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$(1,2)^*$-Pre-Λ-Sets and $(1,2)^*$-Pre-V-Sets

M. Lellis Thivagar and Nirmala Mariappan

Department of Mathematics, Arul Anandar College, Karumathur-625514
Madurai District, Tamilnadu, India.
E-mail:mlthivagar@yahoo.co.in
Department of Mathematics, Lady Doak College, Madurai-625002
Tamil Nadu, India.
E-mail:nirmala_mariappan@yahoo.com

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Abstract

In this paper, we introduce the notions of $(1,2)^*$*-pre-*Λ*-sets and* $(1,2)^*$*-pre-*V*-sets in a bitopological space and study their fundamental properties. We also introduce two new spaces called* $(1,2)^*$*-pre-*T_1 *and* $(1,2)^*$*-pre-*R_0*-spaces.Characterization of these spaces are done using* $(1,2)^*$*-pre-*Λ*-sets and* $(1,2)^*$*-pre-*V*-sets.*

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1 Introduction

In 1986, Maki continued the work of Levine and Dunham on generalized closed sets by introducing the notion of a generalized Λ set in a topological space. Caldas and Dontchev built on Maki's work, by introducing and studying the so called Λ_s-sets and V_s-sets and also other forms called g.Λ_s-sets and g.V_s-sets. In this paper, we introduce the notions of $(1,2)^*$-pre-Λ-sets and $(1,2)^*$-pre-V-sets in a bitopological space and study their fundamental properties. We also introduce two new spaces called $(1,2)^*$-pre-T_1 and $(1,2)^*$-pre-R_0-spaces. Characterization of these spaces are done using $(1,2)^*$-pre-Λ-sets and $(1,2)^*$-pre-V-sets.

2 Preliminaries

Throughout this paper (X, τ_1, τ_2) and (Y, σ_1, σ_2) (briefly X and Y) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1 [11] *A subset S of a bitopological space (X, τ_1, τ_2) is said to be $\tau_{1,2}$-open if $S = A \cup B$ where $A \in \tau_1$, and $B \in \tau_2$. A subset S of X is said to be $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open.*

Definition 2.2 [11] *Let S be a subset of X. Then*

(i) The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined by $\cup\{G/G \subseteq S$ and G is $\tau_{1,2}$-open $\}$.

(ii) The $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined by $\cap\{F/S \subseteq F$ and F is $\tau_{1,2}$-closed $\}$.

Remark 2.3 *$\tau_{1,2}$-open sets need not form a topology.*

We recall the following definitions which are useful in the sequel.

Definition 2.4 [11] *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(i) $(1,2)^*$-semi-open if $A \subseteq \tau_{1,2}$-cl$(\tau_{1,2}$-int$(A))$

(ii) $(1,2)^*$-preopen if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(A))$

(iii) $(1,2)^*$-α-open if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2}$-cl$(\tau_{1,2}$-int$(A)))$

The complements of the sets mentioned above from (i) to (iii) are called their respective closed sets.

3 $(1,2)^*$-pre-Λ-Sets and $(1,2)^*$-pre-V-Sets

First let us introduce the concepts $(1,2)^*$-Λ_p and $(1,2)^*$-V_p.

Definition 3.1 *Let S be a subset of a bitopological space (X, τ_1, τ_2). We define the subsets $(1,2)^*$-$\Lambda_p(S)$ and $(1,2)^*$-$V_p(S)$ as follows: $(1,2)^*$-$\Lambda_p(S) = \cap\{G : G \supseteq S$ and G is $(1,2)^*$-preopen$\}$ and $(1,2)^*$-$V_p(S) = \cup\{F : F \subseteq S$ and F^c is $(1,2)^*$-preopen$\}$.*

First we summarise the fundamental properties of the sets $(1,2)^*$-$\Lambda_p(S)$ and $(1,2)^*$-$V_p(S)$.

Lemma 3.2 *For subsets A,B and* $A_i, i \in I$ *of a bitopological space* (X, τ_1, τ_2) *the following properties hold:*

(1) $A \subseteq (1,2)^*\text{-}\Lambda_p(A)$.

