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## Almost Contra $\theta$ gs-Continuous Functions

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### Abstract

*The aim of this paper is to introduce and study of a new class of function called almost contra  $\theta$ gs-continuous functions using  $\theta$ gs-open set.*

**Keywords:** *Almost contra  $\theta$ gs-continuous,  $\theta$ gs-closed set*

## 1 Introduction

In 1996, Dontchev [6] introduced the notion of contra continuity and strong S-closedness in topological spaces. A new weaker form of this class of functions called contra semi continuous function is introduced and investigated by Dontchev and Noiri [7]. Recently in [10] the notion of  $\theta$ -generalized semi closed (briefly,  $\theta$ gs-closed) set was introduced. The aim of this paper is to introduce and study new generalization of contra continuity called Almost contra  $\theta$ gs-continuous functions utilising  $\theta$ gs-open set.

## 2 Preliminaries

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  (or simply  $X, Y$ ) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a space  $X$  the closure and interior of  $A$  with respect to  $\tau$  are denoted by  $Cl(A)$  and  $Int(A)$  respectively.

**Definition 2.1** A subset  $A$  of a space  $X$  is called

- (1) a semi-open set [9] if  $A \subset Cl(Int(A))$ .
- (2) a semi-closed set [3] if  $Int(Cl(Int(A))) \subset A$ .
- (3) a regular open [23] if  $A = Int(Cl(Int(A)))$

**Definition 2.2** [4] A point  $x \in X$  is called a semi- $\theta$ -cluster point of  $A$  if  $sCl(U) \cap A \neq \phi$ , for each semi-open set  $U$  containing  $x$ . The set of all semi- $\theta$ -cluster point of  $A$  is called semi- $\theta$ -closure of  $A$  and is denoted by  $sCl_\theta(A)$ . A subset  $A$  is called semi- $\theta$ -closed set if  $sCl_\theta(A) = A$ . The complement of semi- $\theta$ -closed set is semi- $\theta$ -open set.

**Definition 2.3** [10] A subset  $A$  of  $X$  is  $\theta$ generalized semi-closed(briefly,  $\theta$ gs-closed)set if  $sCl_\theta(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ . The complement of  $\theta$ gs-closed set is  $\theta$ generalized-semi open (briefly, $\theta$ gs-open).The family of all  $\theta$ gs-closed sets of  $X$  is denoted by  $\theta GSC(X,\tau)$  and  $\theta$ gs-open sets by  $\theta GSO(X,\tau)$ .

**Definition 2.4** [16] A topological space  $X$  is called  $T_{\theta gs}$ -space if every  $\theta$ gs-closed set in it is closed set.

**Definition 2.5** [13] A topological space  $X$  is said to be

- (i)  $\theta$ gs- $T_1$  space if for any pair of distinct points  $x$  and  $y$ , there exist  $\theta$ gs-open sets  $G$  and  $H$  such that  $x \in G$ ,  $y \notin G$  and  $x \notin H$ ,  $y \in H$ .
- (ii)  $\theta$ gs- $T_2$  if for each pair of distinct points  $x$  and  $y$  of  $X$ , there exist disjoint  $\theta$ gs-open sets, one containing  $x$  and the other containing  $y$ .

**Definition 2.6** A function  $f: X \rightarrow Y$  is called:

- (i)  $\theta$ -generalized semi-continuous (briefly, $\theta$ gs-continuous)[11] if  $f^{-1}(F)$  is  $\theta$ gs-closed set in  $X$  for every closed set  $F$  of  $Y$ .
- (ii) contra  $\theta$ gs-continuous [19] if  $f^{-1}(F)$  is  $\theta$ gs-closed set in  $X$  for every open set  $F$  of  $Y$ .

**Definition 2.7** [12] A function  $f: X \rightarrow Y$  is said to be  $\theta$ gs-open (resp.,  $\theta$ gs-closed) if  $f(V)$  is  $\theta$ gs-open (resp.,  $\theta$ gs-closed) in  $Y$  for every open set (resp., closed)  $V$  in  $X$ .

**Definition 2.8** [23] (i)A topological space  $X$  is called Ultra Hausdroff space, if every pair of distinct points of  $x$  and  $y$  in  $X$ , there exist disjoint clopen sets  $U$  and  $V$  in  $X$  containing  $x$  and  $y$  respectively.

