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Edge-Domsaturation Number of a Graph

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Abstract

The edge-domsaturation number $ds'(G)$ of a graph $G = (V, E)$ is the least positive integer k such that every edge of G lies in an edge dominating set of cardinality k. The connected edge-domsaturation number $ds'_c(G)$ of a graph $G = (V, E)$ is the least positive integer k such that every edge of G lies in a connected edge dominating set of cardinality k. In this paper, we obtain several results connecting $ds'(G), ds'_c(G)$ and other graph theoretic parameters.

Keywords: *edge-dominating set, edge-domination number, edge-domsaturation number, connected edge-domsaturation number.*

1 Introduction

Throughout this paper, G denotes a graph with order p and size q. By a graph we mean a finite undirected graph without loops or multiple edges. For graph theoretic terms we refer Harary [2]. In particular, for terminology related to domination theory we refer Haynes et.al [3].

Definition 1.1. *Let $G = (V, E)$ be a graph. A subset D of E is said to be an edge dominating set if every edge in $E - D$ is adjacent to at least one edge in D. An edge dominating set D is said to be a minimal edge dominating set if no proper subset of D is a dominating set of G.*

Acharya [1] introduced the concept of domsaturation number $ds(G)$ of a graph. Arumugam and Kala [4] observed that for any graph G, $ds(G) = \gamma(G)$ or $\gamma(G)+1$ and obtained several results on $ds(G)$. We now extend the concept of domsaturation to edges.

Definition 1.2. *The least positive integer k such that every edge of G lies in an edge dominating set of cardinality k is called the edge-domsaturation number of G and is denoted by $ds'G)$.*

Definition 1.3. *The least positive integer k such that every edge of G lies in a connected edge dominating set of cardinality k is called the connected edge-domsaturation number of G and is denoted by $ds'_c(G)$.*

If G is a graph with edge set E and D is a γ'-set of G, then for any edge $e \in E - D, D \cup \{e\}$ is also an edge dominating set and hence $ds'(G) = \gamma'(G)$ or $\gamma'(G) + 1$.
Thus we have the following definition.

Definition 1.4. *A graph G is said to be of class 1 or class 2 according as $ds'(G) = \gamma'(G)$ or $\gamma'(G) + 1$.*

Definition 1.5. *A tree T of order 3 or more is a caterpillar if the removal of its leaves produces a path.*

Definition 1.6. *A tree containing exactly two vertices that are not leaves (which are necessarily adjacent) is called a double star. Thus a double star is a tree of diameter three.*

We use the following theorems.

Theorem 1.7. *[6] For any tree T of order $p \neq 2$, $\gamma'(G) \leq (p-1)/2$; equality holds if and only if T is isomorphic to the subdivision of a star.*

Theorem 1.8. *[6] Let T be any tree and let $e = uv$ be an edge of maximum degree $\Delta'(T)$. Then $\gamma'(T) = q - \Delta'(T)$ if and only if $diam(T) \leq 4$ and $degw \leq 2$ for every vertex $w \neq u, v$.*

2 Main Results

Theorem 2.1. *The path P_p of order p, $p \geq 4$ is of class 1 if and only if $p \equiv 2 \ (mod\ 3)$.*

Proof. Let $P_p = (1, 2, \ldots, p)$ be of class 1. Let e_i be the edge joining i and $i + 1$. If $p \equiv 0(mod\ 3)$, then e_3 does not lie in an edge dominating set of cardinality $\gamma'(G)$. If $p \equiv 1(mod\ 3)$, then either e_1 or e_3 does not lie in an edge dominating set of cardinality $\gamma'(G)$. Hence if $p \equiv 0$ or $1 (mod\ 3)$, then P_p is of class 2.

Conversely, suppose $p = 3k+2$. Then $\gamma'(G) = k+1$.

$$\begin{aligned} \text{Let } D_1 &= \{e_1, e_3, e_6, \ldots, e_{3k}\} \\ D_2 &= \{e_2, e_5, e_7, \ldots, e_{3k-1}, e_{3k+1}\} \\ \text{and } D_3 &= \{e_1, e_4, e_7, \ldots, e_{3k-2}, e_{3k+1}\}. \end{aligned}$$

Clearly D_1, D_2 and D_3 are $\gamma'(G)$ sets of P_p and $\cup_{i=1}^{3} D_i = E(P_p)$. Hence $ds'(G) = \gamma'(G)$ so that P_p is of class 1.