(2) $A \subseteq B$ *implies that* $(1,2)^*\text{-}\Lambda_p(A) \subseteq (1,2)^*\text{-}\Lambda_p(B)$.

(3) $(1,2)^*\text{-}\Lambda_p((1,2)^*\text{-}\Lambda_p(A)) = (1,2)^*\text{-}\Lambda_p(A)$.

(4) *If* A *is* $(1,2)^*$-*preopen then* $A = (1,2)^*\text{-}\Lambda_p(A)$.

(5) $(1,2)^*\text{-}\Lambda_p(\cup\{A_i : i \in I\}) = \cup\{(1,2)^*\text{-}\Lambda_p(A_i) : i \in I\}$.

(6) $(1,2)^*\text{-}\Lambda_p(\cap\{A_i : i \in I\}) \subseteq \cap\{(1,2)^*\text{-}\Lambda_p(A_i) : i \in I\}$.

(7) $(1,2)^*\text{-}\Lambda_p(X - A) = X$ - $(1,2)^*\text{-}\Lambda_p(A)$.

Proof (1),(2),(4),(6) and (7) are immediate consequences of Definition 3.1. To prove (3): It follows from (1) and (2) that $(1,2)^*\text{-}\Lambda_p(A) \subseteq (1,2)^*\text{-}\Lambda_p((1,2)^*\text{-}\Lambda_p(A))$. If $x \notin (1,2)^*\text{-}\Lambda_p(A)$, then there exists a $(1,2)^*$-preopen set G such that $A \subseteq G$ and $x \notin G$. Hence $(1,2)^*\text{-}\Lambda_p(A) \subseteq G$ and so $x \notin (1,2)^*\text{-}\Lambda_p((1,2)^*\text{-}\Lambda_p(A))$. Thus $(1,2)^*\text{-}\Lambda_p((1,2)^*\text{-}\Lambda_p(A)) \subseteq (1,2)^*\text{-}\Lambda_p(A)$. To prove (5): Let $A = \cup\{A_i : i \in I\}$. By (2) we have that $\cup\{(1,2)^*\text{-}\Lambda_p(A_i) : i \in I\} \subseteq (1,2)^*\text{-}\Lambda_p(A)$. If $x \notin \cup\{(1,2)^*\text{-}\Lambda_p(A_i) : i \in I\}$, then for each $i \in I$, there exists a $(1,2)^*$-preopen set G_i such that $A_i \subseteq G_i$ and $x \notin G_i$. If $G = \cup\{G_i : i \in I\}$ then G is $(1,2)^*$-preopen with $A \subseteq G$ and $x \notin G$. Hence $x \notin (1,2)^*\text{-}\Lambda_p(A)$ and so (5) holds.

Lemma 3.3 *For subsets A,B and* $A_i, i \in I$ *of a bitopological space* (X, τ_1, τ_2) *the following properties hold:*

(1) $(1,2)^*\text{-}V_p(A) \subseteq A$..

(2) $A \subseteq B$ *implies that* $(1,2)^*\text{-}V_p(A) \subseteq (1,2)^*\text{-}V_p(B)$.

(3) $(1,2)^*\text{-}V_p((1,2)^*\text{-}V_p(A)) = (1,2)^*\text{-}V_p(A)$.

(4) *If* A *is* $(1,2)^*$-*preclosed then* $A = (1,2)^*\text{-}V_p(A)$.

(5) $(1,2)^*\text{-}V_p(\cap\{A_i : i \in I\}) = \cap\{(1,2)^*\text{-}V_p(A_i) : i \in I\}$.

(6) $\cup\{(1,2)^*\text{-}V_p(A_i) : i \in I\} \subseteq (1,2)^*\text{-}V_p(\cup\{A_i : i \in I\})$.