(ii)A topological space  $X$  is called Ultra normal if each pair of disjoint closed sets can be separated by disjoint clopen sets.

**Definition 2.9** [19] A space  $X$  is called locally  $\theta$ gs-indiscrete if every  $\theta$ gs-open set is closed in  $X$ .

**Definition 2.10** [17] *A topological space  $X$  is said to be  $\theta$ gs-normal if each pair of disjoint closed sets can be separated by disjoint  $\theta$ gs-open sets.*

**Definition 2.11** [18] *A topological space  $X$  is said to be*  
*(i)  $\theta$ gs-connected if  $X$  cannot be written as union of two non empty disjoint  $\theta$ gs-open sets.*  
*(ii)  $\theta$ gs-compact if every  $\theta$ gs-open cover of  $X$  has a finite subcover.*

**Definition 2.12** [14] *A function  $f : X \rightarrow Y$  is said to be almost continuous if  $f^{-1}(V)$  is open in  $X$  for each regular open set  $V$  of  $Y$ .*

**Definition 2.13** [8] *A topological space  $X$  is said to be hyperconnected if every open set is dense.*

**Definition 2.14** [2] *A function  $f : X \rightarrow Y$  is said to be  $R$ -map if  $f^{-1}(V)$  is regular open in  $X$  for each regular open set  $V$  of  $Y$ .*

**Definition 2.15** [15] *A function  $f : X \rightarrow Y$  is said to be perfectly continuous if  $f^{-1}(V)$  is clopen in  $X$  for each open set  $V$  of  $Y$ .*

**Definition 2.16** [21] *A space  $X$  is said to be weakly Hausdorff if each element of  $X$  is an intersection of regular closed sets.*

**Definition 2.17** *A space  $X$  is said to be*  
*(i) Nearly compact [21] if every regular open cover of  $X$  has a finite subcover.*  
*(ii) Nearly countably compact [21] if every countable cover of  $X$  by regular open sets has a finite subcover.*  
*(iii) Nearly Lindelöf [21] if every regular open cover of  $X$  has a countable subcover.*  
*(iv)  $S$ -Lindelöf [8] if every cover of  $X$  by regular closed sets has a countable subcover.*  
*(v) Countably  $S$ -closed [5] if every countable cover of  $X$  by regular closed sets has a finite subcover.*  
*(vi)  $S$ -closed [1] if every regular closed cover of  $x$  has a finite subcover.*

### 3 Almost Contra $\theta$ gs-Continuous Functions

In this section, new type of continuity called an almost contra  $\theta$ gs-continuity, which is weaker than contra  $\theta$ gs-continuity is introduced and studied some of their properties.

**Definition 3.1** *A function  $f : X \rightarrow Y$  is said to be almost contra  $\theta$ gs-continuous if  $f^{-1}(V)$  is  $\theta$ gs-closed in  $X$  for each regular open set  $V$  in  $Y$ .*

**Theorem 3.2** *If  $X$  is  $T_{\theta gs}$ -space and  $f : X \rightarrow Y$  is almost contra  $\theta gs$ -continuous, then it is contra almost continuous.*

**Proof.** Let  $U$  be a regular open set in  $Y$ . Since  $f$  is almost contra  $\theta gs$ -continuous  $f^{-1}(U)$  is  $\theta gs$ -closed set in  $X$  and  $X$  is  $T_{\theta gs}$ -space, which implies  $f^{-1}(U)$  is closed set in  $X$ . Therefore  $f$  is contra almost continuous.

**Theorem 3.3** *If a function  $f : X \rightarrow Y$  is almost contra  $\theta gs$ -continuous and  $X$  is locally  $\theta gs$ -indiscrete space, then  $f$  is almost continuous.*

**Proof.** Let  $U$  be a regular open set in  $Y$ . Since  $f$  is almost contra  $\theta gs$ -continuous  $f^{-1}(U)$  is  $\theta gs$ -closed set in  $X$  and  $X$  is locally  $\theta gs$ -indiscrete space, which implies  $f^{-1}(U)$  is an open set in  $X$ . Therefore  $f$  is almost continuous.