Definition 2.2. *Let T be a caterpillar. Two supports u and v of T are said to be consecutive if either u and v are adjacent or every vertex in the $u-v$ path in T has degree 2.*

Theorem 2.3. *Let T be a caterpillar. Then T is of class 1 if and only if every support is adjacent to exactly one pendent vertex and for any two consecutive supports u and v, $d(u,v) \equiv 2(mod\ 3)$.*

Proof. Suppose T is a caterpillar of class 1. If there exists two pendent vertices v_1, v_2 which are adjacent to u, then there is no $\gamma'(G)$ -set containing uv_1. Hence every support is adjacent to exactly one pendent vertex. Now, let S denote the set of all supports of T. Suppose there exists two consecutive supports u and v such that $d(u,v) \equiv 0$ *or* $1(mod\ 3)$. Let $P = (u = u_1, u_2, \ldots, u_k = v)$ be the $u-v$ path in T. Then u_2u_3 does not lie in a $\gamma'(G)$- set and hence it follows that for any two consecutive supports u and v, $d(u,v) \equiv 2(mod\ 3)$.

Conversely, let T be a caterpillar in which every support is adjacent to exactly one pendent vertex and $d(u,v) \equiv 2(mod\ 3)$ for any two consecutive supports u and v. Let k denote the number of supports in T. We prove that T is of class 1 by induction on k. If $k = 2$, T is a path P_p with $p \equiv 2(mod\ 3)$ vertices and by the theorem [2.1], T is of class 1. Suppose the theorem is true for all caterpillars with $k-1$ supports. Let T be a caterpillar with k supports $w_1, w_2, \ldots, w_k$ such that w_i and w_{i+1} are consecutive supports. Let x_i be the pendent vertex adjacent to w_i. Let $P_1 = (w_1, v_1, \ldots, v_{3m+1}, w_2)$ be the $w_1 - w_2$ path and let $T_1 = T - \{x_1, w_1, v_1, \ldots, v_{3m+1}\}$. Clearly P_1 is of class 1 and by induction hypothesis T_1 is of class 1. Further the union of any minimum edge dominating set of P_1 and any minimum edge dominating set of T_1 is a minimum edge dominating set of T. Hence T is of class 1.

Theorem 2.4. *If G is a k-regular graph which is edge domatically full, then G is of class 1.*

Proof. Since G is edge domatically full, $d'(G) = \delta'(G) + 1 = k+1$.
Let $\{D'_1, D'_2, \ldots, D'_{k+1}\}$ be an edge domatic partition of G. Any set D'_i either

contains an edge x or exactly one of its neighbours. Hence each D'_i is independent. Also for all $1 \leq j \leq k+1,\ i \neq j$, every edge in D'_i is adjacent to exactly one edge in D'_j. Hence all sets D'_i are of equal cardinality and $|D'_i| = \gamma'(G)$ so that G is of class 1.

Lemma 2.5. *Let G be a path of even order which is of class 1. Then $\gamma'(G) + \beta_1(G) = p - 1$ if and only if $G \cong P_8$.*

Proof. If $G \cong P_8$, clearly $\gamma'(G) + \beta_1(G) = p - 1$. Conversely, suppose $\gamma'(G) + \beta_1(G) = n - 1$. Since G is a path of even order, obviously it is of class 1. By theorem 2.1 , we have $p = 3k + 2$. Obviously $\beta_1(G) = p/2$. Then $\gamma'(G) = p - 2/2$. But P_p is a path and so $\gamma'(G) = \lceil \frac{p-1}{3} \rceil$.Now $\frac{p-2}{2} = \lceil \frac{p-1}{3} \rceil \Rightarrow \frac{3k}{2} = \lceil \frac{3k+1}{3} \rceil \Rightarrow k = 2$. Therefore k=2. Hence $G \cong P_8$.

