Gen. Math. Notes, Vol. 32, No. 1, January 2016, pp.12-20
ISSN 2219-7184; Copyright ©ICSRS Publication, 2016
www.i-csrs.org
Available free online at http://www.geman.in

Boundary Domination in Total Graphs

Mohammed Alatif[1], Puttaswamy[2] and S.R. Nayaka[3]

[1,2,3]Department of Mathematics
P.E.S. College of Engineering, Mandya-571401
Karnataka State, India
[1]E-mail: aabuyasyn@gmail.com
[2]E-mail: prof.puttaswamy@gmail.com
[3]E-mail: nayaka.abhi11@gmail.com

(Received: 18-10-15 / Accepted: 23-12-15)

Abstract

Let $G = (V, E)$ be a connected graph, A subset S of $V(G)$ is called a boundary dominating set if every vertex of $V(G) - S$ is vertex boundary dominated by some vertex of S. The minimum taken over all boundary dominating sets of G is called the boundary domination number of G and is denoted by $\gamma_b(G)$. In this paper we introduce the boundary domination in Total graph.

Keywords: *Boundary dominating set, Boundary domination number, Total graph.*

1 Introduction

By a graph $G = (V, E)$ we mean a finite and undirected graph with no loops and multiple edges. As usual $n = |V|$ and $m = |E|$ denote the number of vertices and edges of a graph G, respectively. Domination in graphs has become an important area of research in graph theory, as evidenced by the many results contained in the two books by Haynes, Hedetniemi and Slater(1998)[4]. In general, we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X. $N(v)$ and $N[v]$ denote the open and closed neighbourhood of a vertex v, respectively. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

One of the recently parameter of Domination is the boundary domination which has been introduced by KM. Kathiresan et[5].

The total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$ if and only if their corresponding elements are either adjacent or incident in G.

Let G be a simple graph $G = (V, E)$ with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$. For $i \neq j$, a vertex v_i is a boundary vertex of v_j if $d(v_j; v_t) \leq d(v_j; v_i)$ for all $v_t \in N(v_i)$ (see [2, 3]).

A vertex v is called a boundary neighbor of u if v is a nearest boundary of u. If $u \in V$, then the boundary neighbourhood of u denoted by $N_b(u)$ is defined as $N_b(u) = \{v \in V : d(u, w) \leq d(u, v) \text{ for all } w \in N(u)\}$. The cardinality of $N_b(u)$ is denoted by $deg_b(u)$ in G. The maximum and minimum boundary degree of a vertex in G are denoted respectively by $\Delta_b(G)$ and $\delta_b(G)$. That is $\Delta_b(G) = \max_{u \in V} |N_b(u)|$, $\delta_b(G) = \min_{u \in V} |N_b(u)|$. A vertex u boundary dominate a vertex v if v is a boundary neighbor of u.

A subset B of $V(G)$ is called a boundary dominating set if every vertex of $V - B$ is boundary dominated by some vertex of B. The minimum taken over all boundary dominating sets of a graph G is called the boundary domination number of G and is denoted by $\gamma_b(G)$ [5]. We need the following theorems.

Theorem 1.1. [8] Let G be a graph with $\Delta < n - 1$, then $\lceil \frac{n}{\Delta+1} \rceil \leq \gamma(G) \leq n - \Delta$.

Theorem 1.2. [7]

a. For any cycle C_n, with $n \geq 4$, $\gamma(T(C_n)) = \lfloor \frac{n}{2} \rfloor$.

b. For any star graph $K_{1,n}$, $\gamma(T(K_{1,n})) = 1$.

The definition of boundary domination motivated us to introduce the boundary domination in Total graph.

2 Main Results

Boundary Domination number of Total graphs of P_n, C_n, K_n and $K_{1,n}$ are obtained in this section.
Let G = (V, E) be a graph with vertices $v_1, v_2, ..., v_n$. Let $e_i = v_i v_{i+1}$ for $1 \leq i \leq n - 1$ be the edges of G. The total graph of G is $T(G)$. Here $V(T(G)) = \{v_i, e_i : 1 \leq i \leq n\}$ and $E(T(G)) = \{e_i e_{i+1}, v_i e_i, e_i v_{i+1} : 1 \leq i \leq n\}$.