**Theorem 3.4** *If  $f : X \rightarrow Y$  is contra  $\theta gs$ -continuous then it is almost contra  $\theta gs$ -continuous.*

**Proof.** Obvious, because every regular open set is open set.

**Remark 3.5** *Converse of the above theorem need not be true in general as seen from the following example.*

**Example 3.6** *Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \phi, \{a\}, \{a, b\}, \{b, c\}\}$  be topologies on  $X$  and  $Y$  respectively. We have  $\theta gs$ -closed sets in  $X$  are  $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ . Define a function  $f : X \rightarrow Y$  by  $f(a) = a$ ,  $f(b) = b$  and  $f(c) = c$ . Then  $f$  is almost contra  $\theta gs$ -continuous function but not contra  $\theta gs$ -continuous, because for the open set  $\{a, b\}$  in  $Y$ ,  $f^{-1}(\{a, b\}) = \{a, b\}$  is not  $\theta gs$ -closed in  $X$ .*

**Theorem 3.7** *If  $f : X \rightarrow Y$  is almost contra  $\theta gs$ -continuous and  $X$  is  $T_{\theta gs}$ -space then  $f$  is contra almost continuous.*

**Proof.** Let  $U$  be a regular open set in  $Y$ . Since  $f$  is almost contra  $\theta gs$ -continuous  $f^{-1}(U)$  is  $\theta gs$ -closed set in  $X$  and  $X$  is  $T_{\theta gs}$ -space, which implies  $f^{-1}(U)$  is an closed set in  $X$ . Therefore  $f$  is contra almost continuous.

**Theorem 3.8** *For a function  $f : X \rightarrow Y$  the followings are equivalent:*

- (i)  $f$  is almost contra  $\theta gs$ -continuous.
- (ii) for every regular closed set  $F$  of  $Y$ ,  $f^{-1}(F)$  is  $\theta gs$ -open set of  $X$ .

**Proof.** (i)  $\Rightarrow$  (ii) Let  $F$  be a regular closed set in  $Y$ , then  $Y - F$  is a regular open set in  $Y$ . By (i),  $f^{-1}(Y - F) = X - f^{-1}(F)$  is  $\theta gs$ -closed set in  $X$ . This implies  $f^{-1}(F)$  is  $\theta gs$ -open set in  $X$ . Therefore, (ii) holds.

(ii)  $\Rightarrow$  (i) Let  $G$  be a regular open set of  $Y$ . Then  $Y - G$  is a regular closed set in  $Y$ . By (ii),  $f^{-1}(Y - G)$  is  $\theta$ -open set in  $X$ . This implies  $X - f^{-1}(G)$  is  $\theta gs$ -open set in  $X$ , which implies  $f^{-1}(G)$  is  $\theta gs$ -closed set in  $X$ . Therefore, (i) holds.

**Theorem 3.9** For a function  $f : X \rightarrow Y$  the followings are equivalent:

- (i)  $f$  is almost contra  $\theta$ gs-continuous.
- (ii)  $f^{-1}(Int(Cl(G)))$  is  $\theta$ gs-closed set in  $X$  for every open subset  $G$  of  $Y$ .
- (iii)  $f^{-1}(Cl(Int(F)))$  is  $\theta$ gs-open set in  $X$  for every closed subset  $F$  of  $Y$ .

**Proof.** (i)  $\Rightarrow$  (ii) Let  $G$  be an open set in  $Y$ . Then  $Int(Cl(G))$  is regular open set in  $Y$ . By (i),  $f^{-1}(Int(Cl(G))) \in \theta GSC(X)$ .

(ii)  $\Rightarrow$  (i) Proof is obvious.

(i)  $\Rightarrow$  (iii) Let  $F$  be a closed set in  $Y$ . Then  $Cl(Int(G))$  is regular closed set in  $Y$ . By (i),  $f^{-1}(Cl(Int(G))) \in \theta GSO(X)$ .

(iii)  $\Rightarrow$  (i) Proof is obvious.

**Theorem 3.10** If  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous injection and  $Y$  is weakly Hausdorff, then  $X$  is  $\theta$ gs- $T_1$ .