Theorem 2.6. *Let G be any connected graph which is of class 1. Then $ds'(G) = q - \beta_1(G)$ (where q is the number of edges)if and only if G is isomorphic to C_4, the subdivision of a star or P_8.*

Proof. Suppose $ds'(G) = q - \beta_1(G)$. Then $ds'(G) = \gamma'(G) = q - \beta_1(G)$. Since $\gamma'(G) \leq p/2$ and $\beta_1(G) \leq p/2$, we have $\gamma'(G) + \beta_1(G) \leq p$ and hence $q \leq p$. If $q = p$, then p is even, $\gamma = \beta_1 = p/2$ and G is unicyclic. Hence it follows from [6] that $G = C_4$. If $q = p - 1$,then we have the following cases:

Case(i). p is odd.
Now$\gamma'(G) = \beta_1(G) = \frac{(p-1)}{2}$ and G is a tree. Hence it follows from theorem 2.6 that G is isomorphic to the subdivision of a star.

Case(ii) p is even.
Now we have $\gamma'(G) = \frac{(p-2)}{2}$, $\beta_1(G) = \frac{p}{2}$ and G is a path. Hence it follows from lemma 2.5 that G is isomorphic to P_8. The converse is obvious.

Theorem 2.7. *For any (p, q) graph G which is of class 1, $ds'(G) + d'(G) = q + 1$ if and only if $G \cong C_3,\ K_{1,p-1}$ or mK_2.*

Proof. Suppose $ds'(G) + d'(G) = q + 1$. Since G is of class 1, we have $ds'(G) = \gamma'(G)$, i.e. $\gamma'(G) + d'(G) = q + 1$. Since $\gamma'(G)d'(G) \leq q$, we have $(d'(G) - 1)(q - d'(G)) \leq 0$. Further, $d'(G) \geq 1$ and $q \geq d'(G)$. So $(q - d'(G))(d'(G) - 1) = 0$. Hence $q = d'(G)$ or $d'(G) = 1$. If $d'(G) = 1$, then G is isomorphic to mK_2. If $q = d'(G)$, then $G = C_3$ or $K_{1,p-1}$. The converse is obvious.

Theorem 2.8. *If T is a bistar, then T is of class 2.*

Proof. Since the non-pendent edge of T is an edge dominating set of T, we have $\gamma'(T) = 1$. There is no γ-set containing any of the pendent edges and so T is of class 2.

Theorem 2.9. *Let T be any tree and let $e = uv$ be an edge of maximum degree $\Delta'(T)$. Then $ds'(T) = q - \Delta'(T) + 1$ if and only if T is isomorphic to bistar or $diam(T) = 4$, $deg w \leq 2$ for every vertex $w \neq u, v$ and there exist at least one pair of end vertices which are distant 3 apart.*

Proof. By theorem 1.8, it is enough to investigate those graphs that are of class 2. If $diam(T) = 1$ or 2, then obviously T is of class 1. If $diam(T) = 3$, then T has exactly one non-pendent edge. Therefore T is of class 2. If $diam(T) = 4$, then each nonpendent edge of T is adjacent to a pendent edge of T and hence the set of all nonpendent edges of T forms a minimum edge dominating set and $\gamma'(T) = q - \Delta'(T)$. Based on the distance between the pendent vertices, we have the following cases:

Case(i). $d(u, v) \neq 3$, for every $u, v \in S$.
Then $d(u, v) = 1, 2$ or 4. Since $diam(T) = 4$, it is impossible that $d(u, v) = 1$ or 2. Hence there exists $u, v \in S$ with $d(u, v) = 4$. In this case T is of class 1.

Case(ii). There exists $u, v \in S$ with $d(u, v) = 3$.
Let e, e' be the pendent edges incident with u, v respectively. Since $diam(T) = 4$, at least one of e, e' should be adjacent to two non-pendent edges. Without loss of generality let e be adjacent to two non-pendent edges. Then there os no two element edge dominating set containing e so that T is of class 2.

Theorem 2.10. *Let G be a graph with $\Delta'(G) = q - 2$. Let e be an edge of degree $q - 2$ and let f be an edge which is non adjacent to e. Then G is of class 1 if and only if for every $g_1 \in E(G) \setminus (N[f] \cup \{e\})$, there exists $g_2 \in N[f]$ such that $N[g_1] \cup N[g_2] = E(G)$.*

Proof. Suppose G is of class 1. Let e be an edge of degree $q - 2$ and let f be an edge non-adjacent to e. Let $g_1 \in E(G) \setminus (N[f] \cup \{e\})$. Since $ds'(G) = \gamma'(G) = 2$, there exists $g_2 \in E(G)$ such that $\{g_1, g_2\}$ is an edge dominating set. Clearly, $g_2 \in N[f]$ and $N[g_1] \cup N[g_2] = E(G)$. The converse is immediate.