Proof (1),(2),(4) and (6) follow from Definitions 3.1. To prove (3): By (1) and (2) we have $(1,2)^*\text{-}V_p((1,2)^*\text{-}V_p(A)) \subseteq (1,2)^*\text{-}V_p(A)$. If $x \in (1,2)^*\text{-}V_p(A)$ then for some $(1,2)^*$-preclosed set $F \subseteq A$, $x \in F$. Then $F \subseteq 1,2)^*\text{-}V_p(A)$ by Definition 3.1. Since F is $(1,2)^*$-preclosed, again by Definition 3.1, $x \in F \subseteq (1,2)^*\text{-}V_p((1,2)^*\text{-}V_p(A))$. To prove (5): Let $A = \cap\{A_i : i \in I\}$. By (2)

we have that $(1,2)^*$-$V_p(A) \subseteq \cap\{(1,2)^*$-$V_p(A_i) : i \in I\}$. If $x \in \cap\{(1,2)^*$-$V_p(A_i) : i \in I\}$, then for each $i \in I$, there exists a $(1,2)^*$-preclosed set F_i such that $F_i \subseteq A_i$ and $x \in F_i$. If $F = \cap\{F_i : i \in I\}$ then F is $(1,2)^*$-preclosed with $F \subseteq A$ and $x \in F$. Hence $x \in (1,2)^*$-$V_p(A)$ and so (5) holds.
Note that in general $(1,2)^*$-$\Lambda_p(A \cap B) \neq (1,2)^*$-$\Lambda_p(A) \cap (1,2)^*$-$\Lambda_p(B)$ as the following example shows.

Example 3.4 *Let* $X = \{a, b, c\}$
$\tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{a, c\}, X\}$;
$\tau_{1,2}$*-open sets* $= \{\phi, \{a, b\}, \{a, c\}, X\}$; $\tau_{1,2}$*-closed sets* $= \{\phi, \{c\}, \{b\}, X\}$;
Let $A = \{b\}$ *and* $B = \{c\}$. *Then* $(1,2)^*$-$\Lambda_p(A \cap B) = \phi$ *but* $(1,2)^*$-$\Lambda_p(A) \cap (1,2)^*$-$\Lambda_p(B) = \{b, c\}$.

Definition 3.5 *A subset* S *of a bitopological space* (X, τ_1, τ_2) *is called a*

(i) $(1,2)^*$*-pre-*Λ*-set or a* $(1,2)^*$-Λ_p*-set (resp.* $(1,2)^*$*-pre-*V*-set or a* $(1,2)^*$-V_p*-set) if* $(1,2)^*$-$\Lambda_p(S)$ *(resp.* $(1,2)^*$-$V_p(S)$*)* $= S$.

(ii) $(1,2)^*$-Λ*-set (resp.* $(1,2)^*$-V*-set) if* $(1,2)^*$-S^Λ *(resp.* $(1,2)^*$-S^V *)* $= S$, *where* $(1,2)^*$-$S^\Lambda = \cap\{O : S \subseteq O, O$ *is* $\tau_{1,2}$*-open* $\}$ *and* $(1,2)^*$-$S^V = \cup\{F : F \subseteq S, F$ *is* $\tau_{1,2}$*-closed*$\}$.

Clearly a subset S is a $(1,2)^*$-pre-Λ-set (resp.$(1,2)^*$-pre-V-set) if and only if it is an intersection (resp. union) of $(1,2)^*$-preopen (resp. $(1,2)^*$-preclosed) sets. A subset S is a $(1,2)^*$-Λ-set (resp.$(1,2)^*$-V-set) if and only if it is an intersection (resp. union) of $\tau_{1,2}$-open (resp.$\tau_{1,2}$-closed) sets. Hence $(1,2)^*$-Λ-sets and $(1,2)^*$-preopen sets are $(1,2)^*$-pre-Λ-sets ; $(1,2)^*$-V-sets and $(1,2)^*$-preclosed sets are $(1,2)^*$-pre-V-sets.

Remark 3.6 *Since* $(1,2)^*$-$\Lambda_p(S) = ((1,2)^*$-$V_p(X-S))^c$, *a subset* S *of a bitopological space* X *is a* $(1,2)^*$*-pre-*Λ*-set if and only if* $X - S$ *is a* $(1,2)^*$*-pre-*V*-set.*

Proposition 3.7 *For a bitopological space* (X, τ_1, τ_2) *the following properties hold:*

(1) ϕ *and* X *are* $(1,2)^*$*-pre-*Λ*-sets and* $(1,2)^*$*-pre-*V*-sets.*

(2) *Arbitrary union of* $(1,2)^*$*-pre-*Λ*-sets (resp.* $(1,2)^*$*-pre-*V*-sets) is a* $(1,2)^*$*-pre-*Λ*-set (resp.* $(1,2)^*$*-pre-*V*-set).*