**Proof.** Suppose  $Y$  is weakly Hausdorff. For any distinct points  $x$  and  $y$  in  $X$ , there exist  $V$  and  $W$  regular closed sets in  $Y$  such that  $f(x) \in V$ ,  $f(y) \notin V$ ,  $f(y) \in W$  and  $f(x) \notin W$ . Since  $f$  is almost contra  $\theta$ gs-continuous,  $f^{-1}(V)$  and  $f^{-1}(W)$  are  $\theta$ gs-open subsets of  $X$  such that  $x \in f^{-1}(V)$ ,  $y \notin f^{-1}(V)$ ,  $y \in f^{-1}(W)$  and  $x \notin f^{-1}(W)$ . This shows that  $X$  is  $\theta$ gs- $T_1$ .

**Corollary 3.11** If  $f : X \rightarrow Y$  is a contra  $\theta$ gs-continuous injection and  $Y$  is weakly Hausdorff, then  $X$  is  $\theta$ gs- $T_1$ .

**Theorem 3.12** If  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous injective function from space  $X$  into a Ultra Hausdorff space  $Y$ , then  $X$  is  $\theta$ gs- $T_2$ .

**Proof.** Let  $x$  and  $y$  be any two distinct points in  $X$ . Since  $f$  is an injective  $f(x) \neq f(y)$  and  $Y$  is Ultra Hausdorff space, there exist disjoint clopen sets  $U$  and  $V$  of  $Y$  containing  $f(x)$  and  $f(y)$  respectively. Then  $x \in f^{-1}(U)$  and  $y \in f^{-1}(V)$ , where  $f^{-1}(U)$  and  $f^{-1}(V)$  are disjoint  $\theta$ gs-open sets in  $X$ . Therefore  $X$  is  $\theta$ gs- $T_2$ .

**Theorem 3.13** If  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous closed injection and  $Y$  is ultra normal, then  $X$  is  $\theta$ gs-normal.

**Proof.** Let  $E$  and  $F$  be disjoint closed subsets of  $X$ . Since  $f$  is closed and injective  $f(E)$  and  $f(F)$  are disjoint closed sets in  $Y$ . Since  $Y$  is ultra normal there exists disjoint clopen sets  $U$  and  $V$  in  $Y$  such that  $f(E) \subset U$  and  $f(F) \subset V$ . This implies  $E \subset f^{-1}(U)$  and  $F \subset f^{-1}(V)$ . Since  $f$  is an almost contra  $\theta$ gs-continuous injection,  $f^{-1}(U)$  and  $f^{-1}(V)$  are disjoint  $\theta$ gs-open sets in  $X$ . This shows  $X$  is  $\theta$ gs-normal.

**Theorem 3.14** If  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous surjection and  $X$  is  $\theta$ gs-connected space, then  $Y$  is connected.

**Proof.** Let  $f : X \rightarrow Y$  be an almost contra  $\theta$ gs-continuous surjection and  $X$  is  $\theta$ gs-connected space. Suppose  $Y$  is a not connected space. Then there exist disjoint open sets  $U$  and  $V$  such that  $Y = U \cup V$ . Therefore  $U$  and  $V$  are clopen in  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\theta$ gs-open sets in  $X$ . Moreover  $f^{-1}(U)$  and  $f^{-1}(V)$  are non empty disjoint and  $X = f^{-1}(U) \cup f^{-1}(V)$ . This is contradiction to the fact that  $X$  is  $\theta$ gs-connected space. Therefore,  $Y$  is connected.

**Theorem 3.15** *For two functions  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$ , let  $g \circ f : X \rightarrow Z$  is a composition function. Then, the following properties hold*

- (i) *If  $f$  is almost contra  $\theta$ gs-continuous and  $g$  is an R-map, then  $g \circ f$  is almost contra  $\theta$ gs-continuous.*
- (ii) *If  $f$  is almost contra  $\theta$ gs-continuous and  $g$  is perfectly continuous, then  $g \circ f$  is  $\theta$ gs-continuous and contra  $\theta$ gs-continuous.*
- (iii) *If  $f$  is contra  $\theta$ gs-continuous and  $g$  is almost continuous, then  $g \circ f$  is almost contra  $\theta$ gs-continuous.*

**Proof.** (i) Let  $V$  be any regular open set in  $Z$ . Since  $g$  is an R-map,  $g^{-1}(V)$  is regular open in  $Y$ . Since  $f$  is an almost contra  $\theta$ gs-continuous  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\theta$ gs-closed set in  $X$ . Therefore,  $g \circ f$  is almost contra  $\theta$ gs-continuous.

