*Gen. Math. Notes, Vol. 8, No. 2, February 2012, pp. 11-17*
*ISSN 2219-7184; Copyright © ICSRS Publication, 2012*
*www.i-csrs.org*
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# Pre-Semi-Closed Sets and Pre-Semi-Separation Axioms in Intuitionistic Fuzzy Topological Spaces

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**Abstract**

*The aim of this paper is to introduce and study different properties of pre-semi closed sets in intuitionistic fuzzy topological spaces. As applications to pre-semi-closed sets we introduce pre-semi $T_{1/2}$-spaces, semi- pre $T_{1/3}$ space and pre-semi $T_{3/4}$-spaces and obtain some of their basic properties.*

**Keywords**: *Intuitionistic Fuzzy (IF) sets, IF semi closed set, IF semi-pre (=$\beta$) closed set, IF generalized closed set, IF regular closed set, IF pre-semi closed set, IF pre- semi $T_{1/2}$ space, IF semi- pre $T_{1/3}$ space, IF pre- semi $T_{3/4}$ space, etc.*

## 1 Introduction

The concept of intuitionistic fuzzy set was introduced by Atanasov [1] in 1983 as a generalization of fuzzy sets. This approach provided a wide field to the generalization of various concepts of fuzzy mathematics. In 1997 Coker [3] defined intuitionistic fuzzy topological spaces. Recently many concepts of fuzzy topological space have been extended in intuitionistic fuzzy(IF) topological spaces. Murugesan and Thangavelu [6] intrduced the concept of pre-semi- closed sets in fuzzy topological spaces. In the present paper we introduce and study different properties of pre-semi-closed sets, pre-semi $T_{1/2}$-spaces, semi- pre $T_{1/3}$ space and pre-semi $T_{3/4}$-spaces in IF topological spaces.

## 2 Preliminaries

**Definition 2.1** *Let X denotes a universe of discourse. Then a fuzzy set A in X is defined as a set of ordered pairs* $A= \{< x, \mu_A(x) > : x \in X \}$*,where* $\mu_A(x): X \to [0, 1]$ *is the grade of belongingness of x into A. Thus the grade of non belongingness of x into A is equal to* $1-\mu_A(x)$*. However ,while expressing the degree of membership of any given element in a fuzzy set, the degree of non membership is not always expressed as a complement to1.Therefore Atanassov [1,2] suggested a generalization of fuzzy set, called an intuitionistic fuzzy set. In the present paper intuitionistic fuzzy will be denoted by IF only.*

*An IF set in X is given by a set of ordered triples* $A = \{< x, \mu_A(x), \nu_A(x)>: x \in X\}$*,where* $\mu_A(x), \nu_A(x) : X \to [0,1]$ *are functions such that* $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$*. The numbers* $\mu_A(x)$ *and* $\nu_A(x)$ *represent the degree of membership and degree of non-membership for each element* $x \in X$ *to* $A \subset X$*, respectively.*

**Definition 2.2[2]** *Let A and B be IF sets of the form* $A= \{< x, \mu_A(x), \nu_A(x)> : x \in X \}$ *and* $B= \{ < y, \mu_B(y), \nu_B(y) > : y \in Y\}$*. Then*

(a) $A \subseteq B$ *if and only If* $\mu_A(x) \le \mu_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$ *for all* $x \in X$*.*

(b) $A^c = \{< x, \nu_A(x), \mu_A(x) > : x \in X \}$*.*

**Definition 2.3[10]** *Two IF sets A and B are said to be quasi-coincident, denoted by* $A_q B$ *if and only if there exists an element* $x \in X$ *such that* $\mu_A(x) > \nu_B(x)$ *or* $\nu_A(x) < \mu_B(x)$*.*

*The expression 'not quasi-coincident' will be abbreviated as* $\urcorner A_q B$*.*

**Proposition 2.1[10]** *For any two IF sets A and B of X,* $\urcorner A_q B$ *if and only if* $A \subseteq B^c$*.*

**Definition 2.4[5]** *An IF set A of an IF topological space* $(X,\tau)$ *is said to be IF semi closed set if* $int( cl (A)) \subseteq A$*.*

**Definition 2.5[5]** *An IF set A of an IF topological space* $(X,\tau)$ *is said to be IF semi-pre (=*$\beta$*)-closed set if* $int( cl( int (A))) \subseteq A$*.*