(3) *Arbitrary intersection of* $(1,2)^*$*-pre-*Λ*-sets (resp.* $(1,2)^*$*-pre-*V*-sets) is a* $(1,2)^*$*-pre-*Λ*-set (resp.* $(1,2)^*$*-pre-*V*-set).*

Proof We shall consider the case of $(1,2)^*$-pre-Λ-sets. (1) is obvious. To prove (2) Let $\{A_i : i \in I\}$ be a family of $(1,2)^*$-pre-Λ-sets in the bitopological space (X, τ_1, τ_2). If $A = \cup\{A_i : i \in I\}$, then by Lemma 3.2, $(1,2)^*$-$\Lambda_p(A) = \cup\{(1,2)^*\text{-}\Lambda_p(A_i) : i \in I\} = \cup\{A_i : i \in I\} = A$. Hence A is a $(1,2)^*$-pre-Λ-set. To prove (3): Let $\{B_i : i \in I\}$ be a family of $(1,2)^*$-pre-Λ-sets in the bitopological space (X, τ_1, τ_2). If $B = \cap\{B_i : i \in I\}$, then by Lemma 3.2, $(1,2)^*$-$\Lambda_p(B) \subseteq \cap\{(1,2)^*\text{-}\Lambda_p(B_i) : i \in I\} = \cap\{B_i : i \in I\} = B$. The other inclusion follows from the definition of $(1,2)^*$-$\Lambda_p(B)$. Hence B is a $(1,2)^*$-pre-Λ-set.

Remark 3.8 *It follows from Proposition 3.7 that the family of all* $(1,2)^*$-*pre-*Λ-*sets (resp.* $(1,2)^*$-*pre-*V-*sets) is a topology on* X *containing all* $(1,2)^*$-*preopen (resp.* $(1,2)^*$-*preclosed) sets. Let* $\tau_{1,2}^{\Lambda_p}$ *(resp.*$\tau_{1,2}^{V_p}$*) denote the family of all* $(1,2)^*$-*pre-*Λ-*sets (resp.*$(1,2)^*$-*pre-*V-*sets). Clearly* $(X, \tau_{1,2}^{\Lambda_p})$ *and* $(X, \tau_{1,2}^{V_p})$ *are Alexandroff spaces, i.e. arbitrary intersection of open sets is open.*

Definition 3.9 *A bitopological space* (X, τ_1, τ_2) *is called* $(1,2)^*$-*pre-*T_1 *if to each pair of distinct points* x, y *of* (X, τ_1, τ_2) *there exist a* $(1,2)^*$-*preopen set* A *containing* x *but not* y *and a* $(1,2)^*$-*preopen set* B *containing* y *but not* x.

Lemma 3.10 *A bitopological space is* $(1,2)^*$-*pre-*T_1 *if and only if every singleton set* $\{x\}$ *in* X *is* $(1,2)^*$-*pre-closed.*

Proof Let (X, τ_1, τ_2) be $(1,2)^*$-pre-T_1. Suppose for some $x \in X$, $\{x\}$ is not $(1,2)^*$-preclosed. Then there exists a point $y \in (1,2)^*\text{-pcl}(\{x\})$ such that $y \neq x$. Since X is $(1,2)^*$-pre-T_1, there exists a $(1,2)^*$-preopen set G containing y but not x. Since $y \in (1,2)^*\text{-pcl}(\{x\})$ and G is a $(1,2)^*$-preopen set containing y, $\{x\} \cap G \neq \phi$.This implies $x \in G$, a contradiction. Conversely let every singleton set $\{x\}$ in X be $(1,2)^*$-preclosed. Let x and y be two distinct points of X. Now $\{x\}^c$ is a $(1,2)^*$-preopen set containing y but not x and $\{y\}^c$, a $(1,2)^*$-preopen set containing x but not y. Hence (X, τ_1, τ_2) is $(1,2)^*$-pre-T_1.