(ii) Let  $V$  be any open set in  $Z$ . Since  $g$  is perfectly continuous,  $g^{-1}(V)$  is clopen in  $Y$ . Since  $f$  is an almost contra  $\theta$ gs-continuous  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\theta$ gs-open and  $\theta$ gs-closed set in  $X$ . Therefore,  $g \circ f$  is  $\theta$ gs-continuous and contra  $\theta$ gs-continuous.

(iii) Let  $V$  be any regular open set in  $Z$ . Since  $g$  is almost continuous,  $g^{-1}(V)$  is open in  $Y$ . Since  $f$  is contra  $\theta$ gs-continuous  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\theta$ gs-closed set in  $X$ . Therefore,  $g \circ f$  is almost contra  $\theta$ gs-continuous.

**Theorem 3.16** *Let  $f : X \rightarrow Y$  is a contra  $\theta$ gs-continuous and  $g : Y \rightarrow Z$  is  $\theta$ gs-continuous. If  $Y$  is  $T_{\theta gs}$ -space, then  $g \circ f : X \rightarrow Z$  is an almost contra  $\theta$ gs-continuous.*

**Proof.** Let  $V$  be any regular open and hence open set in  $Z$ . Since  $g$  is  $\theta$ gs-continuous  $g^{-1}(V)$  is  $\theta$ gs-open in  $Y$  and  $Y$  is  $T_{\theta gs}$ -space implies  $g^{-1}(V)$  open in  $Y$ . Since  $f$  is contra  $\theta$ gs-continuous  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\theta$ gs-closed set in  $X$ . Therefore,  $g \circ f$  is an almost contra  $\theta$ gs-continuous.

**Definition 3.17** *A function  $f : X \rightarrow Y$  is said to be strongly  $\theta$ gs-open (resp. strongly  $\theta$ gs-closed) if image of every  $\theta$ gs-open (resp.  $\theta$ gs-closed) set of  $X$  is  $\theta$ gs-open (resp.  $\theta$ gs-closed) set in  $Y$ .*

**Theorem 3.18** *If  $f : X \rightarrow Y$  is surjective strongly  $\theta$ gs-open (or strongly  $\theta$ gs-closed) and  $g : Y \rightarrow Z$  is a function such that  $g \circ f : X \rightarrow Z$  is an almost contra  $\theta$ gs-continuous, then  $g$  is an almost contra  $\theta$ gs-continuous.*

**Proof.** Let  $V$  be any regular closed (resp. regular open) set in  $Z$ . Since  $g \circ f$  is an almost contra  $\theta$ gs-continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $\theta$ gs-open (resp.  $\theta$ gs-closed) in  $X$ . Since  $f$  is surjective and strongly  $\theta$ gs-open (or strongly  $\theta$ gs-closed),  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $\theta$ gs-open (or  $\theta$ gs-closed). Therefore  $g$  is an almost contra  $\theta$ gs-continuous.

**Definition 3.19** A topological space  $X$  is said to be  $\theta$ gs-ultra-connected if every two non empty  $\theta$ gs-closed subsets of  $X$  intersect.

**Theorem 3.20** If  $X$  is  $\theta$ gs-ultra-connected and  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous surjection, then  $Y$  is hyperconnected.

**Proof.** Let  $X$  be a  $\theta$ gs-ultra-connected and  $f : X \rightarrow Y$  is an almost contra  $\theta$ gs-continuous surjection. Suppose  $Y$  is not hyperconnected. Then there exists an open set  $V$  such that  $V$  is not dense in  $Y$ . Therefore, there exist nonempty regular open subsets  $B_1 = \text{Int}(Cl(V))$  and  $B_2 = Y - Cl(V)$  in  $Y$ . Since  $f$  is an almost contra  $\theta$ gs-continuous surjection,  $f^{-1}(B_1)$  and  $f^{-1}(B_2)$  are disjoint  $\theta$ gs-closed sets in  $X$ . This is contrary to the fact that  $X$  is  $\theta$ gs-ultra-connected. Therefore,  $Y$  is hyperconnected.

