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# Somewhat $\nu-$Continuity

**S. Balasubramanian[1], C. Sandhya[2] and P. Aruna Swathi Vyjayanthi[3]**

[1]Department of Mathematics
Government Arts College (Autonomous), Karur- 639005(T.N.)
E-mail: mani55682@rediffmail.com
[2]Department of Mathematics
C.S.R.Sarma College, Ongole- 523001(A.P.)
E-mail: sandhya_karavadi@yahoo.co.uk
[3]Research Scholar
Dravidian University, Kuppam- 517425(A.P.)
E-mail: vyju_9285@rediffmail.com

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**Abstract**

*The object of the present paper is to study the basic properties of somewhat $\nu-$continuous functions.*

**Keywords:** *Somewhat continuous function; Somewhat b-continuous function.*

# 1 Introduction

b-open[1] sets are introduced by Andrijevic in 1996. K.R.Gentry introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of $\nu-$open sets and $\nu-$continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh introduced somewhat b-continuous functions in the year 2011. Inspired with these developements we introduce in this paper somewhat $\nu-$continuous functions, somewhat $\nu-$irresolute functions, somewhat $\nu-$open and somewhat M-$\nu-$open functions and study its basic properties and interrelation with other type of such functions available in the literature. Throughout the paper $(X, \tau)$ and $(Y, \sigma)$ (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

# 2 Preliminaries

For $A \subset (X;\tau)$, $\overline{A}$ and $A^o$ denote the closure of A and the interior of A in X, respectively. A subset A of X is said to be b-open[1] if $A \subset (\overline{A})^o \cap \overline{A^o}$.

**Definition 2.1:** A function $f$ is said to be
(i) somewhat continuous[7][resp:somewhat b-continuouscite[8]] if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$, $\exists\ V \in \tau$[resp:$V \in bO(\tau)$] $\ni V \neq \phi$ and $V \subset f^{-1}(U)$.
(ii)somewhat open[7][resp: somewhat b-open[8]] provided that if $U \in \tau$ and $U \neq \phi$, $\exists\ V \in \sigma$[resp:$V \in bO(\sigma)$] $\ni V \neq \phi$ and $V \subset f(U)$.

It is clear that every open function is somewhat open and every somewhat open is somewhat b-open. But the converses are not true.

**Definition 2.2:** A topological space $(X, \tau)$ is said to be
(i) resolvable[6]) if there exists a dense set A in $(X, \tau)$ such that $X - A$ is also dense in $(X, \tau)$. Otherwise, $(X, \tau)$ is called irresolvable.
(ii)b-resolvable[8]) if there exists a b-dense set A in $(X, \tau)$ such that $X - A$ is also b-dense in $(X, \tau)$. Otherwise, $(X, \tau)$ is called b-irresolvable.

**Definition 2.3:** If X is a set and $\tau$ and $\sigma$ are topologies on X, then $\tau$ is said to be equivalent [7] to $\sigma$ provided if $U \in \tau$ and $U \neq \phi$, then there is an open [7] set V in $X$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$, then there is an open set V in $(X, \tau)$ such that $V \neq \phi$ and $U \supset V$.

# 3 Somewhat $\nu-$Continuous Functions:

**Definition 3.1:** A function $f$ is said to be somewhat $\nu-$continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$, there exists a non-empty $\nu-$open set V in X such that $V \subset f^{-1}(U)$.

**Example 1:** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f$(a) = a, $f$(b) = c and $f$(c) = b. Then $f$ is somewhat $\nu-$continuous.

**Example 2:** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f$(a) = c, $f$(b) = a and $f$(c) = b. Then $f$ is not somewhat $\nu-$continuous.

**Note 1:** Composition of two somewhat $\nu-$continuous functions need not be somewhat $\nu-$continuous in general.

However, we have the following

**Theorem 3.1:** If $f$ is somewhat $\nu-$continuous and $g$ is continuous, then $g \circ f$ is somewhat $\nu-$continuous.

**Corollary 3.1:** If $f$ is somewhat $\nu-$continuous and $g$ is r-continuous[resp: r-irresolute], then $g \circ f$ is somewhat $\nu-$continuous.

**Definition 3.2:** $A \subset X$ is said to be $\nu-$dense in $X$ if there is no proper $\nu-$closed set C in X such that $M \subset C \subset X$.

**Theorem 3.2:** For a surjective function $f$, the following statements are equivalent:
(i) $f$ is somewhat $\nu-$continuous.
(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper $\nu-$closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a $\nu-$dense subset of X, then $f$(M) is a dense subset of Y.
**Proof:** (i)$\Rightarrow$(ii): Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y - C$ is an open set in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$ By (i), there exists a $V \in \nu O(X) \ni V \neq \phi$ and $V \subset f^{-1}(Y - C) = X - f^{-1}(C)$. This means that $X - V \supset f^{-1}(C)$ and $X - V = D$ is a proper $\nu-$closed set in X.
(ii)$\Rightarrow$(i): Let $U \in \sigma$ and $f^{-1}(U) \neq \phi$ Then $Y - U$ is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq X$. By (ii), there exists a proper $\nu-$closed set D such that $D \supset f^{-1}(Y - U)$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is $\nu-$open and $X - D \neq \phi$.
(ii)$\Rightarrow$(iii): Let M be a $\nu-$dense set in X. Suppose that $f$(M) is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $\nu-$closed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is $\nu-$dense in X.
(iii)$\Rightarrow$(ii): Suppose (ii) is not true. there exists a closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper $\nu-$closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is $\nu-$dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