Lemma 3.11 *Every singleton subset of a bitopological space* (X, τ_1, τ_2) *is* $\tau_{1,2}$-*open or* $(1,2)^*$-*preclosed.*

Proof Let $x \in X$. If $\{x\}$ is not $\tau_{1,2}$-open then $\tau_{1,2}\text{-cl}(X - \{x\}) = X$. Therefore $X - \{x\}$ is $(1,2)^*$-preopen or $\{x\}$ is $(1,2)^*$-preclosed.

Corollary 3.12 *Every singleton subset of a bitopological space* (X, τ_1, τ_2) *is* $(1,2)^*$-*preopen or* $(1,2)^*$-*preclosed.*

Proof Obvious.

We now see additional characterisations of $(1,2)^*$-pre-T_1 spaces.

Theorem 3.13 *For a bitopological space* (X, τ_1, τ_2) *the following are equivalent:*

(1) (X, τ_1, τ_2) *is* $(1,2)^*$*-pre-*T_1.

(2) *Every subset of* X *is a* $(1,2)^*$*-pre-*Λ*-set.*

(3) *Every subset of* X *is a* $(1,2)^*$*-pre-*V*-set.*

(4) *Every* $\tau_{1,2}$ *open subset of* X *is a* $(1,2)^*$*-pre-*V*-set.*

Proof (2) $\Leftrightarrow$ (3) Follows from Remark 3.6. (1) $\Rightarrow$ (3) X is $(1,2)^*$-pre-T_1 implies that every singleton subset $\{x\}$ in X is $(1,2)^*$-preclosed. Therefore every subset of X is a union of $(1,2)^*$-preclosed sets and hence a $(1,2)^*$-pre-V-set. (3) $\Rightarrow$ (4) This is obvious. (4) $\Rightarrow$ (1). Let $x \in X$. By Lemma 3.11, $\{x\}$ is $\tau_{1,2}$-open or $(1,2)^*$-preclosed. If $\{x\}$ is $\tau_{1,2}$-open then by assumption, $\{x\}$ is a $(1,2)^*$-pre-V-set and so $(1,2)^*$-preclosed. Hence each singleton subset of X is $(1,2)^*$-preclosed and X is $(1,2)^*$-pre-T_1.

We now define $(1,2)^*$-generalized pre-Λ-sets and $(1,2)^*$-generalized pre-V-sets in a bitopological space in the following way.

Definition 3.14 *A subset* S *of a bitopological space* (X, τ_1, τ_2) *is called a*

(1) $(1,2)^*$*-generalized pre-*Λ*-set (briefly* $(1,2)^*$*-g-*Λ_p*-set) if* $(1,2)^*$-$\Lambda_p(S) \subseteq F$ *whenever* $S \subseteq F$ *and* F *is* $(1,2)^*$*-preclosed.*

(2) $(1,2)^*$*-generalized pre-*V*-set(briefly* $(1,2)^*$*-g-*V_p*-set) if* $(1,2)^*$-$V_p(S) \supseteq G$ *whenever* $G \subseteq S$ *and* G *is* $(1,2)^*$*-preopen.*

We shall see that we obtain nothing new.

Proposition 3.15 *Let* S *be a subset of the bitopological space* (X, τ_1, τ_2)*. Then*

(1) S *is a* $(1,2)^*$*-generalized pre-*Λ*-set if and only if* S *is a* $(1,2)^*$*-pre-*Λ*-set.*

(2) S *is a* $(1,2)^*$*-generalized pre-*V*-set if and only if* S *is a* $(1,2)^*$*-pre-*V*-set.*

Proof (1) Clearly every $(1,2)^*$-pre-Λ-set is a $(1,2)^*$-generalized pre-Λ-set. Now let S be a $(1,2)^*$-generalized pre-Λ-set. Suppose there exists $x \in (1,2)^*$-$\Lambda_p(S) - S$. Then $S \subseteq X - \{x\}$. We know that $\{x\}$ is $\tau_{1,2}$-open or $(1,2)^*$-preclosed. If $\{x\}$ is $\tau_{1,2}$-open, then $X - \{x\})$ is $\tau_{1,2}$-closed and hence $(1,2)^*$-preclosed. Since S is a $(1,2)^*$-g-Λ_p-set, $(1,2)^*$-$\Lambda_p(S) \subseteq X - \{x\}$, a contradiction. If $\{x\}$ is $(1,2)^*$-preclosed, then $X - \{x\}$ is $(1,2)^*$-preopen and so by Definition 3.1, $(1,2)^*$-$\Lambda_p(S) \subseteq X - \{x\}$, a contradiction. Hence S is a $(1,2)^*$-pre-Λ-set. (2) This is proved in a similar manner.

