(A))) = f(cl(A)). Now scl(f(A)) ⊆ scl(f(cl(A))) = f(cl(A)). Since cl(A) is an IFRCS, we have cl(int(cl(A))) = cl(A). Therefore scl(f(A)) ⊆ f(cl(A)) = f(cl(int(cl(A)))) ⊆ f(A ∪ cl(int(cl(A)))) = f(αcl(A)). Hence scl(f(A)) ⊆ f(αcl(A)).(ii) ⇒ (iii) Since every IFSOS is an IFSPOS, the proof is obvious.(iii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, scl(f(A)) ⊆ f(αcl(A)) ⊆ f(cl(A)) = f(A) ⊆ scl(f(A)). That is scl(f(A)) = f(A). Hence f(A) is an IFSCS and hence is an IFπGSCS in Y. Thus f is an IFAπGSCM.(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFπGSOS in Y. Since Y is an IFπ$T_{1/2}$ space, f(int(cl(A))) is an IFSOS in Y. Therefore f(A) ⊆ f(int(cl(A))) ⊆ sint(f(int(cl(A)))) ⊆ sint(f(A ∪ int(cl(A)))) = sint(f(scl(A))). That is f(A) ⊆ sint(f(scl(A))).
(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ sint(f(scl(A))). This implies f(A) ⊆ sint(f(A ∪ int(cl(A)))) ⊆ sint(f(A ∪ A)) = sint(f(A)) ⊆ f(A). Therefore f(A) is an IFSOS in Y and hence an IFπGSOS in Y. Thus f is an IFAπGSCM, by Theorem 3.6.

Theorem 3.11 *Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IFπ$T_{1/2}$ space.*

(i) f is an IFAπGSCM
(ii) f is an IFAπGSOM
(iii) f(int(A)) ⊆ int(cl(int(f(A)))) for every IFROS A in X.

Proof (i) ⇒ (ii) It is obviously true.
(ii) ⇒ (iii) Let A be any IFROS in X. This implies A is an IFOS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IFπGSOS in Y. Since Y is an IFπ$T_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore f(int(A)) = int(f(int(A)) ⊆ int(cl(int(f(A)))). (iii) ⇒ (i) Let A be an IFRCS in X. Then its complement A^c is an IFROS in X. By hypothesis f(int(A^c)) ⊆ int(cl(int(f(A^c)))). This implies f(A^c) ⊆ int(cl(int(f(A^c)))). Hence f(A^c) is an IFαOS in Y. Since every IFαOS is an IFπGSOS, f(A^c) is an IFπGSOS in Y. Therefore f(A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Theorem 3.12 Let *f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IFπ$T_{1/2}$ space.*

(i) f is an IFAπGSCM
(ii) scl(f(A)) ⊆ f(scl(A)) for every IFSCS A in X

Proof (i) ⇒ (ii) Assume that A is an IFSCS in X. By Definition, int(cl(A)) ⊆ A. This implies cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFπGSCS in Y and hence is an IFπCS in Y, since Y is an IFπ$T_{1/2}$ space. This implies scl(f(cl(A))) = f(cl(A)). Now scl(f(A)) ⊆ scl(f(cl(A))) = f(cl(A)). Since cl(A) is an

IFROS, int(cl(cl(A))) = cl(A). This implies scl(f(A)) ⊆ f(cl(A)) = f(int(cl(cl(A)))) ⊆ f(A ∪ int (cl (cl(A)))) = f(A ∪ int (cl (A))) = f(scl(A)). Hence scl(f(A)) ⊆ f(scl(A)). (ii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSCS in X. By hypothesis, scl(f(A)) ⊆ f(scl(A)) ⊆ f(cl(A)) = f(A) ⊆ scl(f(A)). That is scl(f(A)) = f(A). Hence f(A) is an IFπCS and hence is an IFπGSCS in Y. Thus f is an IFAπGSCM.

Theorem 3.13 *Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IF$_\pi$T$_{1/2}$ space.*

(i) f is an IFAπGSCM

(ii) f(A) ⊆ πint(f(scl(A))) for every IFPOS A in X

Proof (i) ⇒ (ii) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFπGSOS in Y. Since Y is an IF$_\pi$T$_{1/2}$ space, f(int(cl(A))) is an IF$_\pi$OS in Y. Therefore f(A) ⊆ f(int(cl(A))) ⊆ πint(f(int(cl(A)))) ⊆ πint(f(A ∪ int(cl(A)))) = πint(f(scl(A))). That is f(A) ⊆ πint(f(scl(A))). (ii) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ πint(f(scl(A))). This implies f(A) ⊆ πint(f(A ∪ int(cl(A)))) ⊆ πint(f(A ∪ A)) = πint(f(A)) ⊆ f(A). Therefore f(A) is an IFπOS in Y and hence an IFπGOS in Y. Thus f is an IFAπGS closed mapping.