Definition 2.1. *A vertex $v \in V(T(G))$ is said to be a boundary of u if $d(v, w) \leq d(v, u)$ for all $w \in N_b(v)$. A vertex v is a boundary neighbor of* *a vertex u if v is a nearest boundary of u. Two vertices v and u are boundary adjacent if v adjacent to u and there exist another vertex w adjacent to both v and u.*

The boundary neighbourhood of v, denoted by $N_b(v)$ is defined as $N_b(v) = \{u \in V : d(v,w) \leq d(v,u) \text{ for all } w \in N(u)\}$. The cardinality of $N_b(v)$ is denoted by $deg_b(v)$ in $T(G)$. The maximum and minimum boundary degree of a vertex in $T(G)$ are denoted respectively by $\Delta_b(G)$ and $\delta_b(G)$. That is $\Delta_b(G) = \max_{v\in V(T(G))} |N_b(v)|$ and $\delta_b(G) = \min_{v\in V(T(G))} |N_b(v)|$.
A subset S of $V(T(G))$ is called a boundary dominating set if every vertex of $V - S$ is boundary dominated by some vertex of S. The minimum taken over all boundary dominating sets of a graph $T(G)$ is called the boundary domination number of $T(G)$ and is denoted by $\gamma_b(T(G))$. For a real number x; $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x,and we denotes by l to $|V(T(G))|$ in this paper.

Example 2.2. *In Figure 1, we can find the minimum boundary dominating set is $B = \{v_2, v_3\}$ and $\gamma_b(G) = 2$, $V(T(G)) = \{v_1, v_2, v_3, v_4, v_5, v_6, e_1, e_2, e_3, e_4, e_5, e_6\}$ and the minimum boundary dominating sets of $T(G)$ are $\{v_2, e_2, v_3\}, \{v_2, e_6, v_6\}$ therefore $\gamma_b(T(G)) = 3$.*

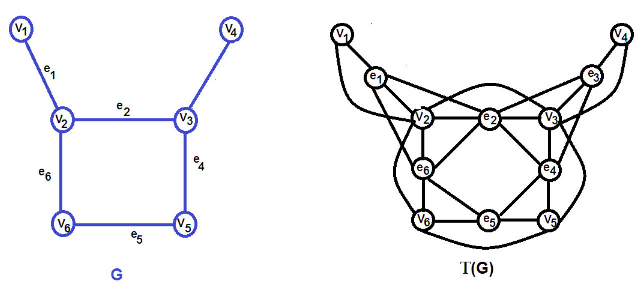

Figure 1: G and T(G)

2.1 Boundary Domination in Total Graph of P_n and C_n

By the definition of Total graph, We have $V[T(G)] = V(G) \cup E(G)$, and

1. $|V[T(P_n)]| = 2n - 1, \Delta_b(T(P_n)) = 4$.
2. $|V[T(C_n)]| = 2n, \Delta_b(T(C_n)) = 4$.

Consider the following example: In Figure 2,

in $T(P_7)$, $\{v_2, e_4, v_7\}$ is a minimum dominating set, $\gamma[T(P_7)] = 3$, and $\{v_1, e_3, v_5, v_6\}$ is a minimum boundary dominating set. Therefore $\gamma_b(T(P_7)) = 4$.

In $T(P_8)$, $\{v_2, e_4, v_7\}$ is a minimum dominating set, $\gamma[T(P_8)] = 3$, and $\{v_1, e_3, v_5, v_6\}$ is a minimum boundary dominating set. Therefore $\gamma_b(T(P_8)) = 4$.