**Theorem 3.3:** Let $f$ be a function and $X = A \cup B$, where $A, B \in RO(X)$. If the restriction functions $f_{|A} : (A; \tau_{|A}) \to (Y, \sigma)$ and $f_{|B} : (B;_{|B}) \to (Y, \sigma)$ are somewhat $\nu-$continuous, then $f$ is somewhat $\nu-$continuous.
**Proof:** Let $U \in \sigma$ such that $f^{-1}(U) \neq \phi$. Then $(f_{|A})^{-1}(U) \neq \phi$ or $(f_{|B})^{-1}(U) \neq$

$\phi$ or both $(f_{|A})^{-1}(U) \neq \phi$ and $(f_{|B})^{-1}(U) \neq \phi$. Suppose $(f_{|A})^{-1}(U) \neq \phi$, Since $f_{|A}$ is somewhat $\nu-$continuous, there exists a $\nu-$open set V in A such that $V \neq \phi$ and $V \subset (f_{|A})^{-1}(U) \subset f^{-1}(U)$. Since V is $\nu-$open in A and A is r-open in X, V is $\nu-$open in X. Thus $f$ is somewhat $\nu-$continuous.

The proof of other cases are similar.

**Definition 3.3:** If X is a set and $\tau$ and $\sigma$ are topologies on X, then $\tau$ is said to be $\nu-$equivalent to $\sigma$ provided if $U \in \tau$ and $U \neq \phi$, then there is a $\nu-$open set V in $X$ such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$, then there is a $\nu-$open set V in $(X, \tau)$ such that $V \neq \phi$ and $U \supset V$.

Now, consider the identity function $f$ and assume that $\tau$ and $\sigma$ are $\nu-$equivalent. Then $f$ and $f^{-1}$ are somewhat $\nu-$continuous. Conversely, if the identity function $f$ is somewhat $\nu-$continuous in both directions, then $\tau$ and $\sigma$ are $\nu-$equivalent.

**Theorem 3.4:** Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\nu-$continuous surjection and $\tau^*$ be a topology for X, which is $\nu-$equivalent to $\tau$. Then $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\nu-$continuous.
**Proof:** Let $V \in \sigma$ such that $f^{-1}(V) \neq \phi$. Since $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\nu-$continuous, there exists a nonempty $\nu-$open set U in $(X, \tau)$ such that $U \subset f^{-1}(V)$. But by hypothesis $\tau^*$ is $\nu-$equivalent to $\tau$. Therefore, there exists a $\nu-$open set $U^* \in (X; \tau^*)$ such that $U^* \subset U$. But $U \subset f^{-1}(V)$. Then $U^* \subset f^{-1}(V)$; hence $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\nu-$continuous.

**Theorem 3.5:** Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\nu-$continuous surjection and $\sigma^*$ be a topology for Y, which is equivalent to $\sigma$. Then $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\nu-$continuous.
**Proof:** Let $V^* \in \sigma^*$ such that $f^{-1}(V^*) \neq \phi$ Since $\sigma^*$ is equivalent to $\sigma$, there exists a nonempty open set V in $(Y, \sigma)$ such that $V \subset V^*$. Now $\phi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\nu-$continuous, there exists a nonempty $\nu-$open set U in $(X, \tau)$ such that $U \subset f^{-1}(V)$. Then $U \subset f^{-1}(V^*)$; hence $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\nu-$continuous.

**Definition 3.4:** A function $f$ is said to be somewhat $\nu-$open provided that if $U \in \tau$ and $U \neq \phi$, then there exists a non-empty $\nu-$open set V in Y such that $V \subset f(U)$.

**Example 3:** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = c$ and $f(c) = b$. Then $f$ is somewhat $\nu-$open and somewhat open.

**Example 4:** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, X\}$. Define a function $f\colon (X, \tau) \to (X, \sigma)$ by $f(a) = b, f(b) = c$ and $f(c) = a$. Then $f$ is not somewhat $\nu-$open and not somewhat open.

**Example 5:** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$, and $\sigma = \{\phi, \{a\}, \{a, b\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is somewhat open but not somewhat$\nu-$open.

**Theorem 3.6:** Let $f$ be an open function and $g$ be somewhat $\nu-$open. Then $g \circ f$ is somewhat $\nu-$open.

**Theorem 3.7:** For a bijective function $f$, the following are equivalent:
(i) $f$ is somewhat $\nu-$open.
(ii) If C is a closed subset of X, such that $f(C) \neq Y$, then there is a $\nu-$closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.
**Proof:** (i)⇒(ii): Let C be any closed subset of X such that $f(C) \neq Y$. Then $X - C$ is open in X and $X - C \neq \phi$. Since $f$ is somewhat $\nu-$open, there exists a $\nu-$open set $V \neq \phi$ in Y such that $V \subset f(X - C)$. Put $D = Y - V$. Clearly D is $\nu-$closed in Y and we claim $D \neq Y$. If D = Y, then $V = \phi$, which is a contradiction. Since $V \subset f(X - C), D = Y - V \supset (Y - f(X - C)) = f(C)$.
(ii)⇒(i): Let U be any nonempty open subset of X. Then $C = X - U$ is a closed set in X and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $\nu-$closed set D of Y such that $D \neq Y$ and $f(C) \subset D$. Clearly $V = Y - D$ is a $\nu-$open set and $V \neq \phi$. Also, $V = Y - D \subset Y - f(C) = Y - f(X - U) = f(U)$.