4 Properties of $(1,2)^*$-pre-Λ-sets and $(1,2)^*$-pre-V-sets

Proposition 4.1 *Let* (X, τ_1, τ_2) *be a bitopological space. Then*

(1) $(X, \tau_{1,2}^{\Lambda_p})$ *and* $(X, \tau_{1,2}^{V_p})$ *are always* $T_{1/2}$ *spaces.*

(2) *If* (X, τ_1, τ_2) *is* $(1,2)^*$-*pre*-T_1, *then both* $(X, \tau_{1,2}^{\Lambda_p})$ *and* $(X, \tau_{1,2}^{V_p})$ *are discrete spaces.*

(3) *The identity function id :* $(X, \tau_{1,2}^{\Lambda_p}) \to (X, \tau_i)$ *is continuous for* i = *1,2.*

(4) *The identity function id :* $(X, \tau_{1,2}^{V_p}) \to (X, \tau_i)$ *is contra-continuous for* i = *1,2.*

Proof (1) Let $x \in X$. Then $\{x\}$ is $\tau_{1,2}$-open or $(1,2)^*$-preclosed in X. If $\{x\}$ is $\tau_{1,2}$-open, then it is $(1,2)^*$-preopen. Therefore $\{x\} \in \tau_{1,2}^{\Lambda_p}$. If $\{x\}$ is $(1,2)^*$-preclosed then $X - \{x\}$ is $(1,2)^*$-preopen and so $X - \{x\} \in \tau_{1,2}^{\Lambda_p}$, i.e., $\{x\}$ is $\tau_{1,2}^{\Lambda_p}$-closed. Hence $(X, \tau_{1,2}^{\Lambda_p})$ is $T_{1/2}$. It can be proved in a similar manner that $(X, \tau_{1,2}^{V_p})$ is $T_{1/2}$. (2) This follows from Theorem 3.13. (3) If A is τ_i-open for i = 1 or 2, then A is $\tau_{1,2}$-open and therefore $(1,2)^*$-preopen. Hence $A \in \tau_{1,2}^{\Lambda_p}$. (4) If A is τ_i-open for i = 1 or 2, then A is $\tau_{1,2}$-open and therefore $(1,2)^*$-preopen. This implies that $X - A$ is $(1,2)^*$-preclosed and hence $X - A$ is $\tau_{1,2}^{V_p}$-open or A is $\tau_{1,2}^{V_p}$-closed.

Definition 4.2 *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is said to be* $(1,2)^*$-*dense if* $\tau_{1,2}$-*cl*$(A) = X$.

Definition 4.3 *A bitopological space* (X, τ_1, τ_2) *is said to be* $(1,2)^*$-*resolvable if* X *is the union of two disjoint* $(1,2)^*$-*dense subsets.*

Proposition 4.4 *If a bitopological space* (X, τ_1, τ_2) *is* $(1,2)^*$-*resolvable, then* $(X, \tau_{1,2}^{\Lambda_p})$ *and* $(X, \tau_{1,2}^{V_p})$ *are discrete spaces.*

Proof By Proposition 4.1, it is enough to show that (X, τ_1, τ_2) is $(1,2)^*$-pre-T_1. Let A and B be disjoint $(1,2)^*$-dense subsets of (X, τ_1, τ_2) such that $X = A \cup B$. Let $x \in X$. Without loss of generality let us assume that $x \in A$. Then $X - \{x\} = (A - \{x\}) \cup B$ is $(1,2)^*$-dense, hence $(1,2)^*$-preopen and $\{x\}$ is $(1,2)^*$-preclosed.

Proposition 4.5 *If* $(X, \tau_{1,2}^{\Lambda_p})$ *is connected then* (X, τ_1, τ_2) *is* $(1,2)^*$-*preconnected, i.e.* X *cannot be represented as a disjoint union of nonempty* $(1,2)^*$-*preopen subsets of* (X, τ_1, τ_2).