**Definition 2.6[9]** *An IF set A of an IF topological space* $(X,\tau)$ *is said to be IF generalized closed if* $cl(A) \subseteq O$ *whenever* $A \subseteq O$ *and O is IF open* $(X,\tau)$*.*

*An IF set A of an IF topological space $(X,\tau)$ is said to be IF generalized open if its complement $A^c$ is IF generalized closed.*

**Proposition2.2[9]** *Every IF open set is IF generalized open but its converse may not be true.*

**Notation 2.1** *Let $(X,\tau)$ be an IF topological space. Then the family of IF regular (respectively semi-pre) closed sets in X, may be denoted by r (respectively sp).*

**Definition2.7[7]** *Let A be an IF set in an IF topological space $(X,\tau)$. Then IF semi pre interior and semi pre closure of A is denoted by spint (A) and spcl(A), defined by*

$$spint(A) = \cup\{G : G \subseteq A, G \text{ is IF semi pre open set in } X\}$$
$$spcl(A) = \cap\{B : A \subseteq B, B \text{ is IF semi pre closed set in } X\}.$$

**Definition 2.8[7]** *An IF set A of an IF topological space $(X,\tau)$ is said to be IF generalized semi-pre closed set if spcl(A) $\subseteq$ O whenever A $\subseteq$ O and O is IF open set in $(X,\tau)$.*
*An IF set A of an IF topological space $(X,\tau)$ is said to be IF generalized semi-pre open if its complement $A^c$ is IF generalized semi-pre closed.*

**Definition 2.9[7]** *An IF topological space $(X,\tau)$ is said to be IF semi-pre $T_{1/2}$ space if every IFGSP closed set in X is an IF semi-pre closed set in X.*

# 3 Pre-Semi-Closed Sets

**Definition 3.1** *Let A be an IF set in an IF topological space $(X,\tau)$. Then A is called an IF pre-semi closed set in X if spcl(A) $\subseteq$O whenever A $\subseteq$ O and O is IF generalized open set in X.*

**Example 3.1** Consider the IF topological space $(X,\tau)$,where X={a, b} and $\tau=\{0_\sim, 1_\sim, U\}$, U = <x,(a/.9,b/.2),(a/.1,b/.8)>. Since every IF open set is IF g-open so $0_\sim$, $1_\sim$, U are IF generalized open sets. Let A = <x,(a/.7,b/.2),(a/.3,b/.8)> be an IF set in X. Then A is an IF pre-semi-closed set in X, for if A $\subseteq$ O and O is IF generalized open set in X, then

$$O = 1_\sim \text{ and hence } spcl(A) \subseteq O.$$

**Theorem 3.1** *Every IF semi-pre-closed set in an IF topological space $(X,\tau)$ is IF pre-semi closed set.*

**Proof.** Let A be an IF semi-pre closed set in an IF topological space $(X,\tau)$. Suppose that A $\subseteq$ O and O is IF generalized open set in X. Since A is an IF semi-pre closed set, hence spcl(A) = A. Thus spcl(A) = A $\subseteq$ O, and hence A is IF pre-semi closed set.

But the converse may not true as shown in the following example.

**Example 3.2:** Consider the IF topological space $(X,\tau)$, where $X=\{a, b\}$ and $\tau = \{0_{\sim}, 1_{\sim}, U\}$, $U = \langle x,(a/.7,b/.3),(a/.3,b/.7)\rangle$. Since every IF open set is IF generalized open so $0_{\sim}, 1_{\sim}$, U are IF g-open sets. Let $A = \langle x,(a/.8,b/.3),(a/.2,b/.7)\rangle$ be an IF set in X. Then A is an IF pre-semi-closed set in X, for if $A \subseteq O$ and O is IF generalized open set in X, then

$O = 1_{\sim}$ and hence $spcl(A) \subseteq O$. But A is not an IF semi-pre-closed set, for $int(A) = U$, so $cl(int(A)) = 1_{\sim} \supset A$.

**Remark 3.1** *The IF pre-semi closed ness is independent from IF generalized closed ness as shown in the following two examples.*

**Example 3.3** In example 3.1 A is an IF pre-semi-closed set in X. But not an IF generalized closed set, for $A \subseteq U$ and U is IF open set in X, but $cl(A) = 1_{\sim} \not\subset U$.