**Theorem 3.8:** The following statements are equivalent:
(i) $f$ is somewhat $\nu-$open.
(ii)If A is a $\nu-$dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.
**Proof:** (i)⇒(ii): Suppose A is a $\nu-$dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists a closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since $f$ is somewhat $\nu-$open and $X - B$ is open, there exists a nonempty $\nu-$open set C in Y such that $C \subset f(X - B)$. Therefore, $C \subset f(X - B) \subset f(f^{-1}(Y - A)) \subset Y - A$. That is, $A \subset Y - C \subset Y$. Now, $Y - C$ is a $\nu-$closed set and $A \subset Y - C \subset Y$. This implies that A is not a $\nu-$dense set in Y , which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.
(ii)⇒(i): Suppose A is a nonempty open subset of X. We want to show that $\nu(f(A))^o \neq \phi$. Suppose $\nu(f(A))^o = \phi$. Then, $\nu\overline{(f(A))} = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y - f(A)) \subset X - A$. Now, $X - A$ is closed. Therefore, $f^{-1}(Y - f(A)) \subset X - A$ gives $X = \overline{(f^{-1}(Y - f(A)))} \subset X - A$. This implies that $A = \phi$, which is contrary to $A \neq \phi$. Therefore, $\nu(f(A))^o \neq \phi$.

Hence $f$ is somewhat $\nu-$open.

**Theorem 3.9:** Let $f$ be somewhat $\nu-$open and A be any r-open subset of X. Then $f_{|A} : (A;\tau_{|A}) \to (Y,\sigma)$ is somewhat $\nu-$open.
**Proof:** Let $U \in \tau_{|A}$ such that $U \neq \phi$. Since U is r-open in A and A is open in X, U is r-open in X and since by hypothesis $f$ is somewhat $\nu-$open function, there exists a $\nu-$open set V in Y, such that $V \subset f(U)$. Thus, for any open set U of A with $U \neq \phi$, there exists a $\nu-$open set V in Y such that $V \subset f(U)$ which implies $f_{|A}$ is a somewhat $\nu-$open function.

**Theorem 3.10:** Let $f$ be a function and $X = A \cup B$, where $A, B \in RO(X)$. If the restriction functions $f_{|A}$ and $f_{|B}$ are somewhat $\nu-$open, then $f$ is somewhat $\nu-$open.
**Proof:** Let U be any open subset of X such that $U \neq \phi$. Since $X = A \cup B$, either $A \cap U \neq \phi$ or $B \cap U \neq \phi$ or both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Since U is open in X, U is open in both A and B.
Case (i): If $A \cap U \neq \phi \in RO(A)$. Since $f_{|A}$ is somewhat $\nu-$open, $\exists\, V \in \nu O(Y)$ $\ni V \subset f(U \cap A) \subset f(U)$, which implies that $f$ is somewhat $\nu-$open.
Case (ii): If $B \cap U \neq \phi \in RO(B)$. Since $f_{|B}$ is somewhat $\nu-$open, $\exists\, V \in \nu O(Y)$ $\ni V \subset f(U \cap B) \subset f(U)$, which implies that $f$ is somewhat $\nu-$open.
Case (iii): If both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Then by case (i) and (ii) f is somewhat $\nu-$open.

**Remark 1:** Two topologies $\tau$ and $\sigma$ for X are said to be $\nu-$equivalent if and only if the identity function $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\nu-$open in both directions.

**Theorem 3.11:** Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat open function. Let $\tau^*$ and $\sigma^*$ be topologies for X and Y, respectively such that $\tau^*$ is equivalent to $\tau$ and $\sigma^*$ is $\nu-$equivalent to $\sigma$. Then $f : (X;\tau^*) \to (Y;\sigma^*)$ is somewhat $\nu-$open.

# 4 Somewhat $\nu-$Irresolute Functions:

**Definition 4.1:** A function $f$ is said to be somewhat $\nu-$irresolute if for $U \in \nu O(\sigma)$ and $f^{-1}(U) \neq \phi$, there exists a non-empty $\nu-$open set V in X such that $V \subset f^{-1}(U)$.

**Example 6** Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f$(a) = c, $f$(b) = a and $f$(c) = b. Then $f$ is somewhat $\nu-$irresolute.

**Note 2:** Composition of two somewhat $\nu-$irresolute functions need not be somewhat $\nu-$irresolute.

However, we have the following

**Theorem 4.1:** If $f$ is somewhat $\nu-$irresolute and $g$ is $\nu-$irresolute, then $g \circ f$ is somewhat $\nu-$irresolute.

**Theorem 4.2:** For a surjective function $f$, the following statements are equivalent:
(i) $f$ is somewhat $\nu-$irresolute.
(ii) If C is a $\nu-$closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper $\nu-$closed subset D of X such that $f^{-1}(C) \subset D$.
(iii)If M is a $\nu-$dense subset of X, then $f$(M) is a $\nu-$dense subset of Y.
**Proof:** (i)⇒(ii): Let C be a $\nu-$closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y-C$ is an $\nu-$open set in Y such that $f^{-1}(Y-C) = X-f^{-1}(C) \neq \phi$ By (i), there exists a $\nu-$open set $V \in \nu O(X) \ni V \neq \phi$ and $V \subset f^{-1}(Y-C) = X - f^{-1}(C)$. This means that $X - V \supset f^{-1}(C)$ and $X - V = D$ is a proper $\nu-$closed set in X.
(ii)⇒(i): Let $U \in \nu O(\sigma)$ and $f^{-1}(U) \neq \phi$ Then $Y - U$ is $\nu-$closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq X$. By (ii), there exists a proper $\nu-$closed set D such that $D \supset f^{-1}(Y - U)$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is $\nu-$open and $X - D \neq \phi$.
(ii)⇒(iii): Let M be a $\nu-$dense set in X. Suppose that $f$(M) is not $\nu-$dense in Y. Then there exists a proper $\nu-$closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper $\nu-$closed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is $\nu-$dense in X.
(iii)⇒(ii): Suppose (ii) is not true, there exists a $\nu-$closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper $\nu-$closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is $\nu-$dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be $\nu-$dense in Y, which is a contradiction to the choice of C.