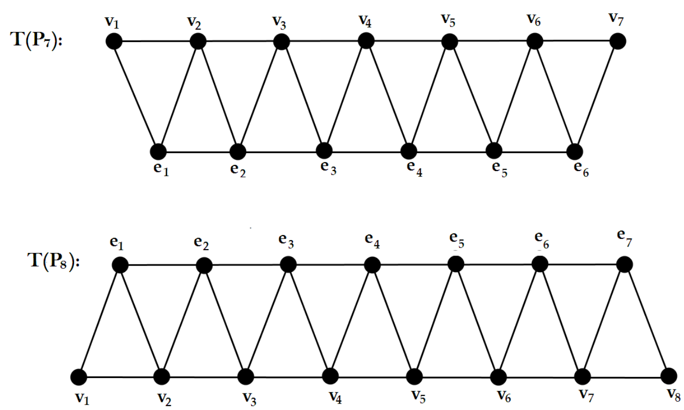

Figure 2: $T(P_7)$ and $T(P_8)$

Theorem 2.3. *For any path P_n, $\gamma(T(P_n)) = \lceil \frac{2n-1}{5} \rceil$.*

Proof. Let $P_n = \{v_1, v_2, ..., v_n\}$ and $e_i = v_i v_{i+1}; 1 \leq i \leq n-1$. Let $u_i \in V(T(P_n))$ be the vertex corresponding to e_i. Let $D_1 = \{v_i \in V(T(Pn)) : i \equiv 2(mod5)\}$ and $D_2 = \{u_i \in V(T(P_n)) : i \equiv 4(mod5)\}$. If $n \equiv 0$ or $3(mod5)$ then $D = D_1 \cup D_2$ is a dominating set of $T(P_n)$ and $|D| = \lceil \frac{2n-1}{5} \rceil$. If $n \equiv 1$ or 2 or $4(mod5)$, then $D = D_1 \cup D_2 \cup \{v_n\}$ is a dominating set of $T(P_n)$ and $|D| = \lceil \frac{2n-1}{5} \rceil$. Hence $\gamma(T(P_n)) \leq \lceil \frac{2n-1}{5} \rceil$. Further since $\gamma(G) \geq \lceil \frac{n}{\Delta+1} \rceil$, it follows that $\gamma(T(P_n)) \geq \lceil \frac{2n-1}{\Delta_t+1} \rceil \geq \lceil \frac{2n-1}{5} \rceil$. Thus $\gamma(T(P_n)) = \lceil \frac{2n-1}{5} \rceil$. $\square$

Theorem 2.4. *For any path P_n,*

$$\gamma_b(T(P_n)) = \begin{cases} \lceil \frac{2n-1}{5} \rceil & \textit{if } n \equiv 1 \ (mod5), \\ \\ \lceil \frac{2n-1}{5} \rceil + 1 & \textit{if } n \equiv 0, 2, 3, 4 \ (mod5) \ . \end{cases}$$

.

Proof. It is easy to observe that $\gamma_b(T(P_4)) = \gamma_b(T(P_5)) = 3$, Let P_n be a path on n vertices where $n > 5$, $V(P_n) = \{v_1, v_2, ..., v_n\}$ and $e_i = v_i v_{i+1}; 1 \leq i \leq n-1$. Let $u_i \in V(T(P_n))$ be the vertex corresponding to e_i. Let $S_1 = \{v_i \in$

$V(T(Pn)) : i \equiv 1(mod5)\}$ and $S_2 = \{u_i \in V(T(P_n)) : i \equiv 3(mod5)\}$. If $n \equiv 1(mod5)$ then $S = S_1 \cup S_2$ is a boundary dominating set of $T(P_n)$. If $n \equiv 0$ or 2 or 3 or $4(mod5)$, then $S = S_1 \cup S_2 \cup \{e_{n-3}\}$ or $S = S_1 \cup S_2 \cup \{v_{n-2}\}$ or $S = S_1 \cup S_2 \cup \{v_{n-3}\}$ or $S = S_1 \cup S_2 \cup \{v_n\}$ respectively is a boundary dominating set of $T(P_n)$ and

