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On Certain Characterizations of Composite Convolution Operators

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Abstract

In this paper I have made an attempt to obtain criterions for hyponormal, quasinormal, binormal composite convolution operators. The characterizations for composite convolution operators to be n-normal and n-binormal are also computed.

Keywords: *Composite convolution operator, hyponormal, quasinormal, binormal, n-normal and n-binormal.*

1 Introduction

Let (X, Ω, μ) be a σ-finite measure space. For each $f \in L^p(\mu)$, $1 \leq p < \infty$, there exists a unique $\phi^{-1}(\Omega)$ measurable function E(f) such that $\int g\, f\, d\mu = \int g\, E(f)\, d\mu$ for every $\phi^{-1}(\Omega)$ measurable function g for which left integral exists. The function E(f) is called conditional expectation of f with respect to the sub- algebra $\phi^{-1}(\Omega)$. For more details about expectation operator, one can refer to Parthasarthy [3].

Given f, $g \in L^2(R)$, then convolution of f and g, $f*g$ is defined by

$$f_* g(x) = \int g(x-y) f(y)\, d\mu(y),$$

where g is fixed, $k(x,y) = g(x-y)$ is a convolution kernel, and the integral operator defined by

$$I_k f(x) = \int k(x-y) f(y)\, d\mu(y)$$

is known as Convolution operator. Suppose $\phi : [0, 1] \rightarrow [0, 1]$ is a measurable transformation, then

$$I_{k,\phi} f(x) = \int k(x-y) f(\phi(y))\, d\mu(y) = \int k_\phi(x-y) f(y)\, d\mu(y)$$

is known as composite convolution operator induced by pair (k,ϕ), where $k_\phi(x-y) = E(f_o(y) k(x-y) \phi^{-1}(y))$, where $f_o = \frac{d\mu\phi^{-1}}{d\mu}$ denotes the Radon-Nikodym derivative of the measure $\mu\phi^{-1}$ with respect to the measure μ.

The adjoint of composite convolution operator $I_{k,\phi}$ is an integral operator induced by the kernel k_ϕ^* and is defined as

$$I^*_{k,\phi} f(x) = \int k^*_\phi(x-y) f(y)\, d\mu(y), \quad \text{where } k^*_\phi(x-y) = \overline{k_\phi(y-x)}$$

Also, $I^n_{k,\phi} f(x) = \int k^n(x-y) f(\phi(y))\, d\mu(y) = \int k^n_\phi(x-y) f(y)\, d\mu(y)$,

where kernel k^n_ϕ is defined as

$$k^n_\phi(x-y) = \int\int\int \ldots \int k_\phi(x-z_1) k_\phi(z_1-z_2) k_\phi(z_2-z_3) \ldots k_\phi(z_{n-1}-y)\, d\mu(z_1)\, d\mu(z_2)\, d\mu(z_3) \ldots d\mu(z_{n-1}) \quad \ldots\ldots (1)$$

For more detail about composition operators, integral operators, convolution operators and composite integral operators, we refer to Singh and Manhas [5], Halmos and Sunder [4], Stepanov ([7], [8]), Gupta and Komal [1], Gupta [2]. Whitley [6] established the Lyubic's [9] conjecture and generalized it to Volterra composition operators on $L^p[0,1]$. The integral operators in particular convolution operators have been the subject matter of extensive study over the last few decades. In this paper some basic operator theoretic properties of composite convolution operators are investigated.

Here, I recall some basic notion in operator theory. Let H be a Hilbert space and B(H) be the algebra of all bounded linear operators acting on H. An operator

$T \in B(H)$ is called normal if $T^* T = T T^*$, and n-normal if $T^* T^n = T^n T^*$, binormal if T^*T commutes with $T T^*$ and n-binormal if $T^* T^n$ commutes with $T^n T^*$. T is hyponormal if $T^* T \leq T T^*$.

Let $L^2(\mu)$ consists of all measurable functions $f : X \rightarrow R$ (or C) such that

$(\int_X |f(x)|^2 d\mu)^{1/2} < \infty$. The space $L^2(X, S, \mu)$ is a Banach space under the norm defined by $\| f \| = (\int_X | f |^2 d\mu)^{1/2}$. Also, $L^2(\mu)$, the space of square-integrable functions is a Hilbert space.