**Theorem 4.3:** Let $f$ be a function and $X = A \cup B$, where $A, B \in RO(X)$. If the restriction functions $f_{|A} : (A; \tau_{|A}) \to (Y, \sigma)$ and $f_{|B} : (B;_{|B}) \to (Y, \sigma)$ are somewhat $\nu-$irresolute, then $f$ is somewhat $\nu-$irresolute.
**Proof:** Let $U \in \nu O(\sigma)$ such that $f^{-1}(U) \neq \phi$. Then $(f_{|A})^{-1}(U) \neq \phi$ or $(f_{|B})^{-1}(U) \neq \phi$ or both $(f_{|A})^{-1}(U) \neq \phi$ and $(f_{|B})^{-1}(U) \neq \phi$. Suppose $(f_{|A})^{-1}(U) \neq \phi$, Since $f_{|A}$ is somewhat $\nu-$irresolute, there exists a $\nu-$open set V in A such that $V \neq \phi$ and $V \subset (f_{|A})^{-1}(U) \subset f^{-1}(U)$. Since V is $\nu-$open in A and A is r-open in X, V is $\nu-$open in X. Thus $f$ is somewhat $\nu-$irresolute.

The proof of other cases are similar.

Now, consider the identity function $f$ and assume that $\tau$ and $\sigma$ are $\nu-$equivalent. Then $f$ and $f^{-1}$ are somewhat $\nu-$irresolute. Conversely, if the identity function $f$ is somewhat $\nu-$irresolute in both directions, then $\tau$ and $\sigma$ are $\nu-$equivalent.

**Theorem 4.4:** Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\nu-$irresolute surjection and $\tau^*$ be a topology for X, which is $\nu-$equivalent to $\tau$. Then $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\nu-$irresolute.
**Proof:** Let $V \in \nu O(\sigma)$ such that $f^{-1}(V) \neq \phi$. Since $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\nu-$irresolute, there exists a nonempty $\nu-$open set U in $(X, \tau)$ such that $U \subset f^{-1}(V)$. But by hypothesis $\tau^*$ is $\nu-$equivalent to $\tau$. Therefore, there exists a $\nu-$open set $U^* \in (X; \tau^*)$ such that $U^* \subset U$. But $U \subset f^{-1}(V)$. Then $U^* \subset f^{-1}(V)$; hence $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat $\nu-$irresolute.

**Theorem 4.5:** Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat $\nu-$irresolute surjection and $\sigma^*$ be a topology for Y, which is equivalent to $\sigma$. Then $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\nu-$irresolute.
**Proof:** Let $V^* \in \nu O(\sigma^*)$ such that $f^{-1}(V^*) \neq \phi$ Since $\nu O(\sigma^*)$ is equivalent to $\nu O(\sigma)$, there exists a nonempty $\nu-$open set V in $(Y, \sigma)$ such that $V \subset V^*$. Now $\phi = f^{-1}(V) \subset f^{-1}(V^*)$. Since $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\nu-$irresolute, there exists a nonempty $\nu-$open set U in $(X, \tau)$ such that $U \subset f^{-1}(V)$. Then $U \subset f^{-1}(V^*)$; hence $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat $\nu-$irresolute.

**Definition 4.4:** A function $f$ is said to be somewhat M-$\nu-$open provided that if $U \in \nu O(\tau)$ and $U \neq \phi$, then there exists a non-empty $\nu-$open set V in Y such that $V \subset f(U)$.

**Example 7:** Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a, f(b) = c$ and $f(c) = b$. Then $f$ is somewhat M-$\nu-$open.

**Example 8:** Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f$(b) = c, $f$(a)=a ,$f$(c) = b. Then $f$ is not somewhat M-$\nu-$open

**Theorem 4.6:** Let $f$ be an r-open function and $g$ somewhat M-$\nu-$open. Then $g \circ f$ is somewhat M-$\nu-$open.

**Theorem 4.7:** For a bijective function $f$, the following are equivalent:
(i) $f$ is somewhat M-$\nu-$open.
(ii) If C is a $\nu-$closed subset of X, such that $f(C) \neq Y$, then there is a $\nu-$closed

subset D of Y such that $D \neq Y$ and $D \supset f(C)$.
**Proof:** (i)⇒(ii): Let C be any $\nu-$closed subset of X such that $f(C) \neq Y$. Then $X - C$ is $\nu-$open in X and $X - C \neq \phi$. Since $f$ is somewhat $\nu-$open, there exists a $\nu-$open set $V \neq \phi$ in Y such that $V \subset f(X-C)$. Put $D = Y-V$. Clearly D is $\nu-$closed in Y and we claim $D \neq Y$. If D = Y, then $V = \phi$, which is a contradiction. Since $V \subset f(X-C), D = Y - V \supset (Y - f(X-C)) = f(C)$.
(ii)⇒(i): Let U be any nonempty $\nu-$open subset of X. Then $C = X - U$ is a $\nu-$closed set in X and $f(X-U) = f(C) = Y - f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a $\nu-$closed set D of Y such that $D \neq Y$ and $f(C) \subset D$. Clearly $V = Y - D$ is a $\nu-$open set and $V \neq \phi$. Also, $V = Y - D \subset Y - f(C) = Y - f(X-U) = f(U)$.