**Definition 3.21** A space  $X$  is said to be

- (i) Countably  $\theta$ gs-compact if every countable cover of  $X$  by  $\theta$ gs-open sets has a finite subcover.
- (ii)  $\theta$ gs-Lindelöf if every  $\theta$ gs-open cover of  $X$  has a countable subcover.
- (iii) mildly  $\theta$ gs-compact if every  $\theta$ gs-clopen cover of  $X$  has a finite subcover.
- (iv) mildly countably  $\theta$ gs-compact if every countable cover of  $X$  by  $\theta$ gs-clopen sets has a finite subcover.
- (v) mildly  $\theta$ gs-Lindelöf if every  $\theta$ gs-clopen cover of  $X$  has a countable subcover.

**Theorem 3.22** Let  $f : X \rightarrow Y$  be an almost contra  $\theta$ gs-continuous surjection. Then, the following properties hold.

- (i) If  $X$  is  $\theta$ gs-compact, then  $Y$  is  $S$ -closed.
- (ii) If  $X$  is countably  $\theta$ gs-closed, then  $Y$  is countably  $S$ -closed.
- (iii) If  $X$  is  $\theta$ gs-Lindelöf, then  $Y$  is  $S$ -Lindelöf.

**Proof.**(i) Let  $\{V_\alpha : \alpha \in I\}$  be any regular closed cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is  $\theta$ gs-open cover of  $X$ . Since  $X$  is  $\theta$ gs-compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$  is finite subcover for  $Y$ . Therefore,  $Y$  is  $S$ -closed.

(ii) Let  $\{V_\alpha : \alpha \in I\}$  be any countable regular closed cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is countable  $\theta$ gs-open cover of  $X$ . Since  $X$  is countably  $\theta$ gs-compact, there exists a finite subset  $I_0$  of  $I$  such

that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$  is finite subcover for  $Y$ . Therefore,  $Y$  is countably S-closed.

(iii) Let  $\{V_\alpha : \alpha \in I\}$  be any regular closed cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is  $\theta$ gs-open cover of  $X$ . Since  $X$  is  $\theta$ gs-Lindelöf, there exists a countable subset  $I_0$  of  $I$  such that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$  is finite subcover for  $Y$ . Therefore,  $Y$  is S-Lindelöf.

**Definition 3.23** A function  $f : X \rightarrow Y$  is said to be almost  $\theta$ gs-continuous if  $f^{-1}(V)$  is  $\theta$ gs-open in  $X$  for each regular open set  $V$  of  $Y$ .

**Theorem 3.24** Let  $f : X \rightarrow Y$  be an almost contra  $\theta$ gs-continuous and almost  $\theta$ gs-continuous surjection. Then, the following properties hold.

- (i) If  $X$  is mildly  $\theta$ gs-closed, then  $Y$  is nearly compact.
- (ii) If  $X$  is mildly countably  $\theta$ gs-compact, then  $Y$  is nearly countably compact.
- (iii) If  $X$  is mildly  $\theta$ gs-Lindelöf, then  $Y$  is nearly Lindelöf.

**Proof.**(i) Let  $\{V_\alpha : \alpha \in I\}$  be any regular open cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous and almost  $\theta$ gs surjection,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is  $\theta$ gs-clopen cover of  $X$ . Since  $X$  is mildly  $\theta$ gs-compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$ , which is finite subcover for  $Y$ . Therefore,  $Y$  is nearly compact.

(ii) Let  $\{V_\alpha : \alpha \in I\}$  be any countable regular open cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous and almost  $\theta$ gs surjection,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is countable  $\theta$ gs-closed cover of  $X$ . Since  $X$  is mildly countably  $\theta$ gs-compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$  is finite subcover for  $Y$ . Therefore,  $Y$  is nearly countably compact.

(iii) Let  $\{V_\alpha : \alpha \in I\}$  be any regular open cover of  $Y$ . Since  $f$  is almost contra  $\theta$ gs-continuous and almost  $\theta$ gs surjection,  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is  $\theta$ gs-closed cover of  $X$ . Since  $X$  is mildly  $\theta$ gs-Lindelöf, there exists a countable subset  $I_0$  of  $I$  such that  $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Since  $f$  is surjective,  $Y = \cup \{V_\alpha : \alpha \in I_0\}$  is finite subcover for  $Y$ . Therefore,  $Y$  is nearly Lindelöf.

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