Theorem 2.11. *Given three positive integers a, b and c with $2 \leq a \leq b \leq c$, there exists a graph G with $\gamma'(G) = a, ds'(G) = a + 1, EIS(G) = b$ and $\beta(G) = c$ if and only if $b \leq 2a - 1$ and $c = b + 1$.*

Proof. If there exists a graph G with $\gamma'(G) = a$, $ds'(G) = a + 1$, $EIS(G) = b$ and $\beta(G) = b + 1$, then it follows from [5] that $b \leq 2a - 1$ and $c = b + 1$.

Conversely, let $b \leq 2a-1$ and $c = b+1$. Let $b = a+k$, where $0 \leq k \leq a-1$. Construct a graph as follows: Let $\{u_1v_1, u_2v_2, ..., u_av_a\}$ be a set of independent edges. Add vertices $x_1, x_2, ..., x_{k+1}$ and $y_1, y_2, ..., y_{k+1}$ and join x_i with

u_i and y_i with v_i for all i, $1 \leq i \leq k+1$. Also add a vertex z and join z with u_i and v_i for all i, $k+2 \leq i \leq a$.

Clearly $\{u_1v_1, u_2v_2, ..., u_av_a\}$ is a minimum edge dominating set of G and hence $\gamma'(G) = a$. But x_iu_i and y_iv_i, $1 \leq i \leq k+2$ does not belong to any γ' set. Therefore $ds'(G) = \gamma'(G) + 1$.Therefore $\{x_1u_1, y_1v_1, u_2v_2, ..., u_av_a\}$ is an edge-domsaturation set with cardinality $a + 1$.
Also, $I = \{u_1x_1, u_2x_2, ..., u_{k+1}x_{k+1}, v_1y_1, v_2y_2, ...v_{k+1}y_{k+1}, u_{k+2}v_{k+2}, ..., u_av_a\}$ is a maximum matching in G. Hence $\beta_1(G) = a + k + 1 = c$. Since $I_1 = I - \{u_1x_1, v_1y_1\} \cup \{u_1v_1\}$ is a maximum matching containing u_1v_1, we have $EIS(u_1v_1) = a + k$ and hence $EIS(G) = \beta_1 - 1 = b$.

3 Connected Edge-Domsaturation Number of a Graph

Definition 3.1. *Let G be a connected graph. The least positive integer k such that every edge of G lies in a connected edge dominating set of cardinality k is called the connected edge-domsaturation number of G and is denoted by $ds'_c(G)$.*

Example 3.2. (i) $ds'_c(K_p) = p - 2$
(ii) $ds'_c(P_p) = p - 2$
(iii) $ds'_c(K_{q,p}) = min\{q, p\}$.

Observation 3.3. *If G is any connected graph with $\Delta'(G) = q - 1$ and $G \ncong K_{1,n}$, then $ds'_c(G) = \gamma'_c(G) + 1$.*

Proof. Since $\Delta'(G) = q - 1$, we have $\gamma'_c(G) = 1$. Further any edge with degree less than $q-1$ does not lie on a $\gamma'_c(G)$-set. Therefore $ds'_c(G) = \gamma'_c(G)+1$.

Observation 3.4. *For any connected graph G with $p \geq 4$ and $\delta'(G) = 1$, we have $ds'_c(G) = \gamma'_c(G) + 1$.*

Proof. Since no pendent edge lies on a $\gamma'_c(G)$-set, the result follows.

We now find an upper bound on connected edge-domsaturation number for trees and unicyclic graphs.