2 Hyponormal, Quasinormal, Binormal Composite Convolution Operators

In this section characterization for hyponormal, quasinormal and binormal composite convolution operators are computed. The conditions for composite convolution operator to be idempotent and projection are explored.

Theorem 2.1: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is hyponormal if and only if*

$$\int k_\phi^*(x-y)\, k_\phi(y-z)\, d\mu(y) \geq \int k_\phi(x-y)\, k_\phi^*(y-z)\, d\mu(y)$$

Proof: Firstly, suppose given condition is true. For $f \in L^2(\mu)$, we have

$$\begin{aligned} \| I_{k,\phi} f \|^2 &= \langle I_{k,\phi}^* I_{k,\phi} f, f \rangle \\ &= \int\int\int k_\phi^*(x-y)\, k_\phi(y-z)\, f(z) d\mu(z)\, d\mu(y)\, \bar{f}(x) d\mu(x) \qquad \text{......(2)} \end{aligned}$$

$$\begin{aligned} \text{and } \| I_{k,\phi}^* f \|^2 &= \langle I_{k,\phi} I_{k,\phi}^* f, f \rangle \\ &= \int\int\int k_\phi(x-y)\, k_\phi^*(y-z)\, f(z) d\mu(z)\, d\mu(y)\, \bar{f}(x) d\mu(x) \qquad \text{.......(3)} \end{aligned}$$

Now,

$$\begin{aligned} &\| I_{k,\phi} f \|^2 - \| I_{k,\phi}^* f \|^2 \\ &= \langle I_{k,\phi}^* I_{k,\phi} f, f \rangle - \langle I_{k,\phi} I_{k,\phi}^* f, f \rangle \\ &= \langle (I_{k,\phi}^* I_{k,\phi} - I_{k,\phi} I_{k,\phi}^*) f, f \rangle \\ &= \int\int \{ \int [k_\phi^*(x-y) k_\phi(y-z) - k_\phi(x-y) k_\phi^*(y-z)] d\mu(y) \}\, f(z) d\mu(z)\, \bar{f}(x) d\mu(x) \\ &= \int\int_{E \times F} \{ \int [k_\phi^*(x-y)\, k_\phi(y-z) - k_\phi(x-y)\, k_\phi^*(y-z)]\, d\mu(y) \} d(\mu \times \mu) \geq 0, \end{aligned}$$

Using (2), (3) and given condition, for any measurable rectangle $E \times F$ of finite measure. Hence, $I_{k,\phi}$ is hyponormal.

Conversely, suppose $I_{k,\phi}$ is hyponormal. Then the required condition trivially follows.

Theorem 2.2: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is quasinormal if and only if*

$$\iint k_\phi^*(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, d\mu(y)\, d\mu(z) = \iint k_\phi(x-y)\, k_\phi^*(y-z)\, k_\phi(z-t) d\mu(y)\, d\mu(z)$$

Proof: Firstly, suppose $I_{k,\phi}$ is quasinormal. Consider $f = \chi_E$ and $g = \chi_F$. For any measurable rectangle $E \times F$ of finite measure, we have

$$\iint_{E\times F} k_\phi^*(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, d\mu(y)\, d\mu(z) = \iint_{E\times F} k_\phi(x-y)\, k_\phi^*(y-z) k_\phi(z-t)\, d\mu(y)\, d\mu(z)$$

Hence, the required condition holds.

Conversely, suppose that given condition is true. For $f,g \in L^2(\mu)$, we have

$$\begin{aligned}\langle I_{k,\phi}^*\, I_{k,\phi}\, I_{k,\phi} f, g \rangle &= \int (I_{k,\phi}^*\, I_{k,\phi}\, I_{k,\phi} f)(x)\, \bar{g}(x) d\mu(x)\\ &= \iint k_\phi^*(x-y)\, (I_{k,\phi}\, I_{k,\phi} f)(y)\, d\mu(y)\, \bar{g}(x) d\mu(x)\\ &= \iiiint k_\phi^*(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, f(t)\, \bar{g}(x)\, d\mu(t)\, d\mu(z)\, d\mu(y) d\mu(x)\end{aligned}$$