**Theorem 4.8:** The following statements are equivalent:
(i) $f$ is somewhat M-$\nu-$open.
(ii)If A is a $\nu-$dense subset of Y, then $f^{-1}(A)$ is a $\nu-$dense subset of X.
**Proof:** (i)⇒(ii): Suppose A is a $\nu-$dense set in Y. If $f^{-1}(A)$ is not $\nu-$dense in X, then there exists a $\nu-$closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since $f$ is somewhat $\nu-$open and $X - B$ is $\nu-$open, there exists a nonempty $\nu-$open set C in Y such that $C \subset f(X-B)$. Therefore, $C \subset f(X-B) \subset f(f^{-1}(Y-A)) \subset Y - A$. That is, $A \subset Y - C \subset Y$. Now, $Y - C$ is a $\nu-$closed set and $A \subset Y - C \subset Y$. This implies that A is not a $\nu-$dense set in Y , which is a contradiction. Therefore, $f^{-1}(A)$ is a $\nu-$dense set in X.
(ii)⇒(i): Suppose A is a nonempty $\nu-$open subset of X. We want to show that $\nu(f(A))^o \neq \phi$. Suppose $\nu(f(A))^o = \phi$. Then, $\nu\overline{(f(A))} = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is $\nu-$dense in X. But $f^{-1}(Y - f(A)) \subset X - A$. Now, $X - A$ is $\nu-$closed. Therefore, $f^{-1}(Y - f(A)) \subset X - A$ gives $X = \overline{(f^{-1}(Y - f(A)))} \subset X - A$. This implies that $A = \phi$, which is contrary to $A \neq \phi$. Therefore, $\nu(f(A))^o \neq \phi$. Hence $f$ is somewhat M-$\nu-$open.

**Theorem 4.9:** Let $f$ be somewhat M-$\nu-$open and A be any r-open subset of X. Then $f_{|A} : (A; \tau_{|A}) \to (Y, \sigma)$ is somewhat M-$\nu-$open.
**Proof:** Let $U \in \nu O(\tau_{|A})$ such that $U \neq \phi$. Since U is r-open in A and A is open in X, U is r-open in X and since by hypothesis $f$ is somewhat M-$\nu-$open function, there exists a $\nu-$open set V in Y, such that $V \subset f(U)$. Thus, for any $\nu-$open set U of A with $U \neq \phi$, there exists a $\nu-$open set V in Y such that $V \subset f(U)$ which implies $f_{|A}$ is a somewhat M-$\nu-$open function.

**Theorem 4.10:** Let $f$ be a function and $X = A \cup B$, where $A, B \in RO(X)$. If the restriction functions $f_{|A}$ and $f_{|B}$ are somewhat M-$\nu-$open, then $f$ is somewhat M-$\nu-$open.
**Proof:** Let U be any $\nu-$open subset of X such that $U \neq \phi$. Since $X = A \cup B$, either $A \cap U \neq \phi$ or $B \cap U \neq \phi$ or both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Since U is

$\nu-$open in X, U is $\nu-$open in both A and B.
Case (i): If $A \cap U \neq \phi \in RO(A)$. Since $f_{|A}$ is somewhat M-$\nu-$open, $\exists$ $V \in \nu O(Y) \ni V \subset f(U \cap A) \subset f(U)$, which implies that $f$ is somewhat M-$\nu-$open.
Case (ii): If $B \cap U \neq \phi \in RO(B)$. Since $f_{|B}$ is somewhat M-$\nu-$open, $\exists$ $V \in \nu O(Y) \ni V \subset f(U \cap B) \subset f(U)$, which implies that $f$ is somewhat M-$\nu-$open.
Case (iii): If both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Then by case (i) and (ii) f is somewhat M-$\nu-$open.

**Remark 2:** Two topologies $\tau$ and $\sigma$ for X are said to be $\nu-$equivalent if and only if the identity function $f\colon (X, \tau) \to (Y, \sigma)$ is somewhat M-$\nu-$open in both directions.

**Theorem 4.11:** Let $f\colon (X, \tau) \to (Y, \sigma)$ be a somewhat open function. Let $\tau^*$ and $\sigma^*$ be topologies for X and Y, respectively such that $\tau^*$ is equivalent to $\tau$ and $\sigma^*$ is $\nu-$equivalent to $\sigma$. Then $f : (X; \tau^*) \to (Y; \sigma^*)$ is somewhat $\nu-$open.

# 5 $\nu$- Resolvable Spaces and $\nu$- Irresolvable Spaces:

**Definition 5.1:** $(X, \tau)$ is said to be $\nu-$resolvable if A and $X - A$ are $\nu-$dense in $(X, \tau)$. Otherwise, $(X, \tau)$ is called $\nu-$irresolvable.

**Example 9:** Let $X = \{a, b, c\}$ and $\tau$ an indiscrete topology on X. Then $(X, \tau)$ is resolvable and $\nu-$resolvable.

**Example 10:** Let $X = \{a, b, c\}$ and$\tau = \{\phi, \{a\}, \{a, b\}, X\}$ on X. Then $(X, \tau)$ is not resolvable but X is $\nu-$resolvable.

**Example 11:** Let $X = \{a, b, c\}$ and$\tau = \{\phi, \{a\}, \{b, c\}, X\}$ on X. Then $(X, \tau)$ is not resolvable and also not $\nu-$resolvable.