Observation 3.5. *For any tree T of order $p \geq 4$, $\gamma'_c(T) = p-3$ if and only if T is a path or $K_{1,3}$.*

Observation 3.6. *For any tree T of order $p \geq 4$, $ds'_c(T) = p - 2$ if and only if $T$is a path.*

Corollary 3.7. *For any tree T of order $p \geq 4$, $ds'_c(T) + \chi(T) \leq p$ and equality holds if and only if T is a path.*

Proof. It follows from observation 3.6 that for any tree T, $ds'_c(G) \leq p-2$. Also $\chi(G) = 2$. Therefore $ds'_c(G) + \chi(G) \leq p$. Further $ds'_c(G) + \chi(G) = p$ if and only if $ds'_c(G) = p - 2$ or equivalently T is a path.

Theorem 3.8. *Let G be a connected unicyclic graph with cycle C. Then $ds'_c(G) = p - 2$ if and only if $G \cong C$ or a cycle C with exactly one pendent edge.*

Proof. Let G be a unicyclic graph with $ds'_c(G) = p - 2$. Let C be the unique cycle in G and suppose $G \neq C$. Let S be the set of all pendent edges of G. We observe that $ds'_c(G) = p - |S|$ if no vertex in C is of degree 2 and $ds'_c(G) = p - |S| - 1$ otherwise. In the former case, $|S| = 2$. But this is impossible as in this case no vertex in C is of degree 2. Therefore $ds'_c(G) = p - |S| - 1$. Now $|S| = 1$ and so G has exactly one pendent edge.

Theorem 3.9. *For any tree T, $T \ncong K_{1,n}$, $ds'_c(T) = q - \Delta'(T) + 1$ if and only if T has at most one vertex of degree greater than 2 or exactly two adjacent vertices of degree greater than 2.*

Proof. We observe that, $ds'_c(T) = q - k + 1$, where k is the number of pendent edges of T. Hence $ds'_c(G) = q - \Delta'(G) + 1$ if and only if $\Delta'(G) = k$. However if T has two non-adjacent vertices of degree greater than 2, then $k > \Delta'(G)$ and hence the result follows.

Theorem 3.10. *Let G be a connected unicyclic graph with cycle C and $G \ncong C$. Then $ds'_c(G) = q - \Delta'(G) + 1$ if and only if one of the following conditions hold.*

1. *G has exactly one vertex of degree greater than 2*
2. *G has exactly two vertices u, v of degree greater than 2 and u, v are adjacent*
3. *$C = C_3$, all the vertices of C are of degree ≥ 3, one vertex of C is of degree 3 and all the vertices not on C have degree one or two.*

Proof. Let G be a connected unicyclic graph with $ds'_c(G) = q - \Delta'(G) + 1$ and as in the proof of theorem 3.8, we have $|S| = \Delta'(G) - 1$ or $|S| = \Delta'(G) - 2$, where S is the number of pendent edges of T.

Case(i). $|S| = \Delta'(G) - 1$.
In this case, every vertex of C is of degree ≥ 3. Now if $C \neq C_3$, then G has at most $\Delta'(G)$ pendent edges. Thus $C = C_3$. It follows that at most one vertex of C is of degree 3 and all vertices not on C have degree 1 or 2. Hence G is of the form described in (3).

Case(ii). $|S| = \Delta'(G) - 2$
In this case, there exists at least one vertex of degree 2 on C. Let $e = uv$ be an edge of maximum degree $\Delta'(G)$. Since $|S| = \Delta'(G) - 2$, at least one of u, v lies on C and all vertices different from u, v have degree one or two. If both u, v have degree at least 3 then G satisfies (2), Otherwise G satisfies (1).

4 Domsaturation Number of a Graph

Theorem 4.1. *Let G be any connected graph and let G' be the graph obtained from G by concatenating a vertex of G with the center of a star $k_{1,n}, (n \geq 2)$. Then $ds(G) = \gamma(G) + 1$.*

Proof. Let $u \in V(G)$ be the support vertex of a star. Suppose u is not dominated by any vertex of G, then clearly u belongs to the γ-set. Suppose u is dominated by some vertices of G. Since number of pendent vertices ≥ 2. So in this case also u belongs to the γ-set.In both these cases the pendent vertices does not belong to any γ-set. So $ds(G) = \gamma(G) + 1$.