…….(4)

Also, $\langle I_{k,\phi}\, I_{k,\phi}^*\, I_{k,\phi} f, g \rangle$

$$\begin{aligned}&= \iint k_\phi(x-y)\, (I_{k,\phi}^*\, I_{k,\phi} f)(y)\, d\mu(y)\, \bar{g}(x) d\mu(x)\\ &= \iiiint k_\phi(x-y)\, k_\phi^*(y-z)\, k_\phi(z-t) f(t)\, \bar{g}(x)\, d\mu(t) d\mu(z) d\mu(y) d\mu(x)\end{aligned}$$

……...(5)

From equation (4) and (5), it follows that $I_{k,\phi}$ is quasinormal if the given condition is true.

Theorem 2.3: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is binormal if and only if*

$$\iiint k_\phi^*(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, k_\phi^*(t-p)\, d\mu(y)\, d\mu(z)\, d\mu(t)$$
$$= \iiint k_\phi(x-y)\, k_\phi^*(y-z)\, k_\phi^*(z-t)\, k_\phi(t-p)\, d\mu(y) d\mu(z)\, d\mu(t).$$

Proof: Firstly, suppose the condition is true. For f, g $\in L^2(\mu)$, we have

$$\langle I^*_{k,\phi} I_{k,\phi} I_{k,\phi} I^*_{k,\phi} f,g\rangle$$

$$= \int (I^*_{k,\phi} I_{k,\phi} I_{k,\phi} I^*_{k,\phi} f)(x)\, \bar{g}(x)d\mu(x)$$

$$= \int\int [k^*_\phi(x-y)(I_{k,\phi} I_{k,\phi} I^*_{k,\phi} f)(y)\, d\mu(y)]\, \bar{g}(x)d\mu(x)$$

$$= \int\int k^*_\phi(x-y)(\int k_\phi(y-z)(I_{k,\phi} I^*_{k,\phi} f)(z)d\mu(z))d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int k^*_\phi(x-y)\, k_\phi(y-z)\, (\int k_\phi(z-t)\, (I^*_{k,\phi} f)(t)d\mu(t))d\mu(z)d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int\int k^*_\phi(x-y)\, k_\phi(y- z)k_\phi(z- t)\int k^*_\phi(t -p)f(p)d\mu(p)d\mu(t)d\mu(z)\, d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int k^*_\phi(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, k^*_\phi(t- p)f(p)d\mu(p))d\mu(t)d\mu(z)\, d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int k^*_\phi(x-y)\, k_\phi(y-z)\, k_\phi(z -t)\, k^*_\phi(t -p)d\mu(y)d\mu(z)d\mu(t)f(p)d\mu(p)\, \bar{g}(x)d\mu(x) \quad \ldots\ldots\ldots(6)$$

and $\langle I_{k,\phi} I^*_{k,\phi} I^*_{k,\phi} I_{k,\phi} f, g\rangle$

$$= \int I_{k,\phi} I^*_{k,\phi} I^*_{k,\phi} I_{k,\phi} f(x)\, \bar{g}(x)d\mu(x)$$

$$= \int\int k_\phi(x-y)\, (I^*_{k,\phi} I^*_{k,\phi} I_{k,\phi} f)(y)\, d\mu(y))\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int k_\phi(x-y)\, k^*_\phi(y-z)\, k^*_\phi(z-t)\, k_\phi(t-p)\, f(p)\, d\mu(p))\, d\mu(t)\, d\mu(z)\, d\mu(y)\, \bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int k_\phi(x-y)\, k^*_\phi(y - z)\, k^*_\phi(z -t)\, k_\phi(t-p)\, d\mu(y)d\mu(z)d\mu(t)\, f(p)\, \bar{g}(x)d\mu(x) \quad \ldots\ldots\ldots(7)$$

It follows from (6) and (7) that $I_{k,\phi}$ is binormal.