Theorem 3.14 Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IF$_\pi$T$_{1/2}$ space.

(i) f is an IFAπGSCM

(ii) If B is an IFROS in X then f (B) is an IFπGSOS in Y

(iii) f (B) ⊆ int(cl(f(B)) for every IFROS B in X.

Proof (i) ⇒ (ii) obviously.

(ii) ⇒ (iii) Let B be any IFROS in X. Then by hypothesis f (B) is an IFπGSOS in Y. Since X is an IF$_\pi$T$_{1/2}$ space, f(B) is an IFOS in Y (Result 2.23). Therefore f (B) = int(f(B)) ⊆ int(cl(f(B))). (iii) ⇒ (i) Let B be an IFRCS in X. Then its complement B^c is an IFROS in X. By hypothesis f(B^c) ⊆ int(cl(f(B^c))). Hence f(B^c) is an IFπOS in Y. Since every IFπOS is an IFπGSOS, f(B^c) is an IFπGSOS in Y. Therefore f (B) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Theorem 3.15 *Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if Y is an IF$_\pi$T$_{1/2}$ space.*

(i) f is an IFAπGSCM.

(ii) int(cl(f(A))) ⊆ f(A) for every IFRCS A in X.

Proof (i) ⇒ (ii) Let A be an IFRCS in X. By hypothesis, f(A) is an IFπGSCS in Y. Since Y is an IF$_\pi$T$_{1/2}$, f(A) is an IFCS in Y (Result 2.23).Therefore cl(f(A) = f (A). Now int(cl(f(A))) ⊆ cl(f(A)) ⊆ f (A). (ii) ⇒ (i) Let A be an IFRCS in X. By hypothesis int(cl(f(A))) ⊆ f(A). This implies f(A) is an IFπCS in Y and hence f(A) is an IFπGSCS in Y. Therefore f is an IFAπGSCM.

Theorem 3.16 *Let f : (X, τ) → (Y, σ) be an IFA closed mapping and g : (Y, σ) → (Z, δ) is IFAπGS closed mapping, then g ∘ f : (X, τ) → (Z, δ) is an IFA closed mapping. if Z is an IFπ$T_{1/2}$ space*

Proof: Let A be an IFRCS in X. Then f(A) is an IFCS in Y. Since g is an IFπGS closed mapping, g(f(A)) is an IFπGSCS in Z. Therefore g(f(A)) is an IFCS in Z, by hypothesis.Hence g ∘ f is an IFA closed mapping.

Theorem 3.17 *Let f : (X, τ) → (Y, σ) be an IFA closed mapping and g : (Y, σ) → (Z, η) be an IFπGS closed mapping. Then g ∘ f : (X, τ) → (Z, η) is an IFAπGS closed mapping.*

Proof: Let A be an IFRCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFπGS closed mapping, g(f(A)) is an IFπGSCS in Z. Hence g ∘ f is an IFAπGS closed mapping.

Theorem 3.18 *If f : (X, τ) → (Y, σ) is an IFAπGS closed mapping and Y is an IF$_{\pi g}T_{1/2}$ space, then f(A) is an IFGCS in Y for every IFRCS A in X.*

Proof: Let f : (X, τ) → (Y, σ) be a mapping and let A be an IFRCS in X. Then by hypothesis f(A) is an IFπGSCS in Y. Since Y is an IF$_{\pi g}T_{1/2}$ space, f(A) is an IFGCS in Y.

Theorem 3.19 *Let c(α, β) be an IFP in X. A mapping f: X → Y is an IFaπGSOM if for every IFOS A in X with f^{-1}(c(α, β)) ∈ A, there exists an IFOS B in Y with c(α, β) ∈ B such that f(A) is IFD in B.*

Proof: Let A be an IFROS in X. Then A is an IFOS in X. Let f^{-1}(c(α, β)) ∈ A, then there exists an IFOS B in Y such that c(α, β) ∈ B and cl(f (A)) = B. Since B is an IFOS, cl(f(A)) = B is also an IFOS in Y. Therefore int(cl(f(A))) = cl(f (A)). Now f(A) ⊆ cl(f (A)) = int(cl(f(A))) ⊆ cl(int(int(cl(f(A)))) = cl(int(cl(f(A)))). This implies f(A) is an IFSOS in Y and hence an IFπGSOS in Y. Thus f is an IFAπGSOM.

Theorem 3.20 *Let f: X → Y be a bijective mapping. Then the following are equivalent.*

(i) f is an IFAπGSOM

(ii) f is an IFAπGSCM

(iii) f^{-1} is an IFAπGS continuous mapping

Proof (i) ⇔ (ii) is obvious from the Theorem 3.7.