**Theorem 5.1:** The following statements are equivalent:
(i) $X$ is $\nu-$resolvable;
(ii)$X$ has a pair of $\nu-$dense sets A and B such that $A \subset B$.
**Proof:** (i)⇒(ii): Suppose that $(X, \tau)$ is $\nu-$resolvable. There exists a $\nu-$dense set A such that $X - A$ is $\nu-$dense. Set B = $X - A$, then we have A = $X - B$.
(ii)⇒(i): Suppose that the statement (ii) holds. Let $(X, \tau)$ be $\nu-$irresolvable. Then $X - B$ is not $\nu-$dense and $\nu\overline{(A)} \subset \nu\overline{(X - B)} \neq X$. Hence A is not $\nu-$dense. This contradicts the assumption.

**Theorem 5.2:** The following statements are equivalent:
(i) $(X, \tau)$ is $\nu-$irresolvable;
(ii) For any $\nu-$dense set A in X, $\nu(A)^o \neq \phi$.
**Proof:** (i)$\Rightarrow$(ii): Let A be any $\nu-$dense set of X. Then we have $\nu\overline{(X-A)} \neq X$; hence $\nu(A)^o \neq \phi$.
(ii)$\Rightarrow$(i): If $X$ is a $\nu-$resolvable space. Then there exists a $\nu-$dense set A in $X$ such that $A^c$ is also $\nu-$dense in X. It follows that $\nu(A)^o = \phi$, which is a contradiction; hence $X$ is $\nu-$irresolvable.

**Definition 5.2:** $(X, \tau)$ is said to be strongly $\nu-$irresolvable if for a nonempty set A in X $\nu(A)^o = \phi$ implies $\nu(\nu\overline{A})^o = \phi$.

**Theorem 5.3:** If $(X, \tau)$ is a strongly $\nu-$irresolvable space and $\nu\overline{A} = X$ for a nonempty subset A of X, then $\nu\overline{(\nu(A)^o)} = X$.

**Theorem 5.4:** If $(X, \tau)$ is a strongly $\nu-$irresolvable space and $\nu(\nu\overline{(A)})^o \neq \phi$ for any nonempty subset A in X, then $\nu(A)^o \neq \phi$.

**Theorem 5.5:** Every strongly $\nu-$irresolvable space is $\nu-$irresolvable.
**Proof:** This follows from Theorems 3.2 and 3.3.

However, the converse of above theorem is not true in general as it can be seen from the following example.

**Example 12:** Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then $(X, \tau)$ is $\nu-$irresolvable but not strongly $\nu-$irresolvable.

**Theorem 5.6:** If $f$ is somewhat $\nu-$open and $\nu(A)^o = \phi$ for a nonempty set A in Y, then $(f^{-1}(A))^o = \phi$.
**Proof:** Let A be a nonempty set in Y such that $\nu(A)^o = \phi$. Then $\nu\overline{(Y-A)} = Y$. Since $f$ is somewhat $\nu-$open and $Y - A$ is $\nu-$dense in Y, by theorem 3.5, $f^{-1}(Y - A)$ is dense in X. Then, $\overline{(X - f^{-1}(A))} = X$; hence $(f^{-1}(A))^o = \phi$.

**Theorem 5.7:** Let $f$ be a somewhat $\nu-$open function. If X is irresolvable, then Y is $\nu-$irresolvable.
**Proof:** Let A be a nonempty set in Y such that $\nu\overline{(A)} = Y$. We show that $\nu(A)^o \neq \phi$. Suppose not, then $\nu\overline{(Y-A)} = Y$. Since $f$ is somewhat $\nu-$open and $Y - A$ is $\nu-$dense in Y, we have by theorem 3.5 $f^{-1}(Y - A)$ is dense in X. Then $(f^{-1}(A))^o = \phi$. Now, since A is $\nu-$dense in Y, $f^{-1}(A)$ is dense in X. Therefore, for the dense set $f^{-1}(A)$, we have $(f^{-1}(A))^o = \phi$, which is a contradiction to Theorem 3.2. Hence we must have $\nu(A)^o \neq \phi$ for all $\nu-$dense sets A in Y. Hence by Theorem 3.2, Y is $\nu-$irresolvable.

# 6 Further Properties:

**Defintion 6.1:** A function $f$ is said to be somewhat semi-continuous[resp: somewhat pre-continuous; somewhat $\beta$-continuous; somewhat r$\alpha$-continuous] if for each $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists $V \in SO(Y)$[resp: $V \in PO(Y); V \in \beta O(Y); V \in r\alpha O(Y)$] $\ni V \neq \phi$ and $V \subset f^{-1}(U)$.

**Theorem 6.1:** The following are equivalent:
(i) $f$ is swt.$\nu$.c.
(ii) $f^{-1}(V)$ is $\nu$-open for every clopen set V in Y.
(iii)$f^{-1}(V)$ is $\nu$-closed for every clopen set V in Y.
(iv) $f(\overline{(A)}) \subseteq \nu\overline{(f(A))}$.

**Corollary 6.1:** The following are equivalent.
(i) $f$ is swt.$\nu$.c.
(ii)For each x in X and each $V \in \sigma(Y, f(x))$ $\exists U \in \nu O(X, x)$ such that $f(U) \subset V$.

**Theorem 6.2:** Let $\Sigma = \{U_i : i \in I\}$ be any cover of X by regular open sets in X, then $f$ is swt.$\nu$.c. iff $f_{/U_i}$: is swt.$\nu$.c., for each $i \in I$.