$$|S| = \begin{cases} \frac{2n-2}{5} + 1 & \text{if } n \equiv 1 \ (mod5), \\ \frac{2n-5}{5} + 2 & \text{if } n \equiv 0 \text{ or } 3(mod5), \\ \frac{2n-3}{5} + 2 & \text{if } n \equiv 2 \text{ or } 4(mod5). \end{cases}$$

$$= \begin{cases} \lceil \frac{2n-1}{5} \rceil & \text{if } n \equiv 1 \ (mod5), \\ \\ \lceil \frac{2n-1}{5} \rceil + 1 & \text{if } n \equiv 0, 2, 3, 4 \ (mod5) \ . \end{cases}$$

Hence

$$\gamma_b(T(P_n)) = \begin{cases} \lceil \frac{2n-1}{5} \rceil & \text{if } n \equiv 1 \ (mod5), \\ \\ \lceil \frac{2n-1}{5} \rceil + 1 & \text{if } n \equiv 0, 2, 3, 4 \ (mod5) \ . \end{cases}$$

□

Theorem 2.5. *For any a cycle C_n,*

$$\gamma_b(T(C_n)) = \begin{cases} \lceil \frac{2n}{5} \rceil + 1 & \text{if } n \equiv 2 \text{ or } 4 \ (mod5), \\ \\ \lceil \frac{2n}{5} \rceil & \text{if } n \equiv 0 \text{ or } 1 \text{ or } 3 \ (mod5). \end{cases}$$

Proof. It is easy to observe that $\gamma_b(T(C_4)) = 3$, Let C_n be a cycle on n vertices where $n \geq 5$, $V(C_n) = \{v_1, v_2, ..., v_n\}$ and $e_i = v_i v_{i+1}; 1 \leq i \leq n-1$ and $e_n = v_1 v_n$. Let $u_i \in V(T(C_n))$ be the vertex corresponding to e_i in $T(C_n)$. Let $S_1 = \{v_i \in V(T(C_n)) : i \equiv 1(mod5)\}$ and $S_2 = \{u_i \in V(T(C_n)) : i \equiv 3(mod5)\}$. If $n \equiv 0$ or $1(mod5)$ then $S = S_1 \cup S_2$ is a boundary dominating set of $T(C_n)$. If $n \equiv 2(mod5)$, then $S = S_1 \cup S_2 \cup \{v_n\}$ is a boundary dominating set of $T(C_n)$. If $n \equiv 3$ or $4(mod5)$, then $S = S_1 \cup S_2 \cup \{e_{n-2}\}$ is a boundary dominating set of $T(C_n)$ and

$$|S| = \begin{cases} \frac{2n-5}{5} + 1 & \text{if } n \equiv 0 \ (mod5), \\ \frac{2n-2}{5} + 1 & \text{if } n \equiv 1 \ (mod5), \\ \frac{2n-4}{5} + 2 & \text{if } n \equiv 2 \ (mod5), \\ \frac{2n-6}{5} + 2 & \text{if } n \equiv 3 \ (mod5), \\ \frac{2n-3}{5} + 2 & \text{if } n \equiv 4 \ (mod5). \end{cases}$$

$$= \begin{cases} \lceil \frac{2n}{5} \rceil + 1 & \text{if } n \equiv 2 \text{ or } 4 \ (mod5), \\ \\ \lceil \frac{2n}{5} \rceil & \text{if } n \equiv 0 \text{ or } 1 \text{ or } 3 \ (mod5). \end{cases}$$

Hence

$$\gamma_b(T(C_n)) = \begin{cases} \lceil \frac{2n}{5} \rceil + 1 & \text{if } n \equiv 2 \text{ or } 4 \ (mod 5), \\ \lceil \frac{2n}{5} \rceil & \text{if } n \equiv 0 \text{ or } 1 \text{ or } 3 \ (mod 5). \end{cases}$$

□

2.2 Boundary Domination in Total Graph of $K_{1,n}$ and K_n

By the definition of Total graph, We have $V[T(G)] = V(G) \cup E(G)$, and

1. $|V[T(K_{1,n})]| = 2n + 1, \Delta_b(T(K_{1,n})) = 2(n-1)$.
2. $|V[T(K_n)]| = \frac{n(n+1)}{2}, \Delta_b(T(K_n)) = \frac{(n-1)(n-2)}{2}$.