**Example 3.4** Consider the IF topological space $(X,\tau)$, where $X=\{a, b\}$ and $\tau = \{0_{\sim}, 1_{\sim}, U\}$, $U = \langle x,(a/1,b/0),(a/0,b/.5)\rangle$. Let $A = \langle x,(a/1,b/.2),(a/0,b/.3)\rangle$ be an IF set in X. Then A is an IF generalized closed set in X but not an IF pre-semi-closed set in X.; for if $A \subseteq A$ and A is an IF generalized open set in X, but $int(cl(int(A))) = 1_{\sim} \supset A$ and hence $spcl(A) = 1_{\sim} \supset A$.

**Theorem 3.2** *Every IF pre-semi closed set in an IF topological space $(X,\tau)$ is IF generalized semi pre closed.*

**Proof.** Let A be an IF pre-semi closed set in an IF topological space $(X,\tau)$. Suppose that $A \subseteq O$ and O is an IF open set in X, then $spcl(A) \subseteq O$ and O is IF generalized open set in X. Hence A is an IF generalized semi pre closed in X.
But the converse may not true as shown in the following example.

**Example 3.5** In example 3.4 A is an IF generalized semi pre closed set in X but not an IF pre-semi closed set.

**Remark 3.2** *Every IF semi pre closed set is IF generalized semi pre closed set but its converse may not be true [7]. Hence the relationships of IF semi pre closed set, IF pre-semi closed set, IF generalized semi pre closed sets are as follows.*

*IF semi pre closed set $\Rightarrow$ IF pre-semi closed set $\Rightarrow$ IF generalized semi pre closed set.*
*However the converses are not true in general.*

**Theorem 3.3** *Let A be an IF set in an IF topological space $(X,\tau)$. Then A is an IF pre-semi closed if and only if $\rceil(A\ _q\ F) \Rightarrow \rceil(spcl(A)\ _q F)$ for every IF generalized closed set F of X.*

**Proof.**

**Necessity** Let F is an IF generalized closed set of X and $\rceil$ (A $_q$ F). Then by proposition 2.1 A $\subseteq F^c$ and $F^c$ is IF generalized open in X. Now since A is IF pre-semi closed, spcl (A) $\subseteq F^c$. Hence $\rceil$(spcl(A) $_q$F).

**Sufficiency** Let O is an IF generalized open set of X such that A$\subseteq$ O that is A $\subseteq$ $(O^c)^c$. Hence by proposition 2.1 $\rceil$(A $_q$ $O^c$) and $O^c$ is IF generalized closed set in X. Hence by hypothesis $\rceil$(spcl(A) $_q$ $O^c$). Therefore spcl(A) $\subseteq$ $(O^c)^c$. That is spcl(A) $\subseteq$ O. Hence A is IF pre-semi closed in X.

**Lemma 3.1** *Let A be an IF set in an IF topological space (X,$\tau$ ). Then A$\cup$ int(cl(int(A))) $\subseteq$ spcl(A).*

**Theorem 3.4** *Let A be an IF set in an IF topological space (X,$\tau$ ). If A is IF generalized open and IF pre-semi closed, then A is IF semi-pre closed.*

**Proof.** Since A is IF generalized open and IF pre-semi closed, it follows that A $\cup$ int(cl(int(A))) $\subseteq$ spcl(A) $\subseteq$ A. Hence int(cl(int(A))) $\subseteq$ A and A is IF semi-pre closed.

**Theorem 3.5** *Let A be an IF set in an IF topological space (X,$\tau$ ). Then the following are equivalent:*

*(i)* *A is IF regular open.*
*(ii)* *A is IF open and IF pre-semi closed.*
*(iii)* *A is IF open and IF generalized semi pre closed set.*

**Proof** (i) $\Rightarrow$ (ii) Let A be an IF regular open set in an IF topological space ( X,$\tau$ ). Then A is both IF open and IF semi closed. Now since every IF semi closed set is IF semi pre closed set, hence by theorem 3.1 A is an IF pre-semi closed set.
(ii) $\Rightarrow$ (iii) Let A be IF open and IF pre-semi closed. Then by theorem 3.2 A is IF generalized semi pre closed set.
(iii) $\Rightarrow$ (i) Let A be IF open and IF generalized semi pre closed set. Then A $\subseteq$ A and A is an IF open set in X and so A $\cup$ int(cl(int(A))) $\subseteq$ spcl(A) $\subseteq$ A. Hence int(cl(int(A))) $\subseteq$ A. Since A is IF open it follows that int(cl(A)) $\subseteq$ A = int(A) $\subseteq$ int(cl(A)). Hence A is IF regular open.