**Theorem 6.3:** If $f$ is $\nu$-irresolute[resp: $\nu$-continuous] and $g$ is swt.$\nu$.c.,[resp: swt.c.,] then $g \circ f$ is swt.$\nu$.c.

**Theorem 6.4:** If $f$ is $\nu$-irresolute, $\nu$-open and $\nu O(X) = \tau$ and $g$ be any function, then $g \circ f$ is swt.$\nu$.c iff $g$ is swt.$\nu$.c.

**Corollary 6.2:** If $f$ is $\nu$-irresolute, $\nu$-open and bijective, $g$ is a function. Then $g$ is swt.$\nu$.c. iff $g \circ f$ is swt.$\nu$.c.

**Theorem 6.5:** If $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for all $x \in X$ be the graph function of $f$. Then $g$ is swt.$\nu$.c iff $f$ is swt.$\nu$.c.
**Proof:** Let $V \in \sigma(Y)$, then $X \times V$ is open in $X \times Y$. Since $g : X \to X \times Y$ swt.$\nu$.c., $f^{-1}(V) = f^{-1}(X \times V) \in \nu O(X)$. Thus $f$ is swt.$\nu$.c.
Conversely, let $x \in X$ and $F \in \sigma(X \times Y, g(x))$. Then $F(\{x\} \times Y) \in \sigma(x \times Y, g(x))$. Also $x \times Y$ is homeomorphic to $Y$. Hence $\{y \in Y : (x, y) \in F\} \in \sigma(Y)$. Since $f$ is swt.$\nu$.c. $\{f^{-1}(y) : (x, y) \in F\} \in \nu O(X)$. Further $x \in \{f^{-1}(y) : (x, y) \in F\} = g^{-1}(F)$. Hence $g^{-1}(F)$ is $\nu$-open. Thus $g$ is swt.$\nu$.c.

**Theorem 6.6:** (i) If $f : X \to \Pi Y_\lambda$ is swt.$\nu$.c, then $P_\lambda \circ f : X \to Y_\lambda$ is swt.$\nu$.c for each $\lambda \in \Lambda$, where $P_\lambda$:$\Pi Y_\lambda$ onto $Y_\lambda$.
(ii) $f$: $\Pi X_\lambda \to \Pi Y_\lambda$ is swt.$\nu$.c, iff $f_\lambda : X_\lambda \to Y_\lambda$ is swt.$\nu$.c for each $\lambda \in \Lambda$.

**Remark 1:** Algebraic sum and product of swt.$\nu$.c functions is not in general swt.$\nu$.c.
The pointwise limit of a sequence of swt.$\nu$.c functions is not in general swt.$\nu$.c.

However we can prove the following:

**Theorem 6.7:** The uniform limit of a sequence of swt.$\nu$.c functions is swt.$\nu$.c.

**Note 1** Pasting Lemma is not true for swt.$\nu$.c functions. However we have the following weaker versions.

**Theorem 6.8: Pasting Lemma** Let X and Y be topological spaces such that $X = A \cup B$ and let $f_{/A}$ and $g_{/B}$ are swt.$\nu$.c[resp: swt.r.c] maps such that $f(x) = g(x)$ for all $x \in A \cap B$. If $A; B \in RO(X)$ and $\nu$O(X)[resp: RO(X)] is closed under finite unions, then the combination $\alpha : X \to Y$ is swt.$\nu$.c.
**Proof:** Let $F \in \sigma(Y)$, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$ where $f^{-1}(F) \in \nu O(A)$ and $g^{-1}(F) \in \nu O(B) \Rightarrow f^{-1}(F) \in \nu O(X)$ and $g^{-1}(F) \in \nu O(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) \in \nu O(X) \Rightarrow \alpha^{-1}(F) \in \nu O(X)$. Hence $\alpha$ is swt.$\nu$.c.

**Theorem 6.9:** (i) If $f$ is swt.s.c, then $f$ is swt.$\nu$.c.
(ii) If $f$ is swt.r.c, then $f$ is swt.$\nu$.c.
(iii)If $f$ is swt.r$\alpha$.c, then $f$ is swt.$\nu$.c.

# 7 Covering and Separation Properties:

**Theorem 7.1:** If $f$ is swt.$\nu$.c. surjection and X is $\nu$-compact, then Y is compact.
**Proof:** Let $\{G_i : i \in I\}$ be any open cover for Y and $f$ is swt.$\nu$.c., $\exists U_i \in \nu O(X) \ni U_i \subset f^{-1}(G_i)$. Thus $\{U_i\}$ forms a $\nu$-open cover for X such that $\{U_i\} \subset \{f^{-1}(G_i)\}$ and hence have a finite subcover, since X is $\nu$-compact. Since $f$ is surjection, $Y = f(X) = \cup_{i=1}^{n} f(U_i) \subset \cup_{i=1}^{n} G_i$. Therefore Y is compact.

**Theorem 7.2:** If $f$ is swt.$\nu$.c., surjection and X is $\nu$-compact[$\nu$-lindeloff] then Y is mildly compact[mildly lindeloff].
**Proof:** Let $\{U_i : i \in I\}$ be clopen cover for Y. For each $x \in X, \exists U_x \in I \ni f(x) \in U_x$ and $V_x \in \nu O(X, x) \ni f(V_x) \subset U_x$. Since $\{V_i : i \in I\}$ is a cover of X by $\nu$-open sets of X, $\exists$ a finite subset $I_0$ of $I \ni X = \cup\{V_x : x \in I_0\}$. Therefore $Y = \cup\{f(V_x) : x \in I_0\} \subset \cup\{U_x : x \in I_0\}$. Hence Y is mildly compact.