Consider the following examples:

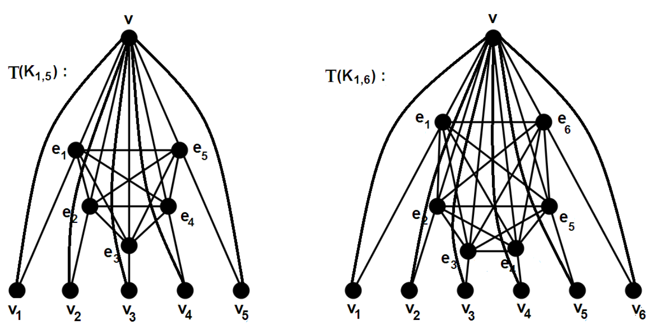

Figure 3: $T(K_{1,5})$ and $T(K_{1,6})$

In $T(K_{1,5})$, from Theorem 1.1.c, we have $\gamma[T(K_{1,5})] = 1$ and $\{v, v_1, v_5\}$ is a minimum boundary dominating set. Therefore $\gamma_b(T(K_{1,5})) = 3$.

In $T(K_{1,6})$, from Theorem 1.1.c, we have $\gamma[T(K_{1,6})] = 1$ and $\{v, v_1, v_6\}$ is a minimum boundary dominating set. Therefore and $\gamma_b(T(K_{1,6})) = 3$.

Theorem 2.6. *If $G \cong T(K_n)$ or $T(K_{1,n})$, $n \geq 3$, then $\gamma_b(G) = 3$.*

Proof. Let $G \cong T(K_n)$ and let $u_i \in V(T(K_n))$ be the vertex corresponding to e_i. Suppose $v_i \in V(G)$, since $d(v_i, v_j) \leq d(v_i, v_t)$ for all $j \neq t$, then there exists $\frac{(n-1)(n-2)}{2} \in N_b(v_i)$ and $2(n-1) \in N(v_i)$. If a vertex $u_i \in V(G)$ is adjacent to v_i, then there exists $\frac{(n-1)(n-2)}{2} \in N_b(u_i)$ and $2(n-1) \in N(u_i)$. Now we take a vertex $v_j \in S$ such that $i \neq j$, then there exists $\frac{(n-1)(n-2)}{2} \in N_b(v_j)$ and $2(n-1) \in N(v_j)$, also there exists $\frac{(n-2)(n-3)}{2}$ boundary neighbor vertices are common between v_i, u_i and v_j, and $|N_{tb}(v_i) \cup N_{tb}(v_j) \cup N_{tb}(u_i) \cup \{v_i, u_i, v_j\}|$

$$= \frac{3(n-1)(n-2)}{2} - \frac{2(n-2)(n-3)}{2} + 3 = \frac{n(n+1)}{2} = |V(G)|.$$

□

Therefore the boundary dominating set is $S = \{v_i, u_i, v_j\}$, which contained any path P_3 in G. Similarly we can prove that if $G \cong T(K_{1,n})$. Hence $\gamma_b(G) = 3$.

Proposition 2.7. *Let u be a vertex of a total graph of G. Then $V(T(G)) - N_b(u)$ is a boundary dominating set for $T(G)$.*

Theorem 2.8. *If G is a connected graph of order $n \geq 3$ and $T(G)$ is a total graph of G of order $l \geq 5$, then $\gamma_b(T(G)) \leq l - \Delta_b(T(G))$.*

Proof. Let u be a vertex of a total graph of G. Then by the above proposition, $V(T(G)) - N_b(u)$ is a boundary dominating set for $T(G)$ and $|N_b(u)| = \Delta_b(u)$. But $|N_b(u)| \geq 1$. Thus $\gamma_b(T(G)) \leq l - 1$. Suppose $\gamma_b(T(G)) = l - 1$, then there exists a unique vertex u^* in $T(G)$ such that u^* is a boundary neighbour of every vertex of $V(T(G)) - \{u^*\}$, this is a contradiction to the fact that in a graph there exist at least two boundary vertices. Thus $\gamma_b(T(G)) \leq l - 2$. Hence $\gamma_b(T(G)) \leq l - \Delta_b(T(G))$.