Conversely, suppose $I_{k,\phi}$ is binormal. Take f = χ_E and g = χ_F, we see that from (6) and (7)

$$\int\int_E\int_F k^*_\phi(x-y)\, k_\phi(y-z)\, k_\phi(z-t)\, k^*_\phi(t-p)\, d\mu(y)\, d\mu(z)\, d\mu(t)$$

$$= \int\int_E\int_F k_\phi(x-y)\, k^*_\phi(y-z)\, k^*_\phi(z-t)\, k_\phi(t-p)d\mu(y)\, d\mu(z)\, d\mu(t)$$

for all E, F $\in$ S$\times$S. Hence the required condition holds.

Theorem 2.4: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is an idempotent if and only if k_ϕ is an idempotent, i.e.*

$k_\phi^2 = k_\phi$ a.e., where $k_\phi^2(x-y) = \int k_\phi(x-z)\, k_\phi(z-y)\, dz$.

Proof: Firstly, suppose k_ϕ is idempotent. For f, g $\in L^2(\mu)$, we have

$$\begin{aligned}
\langle I_{k,\phi}^2 f, g\rangle &= \langle I_{k,\phi} f, I_{k,\phi}^* g\rangle \\
&= \int (I_{k,\phi} f)(x)\, \overline{(I_{k,\phi}^* g)(x)}\, d\mu(x) \\
&= \iint k_\phi(x-y) f(y) d\mu(y) \overline{\int k_\phi^*(x-z) g(z) d\mu(z)}\, d\mu(x) \\
&= \iiint k_\phi(x-y) k_\phi(z-x) f(y) \bar{g}(z) d\mu(z)\, d\mu(y)\, d\mu(x). \\
&= \iiint k_\phi(z-x) k_\phi(x-y) d\mu(x)\, f(y) \bar{g}(z) d\mu(y)) d\mu(z) \\
&= \iint k_\phi^2(z-y) f(y) d\mu(y)\, \bar{g}(z) d\mu(z) \\
&= \int (I_{k,\phi} f)(z) \bar{g}(z) dz = \langle I_{k,\phi} f, g\rangle .
\end{aligned}$$

Hence, $I_{k,\phi}$ is an idempotent.

Conversely, suppose $I_{k,\phi}$ is an idempotent, i.e.,

$$\langle I_{k,\phi}^2 f, g\rangle = \langle I_{k,\phi} f, g\rangle \qquad \text{for all f, g} \in L^2(\mu)$$

Then, $\iiint k_\phi(z-x)\, k_\phi(x-y)\, d\mu(x)\, f(y)\, d\mu(y)\, \bar{g}(z)\, d\mu(z)$

$$\begin{aligned}
&= \iint k_\phi(z-y) f(y)\, \bar{g}(z)\, d\mu(x) d\mu(z) \\
&= \iint k_\phi^2(z-y) f(y) d\mu(y) \bar{g}(z) d\mu(z) = \iint k_\phi(z-y) f(y)\, \bar{g}(z)\, d\mu(y) d\mu(z)
\end{aligned}$$

Taking $f = \chi_E$ and $g = \chi_F$ for E, F $\in$ **Ω**, we have

$$\int_{E\times F} k_\phi^2 (z-y)\, d\mu(y)\, d\mu(z) = \int_{E\times F} k_\phi (z-y) d\mu(y)\, d\mu(z).$$

This proves that $k_\phi^2 = k_\phi$ a.e. Hence, k_ϕ is an idempotent.

Corollary 2.5: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is an projection if and only if k_ϕ is an idempotent and $\overline{k_\phi(y-x)} = k_\phi(x-y)$.*

Theorem 2.6: *The product of two composite convolution operators ($I_{k,\phi} . I_{h,\phi}$) is a composite convolution operators $I_{k_\phi h_\phi}$,* if $(k_\phi h_\phi)(x\text{-}z) = \int k_\phi(x\text{-}y)\, h_\phi(y\text{-}z)d\mu(y)$.