**Theorem 7.3:** If $f$ is swt.$\nu$.c., surjection and $X$ is s-closed then $Y$ is mildly compact[mildly lindeloff].

**Proof:**Let $\{V_i : V_i \in \sigma(Y); i \in I\}$ be a cover of $Y$, then $\{f^{-1}(V_i) : i \in I\}$ is $\nu$-open cover of X[by Thm 3.1] and so there is finite subset $I_0$ of I, such that $\{f^{-1}(V_i) : i \in I_0\}$ covers $X$. Therefore $\{V_i : i \in I_0\}$ covers $Y$ since $f$ is surjection. Hence $Y$ is mildly compact.

**Corollary 7.1:** (i) If $f$ is swt.$\nu$.c[resp: swt.r.c] surjection and X is $\nu$-lindeloff then Y is mildly lindeloff.
(ii) If $f$ is swt.$\nu$.c.[resp: swt.$\nu$.c.; swt.r.c] surjection and X is locally $\nu$-compact[resp:$\nu$-Lindeloff; locally $\nu$-lindeloff], then Y is locally compact[resp: Lindeloff; locally lindeloff].
(iii)If $f$ is swt.$\nu$.c., surjection and X is semi-compact[semi-lindeloff; $\beta-$compact; $\beta-$lindeloff] then Y is mildly compact[mildly lindeloff].
(iv) If $f$ is swt.r.c., surjection and X is $\nu$-compact[s-closed], then Y is compact[mildly compact; mildly lindeloff].

**Theorem 7.4:** If $f$ is swt.$\nu$.c.,[resp: swt.r.c.] surjection and $X$ is $\nu$-connected, then $Y$ is connected.

**Corollary 7.2:** The inverse image of a disconnected space under a swt.$\nu$.c.,[resp: swt.r.c.] surjection is $\nu$-disconnected.

**Theorem 7.5:** If $f$ is swt.$\nu$.c.swt.$\nu$.c.[resp: swt.r.c.], injection and $Y$ is $UT_i$, then X is $\nu - T_i$; i = 0, 1, 2.
**Proof:** Let $x_1 \neq x_2 \in X$. Then $f(x_1) \neq f(x_2) \in Y$ since $f$ is injective. For $Y$ is $UT_2$ $\exists V_j \in CO(Y) \ni f(x_j) \in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 7.1, $\exists U_j \in \nu O(X, x_j) \ni x_j \in U_j \subset f^{-1}(V_j)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is $\nu - T_2$.

**Theorem 7.6:** If $f$ is swt.$\nu$.c.[resp: swt.r.c.], injection; closed and Y is $UT_i$, then X is $\nu - T_i$ ; i = 3, 4.
**Proof:**(i) Let x in X and F be a closed subset of X not containing x, then $f$(x) and $f$(F) be a closed subset of Y not containing $f$(x), since $f$ is closed and injection. Since Y is ultraregular, $f$(x) and $f$(F) are separated by disjoint clopen sets U and V respectively. Hence $\exists A, B \in \nu O(X) \ni x \in A \subset f^{-1}(U); F \subset B \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is $\nu - T_3$.
(ii) Let $F_j$ and $f(F_j)$ are disjoint closed subsets of X and Y respectively for j = 1,2, since $f$ is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint clopen sets $V_j$ respectively for j = 1,2. Hence $\exists U_j \in \nu O(X) \ni F_j \subset U_j \subset f^{-1}(V_j)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is $\nu - T_4$.

**Theorem 7.7:** If $f$ is swt.$\nu$.c.[resp: swt.r.c.], injection and
(i) Y is $UC_i$[resp: $UD_i$] then X is $\nu C_i$[resp: $\nu D_i$] i = 0, 1, 2.

(ii) Y is $UR_i$, then X is $\nu - R_i$ i = 0, 1.
(iii)Y is $UT_2$, then the graph $G(f)$ of $f$ is $\nu$-closed in $X \times Y$.
(iv) Y is $UT_2$, then $A = \{(x_1, x_2)|f(x_1) = f(x_2)\}$ is $\nu$-closed in $X \times X$.

**Theorem 7.8:** If $f$ is swt.r.c.[resp: swt.c.]; $g$ is swt.$\nu$.c[resp: swt.r.c]; and Y is $UT_2$, then $E = \{x \in X : f(x) = g(x)\}$ is $\nu$-closed in X.

# References


[1] D. Andrijevic, On b-open sets, *Math. Vesnik*, 48(1996), 59-64.

[2] A.A. El-Atik, A study of some types of mappings on topological spaces, *M.Sc. Thesis*, (1997), Tanta University, Egypt.

[3] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, On $\nu - T_i, \nu - R_i$ and $\nu - C_i$ axioms, *Scientia Magna*, 4(4) (2008), 86-103.

[4] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, On $\nu-$compact spaces, *Scientia Magna*, 5(1) (2009), 78-82.

[5] S. Balasubramanian and P.A.S. Vyjayanthi, On $\nu D-$sets and separation axioms, *Int. Journal of Math. Analysis*, 4(19) (2010), 909-919.

[6] M. Ganster, Preopen sets and resolvable spaces, *Kyungpook Math. J.*, 27(2) (1987), 135-143.

[7] K.R. Gentry and H.B. Hoyle, Somewhat continuous functions, *Czechslovak Math. J.*, 21(96) (1971), 5-12.

[8] T. Noiri and N. Rajesh, Somewhat b-continuous functions, *J. Adv. Res. in Pure Math.*, 3(3) (2011), 1-7.