**Lemma 3.2** *Let A be an IF set in an IF topological space (X,$\tau$ ). Then spcl( spcl(A)) = spcl(A).*

**Theorem 3.6** *Let A be an IF pre-semi closed set in an IF topological space (X,$\tau$). If B is an IF set in X such that A $\subseteq$B $\subseteq$spcl(A),then B is also IF pre-semi closed.*
**Proof.** Let B be an IF set in an IF topological space (X,$\tau$ ) such that B$\subseteq$ O and O is an IF generalized open set in X. Then A $\subseteq$ O, since A is IF pre-semi closed, hence by lemma 3.2 spcl(B) $\subseteq$ spcl( spcl(A)) = spcl(A) $\subseteq$ O. Hence, B is an IF pre-semi closed in X.

**Definition 3.2** *An IF set A in an IF topological space $(X,\tau)$ is called an IF pre-semi open if and only if its complement $A^c$ is IF pre-semi closed.*

**Theorem 3.7** *An IF set A in an IF topological space $(X,\tau)$ is called an IF pre-semi open if $F \subseteq spint(A)$ whenever $F \subseteq A$ and F is IF generalized closed set in X.*

**Proof.** Let an IF set A in an IF topological space $(X,\tau)$ is IF pre-semi open and F is an IF generalized closed set in X such that $F \subseteq A$. Then $A^c \subseteq F^c$, where $A^c$ is IF pre-semi closed and $F^c$ is an IF generalized open set in X. Hence from definition 3.1 $spcl(A^c) \subseteq F^c$. Hence $(F^c)^c \subseteq (spcl(A^c))^c$. That is $F \subseteq spint(A^c)^c = spint(A)$.

**Theorem 3.6** *Let A be an IF pre-semi closed set in an IF topological space $(X,\tau)$. If B is an IF set in X such that $spint(A) \subseteq B \subseteq A$, then B is also IF pre-semi open.*

**Proof** Let B be an IF set in an IF topological space $(X,\tau)$ such that $F \subseteq B$ and F is IF generalized closed set in X. Then $F \subseteq A$, since A is IF pre-semi closed, hence $F \subseteq spint(A)$. Therefore $F \subseteq spint(A) \subseteq B$. Hence, B is an IF pre-semi open in X.

# 4 Pre-Semi-Separation Axioms

**Definition 4.1** *An IF topological space $(X,\tau)$ is said to be IF pre- semi $T_{1/2}$ space if every IF pre-semi closed set in X is IF semi-pre closed set in X.*

**Theorem 4.1** *Every IF semi-pre $T_{1/2}$ space is an IF pre- semi $T_{1/2}$ space.*

**Proof.** Let $(X,\tau)$ be an IF semi-pre $T_{1/2}$ space and A be an IF pre-semi closed set in X. Now by theorem 3.2 every IF pre-semi closed set is an generalized semi pre closed set in X. Hence, A is an generalized semi pre closed set. in X and consequently A is IF semi-pre closed set in X. Thus $(X,\tau)$ is IF pre- semi $T_{1/2}$ space.

**Definition 4.2** *An IF topological space $(X,\tau)$ is said to be IF semi- pre $T_{1/3}$ space if every generalized semi pre closed set in X is IF pre- semi closed set in X.*

**Theorem 4.2** *Every IF semi-pre $T_{1/2}$ space is an IF semi- pre $T_{1/3}$ space.*

**Proof.** Let $(X,\tau)$ be an IF semi-pre $T_{1/2}$ space and A be an generalized semi pre closed set in X. Then, A is IF semi-pre closed set in X. Now by theorem 3.1 every IF semi-pre-closed set in X is IF pre-semi closed. Thus $(X,\tau)$ is IF semi- pre $T_{1/3}$ space.

**Definition 4.3** *An IF topological space $(X,\tau)$ is said to be IF pre- semi $T_{3/4}$ space if every IF pre-semi closed set in X is IF pre closed set in X.*

**Theorem 4.3** *Every IF pre- semi $T_{3/4}$ space is an IF pre- semi $T_{1/2}$ space.*

**Proof.** Let ( X,τ ) be an IF pre- semi $T_{3/4}$ space and A be an IF pre-semi closed set in X. Then A is IF pre closed set in X. Now every IF pre closed set in X is IF semi-pre closed set in X. Hence, A is an IF semi-pre closed set in X. Thus ( X,τ ) is IF pre- semi $T_{1/2}$ space.

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