Moreover, $I_{k,\phi}\ I_{h,\phi} = I_{h,\phi}\ I_{k,\phi}$, if $(k_\phi h_\phi)(x\text{-}z) = (h_\phi k_\phi)(x\text{-}z)$

Proof: For every $f \in L^2[0,1]$,

$$\begin{aligned}
I_{k,\phi}\ I_{h,\phi} f(x) &= \int k_\phi(x\text{-}y)\,(I_{h,\phi}f)(y)d\mu(y) \\
&= \int\int k_\phi(x\text{-}y)\, h_\phi(y\text{-}z)\, f(z)\, d\mu(z)\, d\mu(y) \\
&= \int \left[\int k_\phi(x\text{-}y)\, h_\phi(y\text{-}z)d\mu(y)\right] f(z)\, d\mu(z) \\
&= \int (k_\phi h_\phi)(x\text{-}z)\, f(z)d\mu(z) \qquad \ldots\ldots..(8)
\end{aligned}$$

Hence, the product of two composite convolution operators, i.e., $I_{k,\phi}\ .\ I_{h,\phi}$ is again a composite convolution operator $I_{k_\phi h_\phi}$ induced by convolution kernel $k_\phi h_\phi$.

$$\begin{aligned}
\text{Again, } I_{h,\phi}\ I_{k,\phi} f(x) &= \int\int h_\phi(x\text{-}y)k_\phi(y\text{-}z)\, f(z)\, d\mu(z)\, d\mu(y) \\
&= \int (h_\phi k_\phi)(x\text{-}z)\, f(z)d\mu(z) \qquad \ldots\ldots\ldots(9)
\end{aligned}$$

The equation (8) and (9) gives desired conclusion.

Corollary 2.7: *The product of two composite convolution operators is zero if atleast one of them is zero.*

3 n- Normal and n- Binormal Composite Convolution Operators

Theorem 3.1: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is n-normal if and only if*

$$\int k_\phi^*(x\text{-}y)\ k_\phi^n(y\text{-}z)\, d\mu(y) = \int k_\phi^n(x\text{-}y)\ k_\phi^*(y\text{-}z)\, d\mu(y)$$

Proof: Firstly, suppose $I_{k,\phi}$ is n- normal. Then for any measurable rectangle $E \times F$ of finite measure, we have

$$\begin{aligned}
\langle I_{k,\phi}^*\ I_{k,\phi}^n\ \chi_E, \chi_F\rangle &= \int I_{k,\phi}^*\ I_{k,\phi}^n\ \chi_E(x)\, \chi_F(x)d\mu(x) \\
&= \int\int\int k_\phi^*(x\text{-}y)\ k_\phi^n(y\text{-}z)\, \chi_E(y)\, \chi_F(x)\, d\mu(z)d\mu(y)\, d\mu(x) \\
&= \iint_{E\times F} \int k_\phi^n(y\text{-}z)\ k_\phi^*(x\text{-}y)\, d\mu(y)\, d(\mu\times\mu)
\end{aligned}$$

and similarly

$$\langle I^n_{k,\phi}\, I^*_{k,\phi}\, \chi_E, \chi_F \rangle = \iint_{E\times F} \int k^n_\phi(x-y)\; k^*_\phi(y-z)\, d\mu(y)\, d(\mu\times\mu)$$

Hence, the condition follows.

Conversely, if the condition is true, then $I_{k,\phi}$ is n- normal as the proof is straight forward.

Theorem 3.2: *Let $I_{k,\phi} \in B(L^2(\mu))$. Then $I_{k,\phi}$ is n-binormal if and only if*

$$\iiint k^*_\phi(x-y)\; k^n_\phi(y-z)\; k^n_\phi(z-t)\, k^*_\phi(t-p)\, d\mu(y)\, d\mu(z)\, d\mu(t)$$
$$= \iiint k^n_\phi(x-y)\; k^*_\phi(y-z)\, k^*_\phi(z-t)\, k^n_\phi(t-p)\, d\mu(y)d\mu(z)\, d\mu(t).$$