□

Theorem 2.9. *If G is a connected graph of order $n \geq 3$ and $T(G)$ its a total graph of order $l \geq 5$, then $\gamma_b(T(G)) \geq \lceil \frac{l}{1+\Delta_b} \rceil$.*

Proof. We have four cases:

Case 1: If $G \cong P_n$, since $l = |V[T(P_n)]| = 2n - 1$ and $\Delta_b(T(P_n))) = 4$, then

$$\lceil \frac{l}{1+\Delta_b} \rceil = \lceil \frac{2n-1}{5} \rceil \leq \begin{cases} \lceil \frac{2n-1}{5} \rceil & \text{if } n \equiv 1 \ (mod 5), \\ \lceil \frac{2n-1}{5} \rceil + 1 & \text{if } n \equiv 0, 2, 3, 4 \ (mod 5) \ . \end{cases} = \gamma_b(T(P_n)).$$

Case 2: If $G \cong C_n$, since $l = |V[T(C_n)]| = 2n$ and $\Delta_b(T(C_n)) = 4$, then

$$\lceil \frac{l}{1+\Delta_b} \rceil = \lceil \frac{2n}{5} \rceil \leq \begin{cases} \lceil \frac{2n}{5} \rceil + 1 & \text{if } n \equiv 2 \text{ or } 4 \ (mod 5), \\ \lceil \frac{2n}{5} \rceil & \text{if } n \equiv 0 \text{ or } 1 \text{ or } 3 \ (mod 5). \end{cases} = \gamma_b(T(C_n)).$$

Case 3: If $G \cong K_{1,n}$, since $l = |V[T(K_{1,n})]| = 2n+1$ and $\Delta_b(T(K_{1,n})) = 2n-2$, then $\lceil \frac{l}{1+\Delta_b} \rceil = \lceil \frac{2n+1}{2n-1} \rceil = 1 + \lceil \frac{2}{2n-1} \rceil \leq 3 = \gamma_b(T(K_{1,n}))$.

Case 4: If $G \cong K_n$, since $l = |V[T(K_n)]| = \frac{n(n+1)}{2}$ and $\Delta_b(T(K_n)) = \frac{(n-1)(n-2)}{2}$, then $\lceil \frac{l}{1+\Delta_b} \rceil = \lceil \frac{\frac{n(n+1)}{2}}{\frac{(n-1)(n-2)}{2}+1} \rceil = 1 + \lceil \frac{4(n-1)}{(n-1)(n-2)+2} \rceil \leq 3 = \gamma_b(T(K_n))$.

Hence $\lceil \frac{l}{1+\Delta_{tb}} \rceil \leq \gamma_b(T(G))$.

□

From the Theorems 2.8 and 2.9, we obtained the upper and lower bounds of the boundary domination of the total graphs of P_n, C_n, K_n and $K_{1,n}$ as the following

Observation 2.10. *For any graph G, we have, $\lceil \frac{l}{1+\Delta_b} \rceil \leq \gamma_b(T(G)) \leq l - \Delta_b(T(G))$.*