Theorem 4.2. *Given any three positive integers a, b, and c with $3 \leq a \leq b \leq c$, their exists a graph G with $ds(G) = a, IS(G) = b$ and $\Gamma(G) = c$.*

Proof. Case(i). $a = 3$
Let $k = \begin{cases} 0 & \text{if } c \leq 2b-2 \\ c-2b+2 & \text{if } c > 2b-2 \end{cases}$ and
let $\alpha = \begin{cases} 2b-2-c & \text{if } c \leq 2b-2 \\ 0 & \text{if } c > 2b-2. \end{cases}$

Let $P_4 = (v_1, v_2, v_3, v_4)$ be a path on 4 vertices. Attach $b-2$ pendent vertices $u_1, u_2, \ldots, u_{b-2}$ to v_2 and $b-2+k$ pendent vertices $w_1, w_2, \ldots, w_{b-2+k}$ to v_3. Add the edges $u_1w_1, u_2w_2, \ldots, u_\alpha w_\alpha$. For the resulting graph G, we have $\gamma(G) = 2$. But the pendent vertices does not lie in any dominating set of cardinality 2. Thus $ds(G) = 3 = a$.

If $b = c$, then clearly $IS(G) = IS(v_2)$ or $IS(v_3)$.

If $b < c$, then $IS(G) = IS(v_3)$. Since v_3 is the only vertex which is the minimum of all $IS(v)'s$, for every $v \in V(G)$. In both the cases, $\{v_3, u_1, u_2, \ldots, u_{b-2}, v_1\}$ is the desired IS-set of G. Hence $IS(G) = b - 2 + 2 = b$.

Also $\{u_1, u_2, \ldots, u_{b-2}, w_{\alpha+1}, w_{\alpha+2}, \ldots, w_{b-2+k}, v_1, v_4\}$ is the maximum cardinality of a minimal dominating set and hence $\Gamma(G) = 2b - 2 + k - \alpha$.

If $c \leq 2b - 2$, then $2b - 2 + k - \alpha = 2b - 2 - (2b - 2 - c) = c$.

If $c > 2b-2$, then $2b-2+k-\alpha = 2b-2+c-2-2b = c$. Hence $\Gamma(G) = c$.

Case(ii). $a \geq 4$

Let $k = \begin{cases} 0 & \text{if } c \leq 2b - a \\ c - 2b + a & \text{if } c > 2b - a \end{cases}$ and

let $\alpha = \begin{cases} 2b - a - c & \text{if } c \leq 2b - 2 \\ 0 & \text{if } c > 2b - a \end{cases}$

Let $P = (v_1, v_2, \ldots, v_a)$ be a path on a vertices. Attach pendent vertices $u_1, u_4, \ldots, u_a$ to $v_1, v_4, \ldots, v_a$ respectively. Attach $b - (a - 1)$ pendent vertices $s_1, s_2, \ldots, s_{b-(a-1)}$ to v_2, attach $b-(a-1)+k$ pendent vertices $t_1, t_2, \ldots, t_{b-(a-1)+k}$ to v_3 add the edge u_1u_a and the edges $s_1t_1, s_2t_2, \ldots s_\alpha t_\alpha$.
Clearly $\{u_1, v_2, v_3, ..., v_{a-1}\}$ is a γ- set.But the pendent vertices adjacent to v_2, v_3 and the vertices v_1, v_a does not belong to any γ set.Therefore $ds(G) = a$. If $a = b = c$,then $k = 0$ and $\alpha = 0$. Hence $IS(G) = IS(i) = a$ for all $i \in V$ If $a < b$ and $b = c$, then $IS(v_2)$ or $IS(v_3)$ is the IS-set of G. If $a < b < c$, then $IS(v_3)$ is the only set having minimum cardinality among all IS-sets. From these three cases, $\{v_3, s_1, s_2, \ldots, s_{b-(a-1)}, u_1, u_4, u_5, \ldots, u_{a-1}, v_a\}$ is the desired IS-set. Hence $IS(G) = b - (a - 1) + 1 + 1 + a - 3 = b$. Also $\{s_1, s_2, \ldots, s_{b-(a-1)}, t_{\alpha+1}, t_{\alpha+2}, \ldots, t_{b-(a-1)+k}, u_4, u_5, \ldots, u_{a-1}, v_a, v_1\}$ is a dominating set of maximum cardinality and hence $\Gamma(G) = 2b - a + k - \alpha$. As in case(i), we have $\Gamma(G) = c$.

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