Proof: Firstly, suppose the condition is true. For f, g $\in L^2(\mu)$, we have

$$\langle I^*_{k,\phi}\, I^n_{k,\phi}\, I^n_{k,\phi}\, I^*_{k,\phi}\, f, g\rangle$$

$$= \int (I^*_{k,\phi}\, I^n_{k,\phi}\, I^n_{k,\phi}\, I^*_{k,\phi}\, f)(x)\; \bar{g}(x)d\mu(x)$$

$$= \iint [k^*_\phi(x-y)(I^n_{k,\phi}\, I^n_{k,\phi}\, I^*_{k,\phi}\, f)(y)\, d\mu(y)]\; \bar{g}(x)d\mu(x)$$

$$= \iint k^*_\phi(x-y)(\int k^n_\phi(y-z)\,(I^n_{k,\phi}\, I^*_{k,\phi}\, f)(z)d\mu(z))\, d\mu(y)\,\bar{g}(x)d\mu(x)$$

$$= \iiint k^*_\phi(x-y)\; k^n_\phi(y-z)\,(\int k^n_\phi(z-t)\,(I^*_{k,\phi}\, f)(t)d\mu(t))d\mu(z)d\mu(y)\; \bar{g}(x)d\mu(x)$$

$$= \iiiint k^*_\phi(x-y)\, k^n_\phi(y-z)\, k^n_\phi(z-t)\int k^*_\phi(t-p)f(p)d\mu(p)d\mu(t)d\mu(z)d\mu(y)\,\bar{g}(x)d\mu(x)$$

$$= \int\!\!\int\!\!\int\!\!\int\!\!\int k^*_\phi(x-y)\, k^n_\phi(y-z)\, k^n_\phi(z-t)\, k^*_\phi(t-p)f(p)d\mu(p))d\mu(t)d\mu(z)\, d\mu(y)\,\bar{g}(x)d\mu(x)$$

$$= \int\!\!\int\!\!\int\!\!\int\!\!\int k^*_\phi(x-y)\; k^n_\phi(y,z)\; k^n_\phi(z,t)\, k^*_\phi(t-p)d\mu(y)d\mu(z)d\mu(t)f(p)\, d\mu(p)\,\bar{g}(x)d\mu(x)$$

…......…(10)

and $\langle I_{k,\phi}\, I^*_{k,\phi}\, I^*_{k,\phi}\, I_{k,\phi}\, f, g\rangle$

$$= \int I_{k,\phi}\, I^*_{k,\phi}\, I^*_{k,\phi}\, I_{k,\phi}\, f(x)\; \bar{g}(x)d\mu(x)$$

$$= \iint k^n_\phi(x-y)\,(I^*_{k,\phi}\, I^*_{k,\phi}\, I_{k,\phi}\, f)(y)\, d\mu(y))\,\bar{g}(x)d\mu(x)$$

$$= \int\!\!\int\!\!\int\!\!\int\!\!\int k^n_\phi(x-y)\, k^*_\phi(y-z)\, k^*_\phi(z-t)\, k^n_\phi(t-p)f(p)d\mu(p)d\mu(t)\, d\mu(z)d\mu(y)\,\bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int \; k_\phi^n(x\text{-}y)\, k_\phi^*(y - z)\, k_\phi^*(z - t)\; k_\phi^n(t\text{-}p)\; d\mu(y)d\mu(z)d\mu(t)\; f(p)\; \bar{g}\,(x)d\mu(x)$$

…….…(11)

Hence, $I_{k,\phi}$ is n- binormal using (10) and (11).

Conversely, suppose $I_{k,\phi}$ is n-binormal. For f = χ_E and g = χ_F, we get from (10) and (11)

$$\int \int_E \int_F \; k_\phi^*(x\text{-}y)\; k_\phi^n(y\text{-}z)\; k_\phi^n(z\text{-}t)\, k_\phi^*(t\text{-}p)\; d\mu(y)\; d\mu(z)\; d\mu(t)$$

$$= \int \int_E \int_F \; k_\phi^n(x\text{-}y)\; k_\phi^*(y\text{-}z)\; k_\phi^*(z\text{-}t)\; k_\phi^n(t\text{-}p) d\mu(y)\; d\mu(z)\; d\mu(t)$$

for all E, F $\in$ S$\times$S. Hence the required condition holds.

4 Conclusion

In this paper major thrust has been made to obtain criterions for hyponormal, quasinormal, binormal composite convolution operators. The characterizations for composite convolution operators to be n-normal and n-binormal are also major findings of the research paper. Above paper is extension of my research paper "Composite Convolution Operators on $L^2(\mu)$" published in International Journal of Innovation in Sciences and Mathematics (IJISM), vol.2, Issue 4, (2014), 364-366.

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