Theorem 2.11. *Let G and $\overline{G}$ be connected complementary graphes. Then,*

$$\gamma_b(T(G)) + \gamma_b(\overline{T(G)}) \leq n.$$

$$\gamma_b(T(G)) \cdot \gamma_b(\overline{T(G)}) \leq 3(n-3).$$

Proof. If $G \cong T(K_n)$, then G is the strongly regular graph with parameters $(\frac{n(n+1)}{2}, (n-2), n-1, 4)$, which is graph of $\gamma_b(T(K_n)) = 3$. And $\overline{G}$ also the strongly regular graph of parameters $(\frac{n(n+1)}{2}, \frac{(n-1)(n-2)}{2}, \frac{(n-3)(n-4)}{2}, \frac{(n-2)(n-3)}{2})$. Suppose $v \in \overline{G}$ then $deg(v) = \frac{(n-1)(n-2)}{2}$, and there exists $\frac{n(n+1)}{2} - \frac{(n-1)(n-2)}{2} = 2(n-1) \in N_b(v)$, and there exists $\frac{(n-2)(n-3)}{2}$ are common vertices between v and any vertex u is not adjacent to v. Similarly we can proceed up to all the n vertices. Finally we get a boundary domination of $\overline{G}$ is $\gamma_b(\overline{G}) = \frac{(n-1)(n-2)}{2} - \frac{(n-2)(n-3)}{2} = n-2$. Hence

$$\gamma_b(T(G)) + \gamma_{tb}(\overline{T(G)}) \leq n+1.$$

$$\gamma_b(T(G)) \cdot \gamma_b(\overline{T(G)}) \leq 3(n-3).$$

□

Theorem 2.12. *For any graph G and $T(G)$ its total graph of order l ,*

$$\gamma(T(G)) + \gamma_b(T(G)) \leq l+1.$$

Proof. Let $v \in V(T(G))$, then $N(v) \cup N_b(v) \cup \{v\} = V(T(G))$, $|N(v)| + |N_b(v)| + 1 = l$ and $\Delta(T(G)) + \Delta_b(T(G)) + 1 = l$. But we have $\gamma(T(G)) \leq l - \Delta(T(G))$ and $\gamma_b(T(G)) \leq l - \Delta_b(T(G))$. Therefore $\gamma(T(G)) + \gamma_b(T(G)) \leq 2l - \Delta(T(G)) + \Delta_b(T(G)) = 2l - l + 1 = l + 1$.
Hence $\gamma(T(G)) + \gamma_b(T(G)) \leq l+1$. □

3 Conclusion

In this paper we computed the exact value of the boundary domination number for total graphs of paths, cycles, complete graphs, star graphs and some special graphs. Also we found some upper bounds for boundary domination number for total graph of a graph.

References

[1] D. Michalak, On middle and total graphs with coarseness number equal 1, *Graph Theory, Proc. Conf, Lagow/Pol.*, (1981), *Lect. Notes Math.*, Springer, Berlin, 1018(1983), 139-150.

[2] G. Chartrand, D. Erwin, G.L. Johns and P. Zhang, On boundary vertices in graphs, *J. Combin. Math. Combin. Comput.*, 48(2004), 39-53.

[3] G. Chartrand, D. Erwin, G.L. Johns and P. Zhang, Boundary vertices in graphs, *Disc. Math.*, 263(2003), 25-34.

[4] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel-Dekker, Inc., (1997).

[5] K.M. Kathiresan, G. Marimuthu and M.S. Saraswathy, Boundary domination in graphs, *Kragujevac J. Math.*, 33(2010), 63-70.

[6] J.V. Vivin and M.M.A. Ali, On harmonious coloring of middle graph of $C(C_n)$, $C(K_{1,n})$ and $C(P_n)$, *Note di Matematica*, 29(2) (2009), 203-214.

[7] N. Murugesan and D.S. Nair, (1,2) - domination in total graph of $K_{1,n}$, C_n and P_n, *Advances in Mathematics*, 4(3) (2013), 516-526.

[8] E. Sampathkumar and H.B. Walikar, *Recent Developments in the Theory of Domination in Graphs*, J. Math Researc Instit. Allahabad, 1(1979).

[9] Y.B. Venketakrishnan and V. Swaminathan, Colorclass domination number of middle graph and central graph $K_{1,n}$, C_n and P_n, *AMO*, 12(2) (2010), 233-237.