(ii) $\Rightarrow$ (iii) Let A $\subseteq$ X be an IFRCS. Then by hypothesis, f(A) is an IFπGSCS in Y. That is $(f^{-1})^{-1}$(A) is an IFπGSCS in Y. This implies f^{-1} is an IFAπGS continuous mapping. (iii) $\Rightarrow$ (ii) Let A $\subseteq$ X be an IFRCS. Then by hypothesis $(f^{-1})^{-1}$(A) is an IFπGSCS in Y. That is f(A) is an IFπGSCS in Y. Hence f is an IFAπGSCM.

Theorem 3.21 *Let f: X $\rightarrow$ Y be a mapping. If f(sint(B)) $\subseteq$ sint(f(B)) for every IFS B in X, then f is an IFAπGSOM.*

Proof: Let B $\subseteq$ X be an IFROS. By hypothesis, f(sint(B)) $\subseteq$ sint(f(B)). Since B is an IFROS, it is an IFSPOS in X. Therefore sint(B) = B. Hence f(B) = f(sint(B)) $\subseteq$ sint(f(B)) $\subseteq$ f(B). This implies f(B) is an IFSOS and hence an IFπGSOS in Y. Thus f is an IFAπGSOM.

Theorem 3.22 *Let f: X $\rightarrow$ Y be a mapping. If scl(f(B)) $\subseteq$ f(scl(B)) for every IFS B in X, then f is an IFAπGSCM.*

Proof: Let B $\subseteq$ X be an IFRCS. By hypothesis, scl(f(B)) $\subseteq$ f(scl(B)). Since B is an IFRCS, it is an IFSCS in X. Therefore scl(B) = B. Hence f(B) = f(scl(B)) $\supseteq$ scl(f(B)) $\supseteq$ f(B). This implies f(B) is an IFSCS and hence an IFπGSCS in Y. Thus f is an IFAπGSCM.

Theorem 3.23 *Let f: X $\rightarrow$ Y be a mapping where Y is an IF$_\pi T_{1/2}$ space. If f is an IFAπGSCM, then f(sint(B)) $\subseteq$ cl(int(cl(f(B))) for every IFROS B in X.*

Proof: This theorem can be easily proved by taking complement in Theorem 3.21.

Theorem 3.24 *Let f: X $\rightarrow$ Y be an IFAπGSOM, where Y is an IF$_\pi T_{1/2}$ space. Then for each IFP c(α, β) in Y and each IFROS B in X such that f^{-1} (c(α, β)) $\in$ B, cl(f(cl(B))) is an IFSN of c(α, β) in Y.*

Proof: Let c(α, β) $\in$ Y and let B be an IFROS in X such that f^{-1}(c(α, β)) $\in$ B. That is c(α, β) $\in$ f(B). By hypothesis, f(B) is an IFπGSOS in Y. Since Y is an IF$_\pi T_{1/2}$ space, f(B) is an IFSOS in Y. Now c(α, β) $\in$ f(B) $\subseteq$ f(cl(B)) $\subseteq$ cl(f(cl(B))). Hence cl(f(cl(B))) is an IFSN of c(α, β) in Y.

Remark 3.25 *If an IFS A in an IFTS (X, τ) is an IFπGSCS in X, then πgscl(A) = A. But the converse may not be true in general, since the intersection does not exist in IFπGSCSs.*

Remark 3.26 *If an IFS A in an IFTS (X, τ) is an IFπGSOS in X, then πgsint(A) = A. But the converse may not be true in general, since the union does not exist in IFπGSOSs.*

Theorem 3.27 *Let f: X $\rightarrow$ Y be a mapping. If f is an IFAπGSCM, then πgscl(f(A)) $\subseteq$ f(cl(A)) for every IFSOS A in X.*

Proof: Let A be an IFSOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFπGSCS in Y. Then πgscl(f(cl(A)) = f(cl(A)). Now πgscl(f(A)) $\subseteq$ πgscl(f(cl(A))) = f(cl(A)). That is πgscl(f(A)) $\subseteq$ f(cl(A)).

Corollary 3.28 *Let f: X $\rightarrow$ Y be a mapping. If f is an IFAπGSCM, then πgscl(f(A) $\subseteq$ f(cl(A)) for every IFGSOS A in X.*

Proof: Since every IFSOS is an IFGSOS, the proof is obvious from the Theorem 3.27.

Corollary 3.29 *Let f: X $\rightarrow$ Y be a mapping. If f is an IFAπGSCM, then πgscl(f(A) $\subseteq$ f (cl(A)) for every IFGOS A in X.*

Proof: Since every IFGOS is an IFGSOS, the proof is obvious from the Theorem 3.27.

Theorem 3.30 *Let f: X $\rightarrow$ Y be a mapping. If f is an IFAπGSCM, then πgscl(f(A)) $\subseteq$ f(cl(sint(A))) for every IFSOS A in X.*

Proof: Let A be an IFSOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFπGSCS in Y. Then πgscl(f(A)) $\subseteq$ πgscl(f(cl(A))) = f(cl(A)) $\subseteq$ f(cl(sint(A))), since sint(A) = A.

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