Proof Suppose that (X, τ_1, τ_2) is not $(1,2)^*$-preconnected. Then there exist nonempty disjoint $(1,2)^*$-preopen subsets A and B of X such that $X = A \cup B$. Since A and B are open in $(X, \tau_{1,2}^{\Lambda_p})$, we have a contradiction.

Remark 4.6 *The topological space* $(X, \tau_{1,2}^{\Lambda_p})$ *is connected if and only if* $(X, \tau_{1,2}^{V_p})$ *is connected.*

5 Applications

Definition 5.1 *A bitopological space* (X, τ_1, τ_2) *is called a*

(1) $(1,2)^*$-R_0*-space if for each* $\tau_{1,2}$*-open set* U *of* (X, τ_1, τ_2) *and each* $x \in U$, $\tau_{1,2}$-$cl(\{x\}) \subseteq U$.

(2) $(1,2)^*$*-pre-*R_0 *space if for each* $(1,2)^*$*-preopen set* U *of* (X, τ_1, τ_2) *and each* $x \in U$, $(1,2)^*$-$pcl(\{x\}) \subseteq U$.

Definition 5.2 *A bitopological space* (X, τ_1, τ_2) *is called a* $(1,2)^*$-T^{V_P}*-space if* $\tau_{1,2}^{\Lambda_p} = \tau_{1,2}^{V_p}$.

Lemma 5.3 *A bitopological space* (X, τ_1, τ_2) *is* $(1,2)^*$*-pre-*R_0 *if and only if it is* $(1,2)^*$*-pre-*T_1.

Proof Necessity. Let $x \in X$. By Lemma 3.11, $\{x\}$ is $\tau_{1,2}$-open or $(1,2)^*$-preclosed in X. If $\{x\}$ is $\tau_{1,2}$-open, then it is $(1,2)^*$-preopen. Since X is $(1,2)^*$-pre-R_0, $(1,2)^*$-pcl$(\{x\}) \subseteq \{x\}$ which implies $\{x\}$ is $(1,2)^*$-preclosed. Hence X is pre-T_1. Sufficiency. If (X, τ_1, τ_2) is $(1,2)^*$-pre-T_1 then every singleton subset of X is $(1,2)^*$-preclosed. Therefore every $(1,2)^*$-preopen set contains the $(1,2)^*$-preclosure of its singleton subsets and X is $(1,2)^*$-pre-R_0

Theorem 5.4 *For a bitopological space* (X, τ_1, τ_2) *the following are equivalent:*

(1) (X, τ_1, τ_2) *is* $(1,2)^*$*-pre-*R_0.

(2) $(X, \tau_{1,2}^{\Lambda_p})$ *is discrete.*

(3) $(X, \tau_{1,2}^{V_p})$ *is discrete.*

(4) *For each* $x \in X$, $\{x\}$ *is a* $(1,2)^*$-Λ_p*-set of* (X, τ_1, τ_2).

(5) *For each* $(1,2)^*$*-preopen set* P *of* X, $P = (1,2)^*$-$V_p(P)$.

(6) (X, τ_1, τ_2) *is a* $(1,2)^*$-T^{V_P}*-space.*

(7) $(X, \tau_{1,2}^{\Lambda_p})$ *is an* R_0*-space.*

Proof (1) $\Rightarrow$ (2) Follows from Lemma 5.3 and Proposition 4.1. (2) $\Leftrightarrow$ (3) Follows from Remark 3.6. (3) $\Rightarrow$ (4) For each $x \in X$, $\{x\}$ is $\tau_{1,2}^{\Lambda_p}$-open and $\{x\}$ is a $(1,2)^*$-Λ_p-set of (X, τ_1, τ_2). (4) $\Rightarrow$ (5) Let P be a $(1,2)^*$-preopen set. Let $x \in P^c$. By assumption $\{x\} = (1,2)^*$-$\Lambda_p(\{x\})$ and therefore $(1,2)^*$-$\Lambda_p(\{x\}) \subseteq P^c$. Hence $P^c \supseteq \cup\{(1,2)^*\text{-}\Lambda_p(\{x\}) : x \in P^c\} = (1,2)^*\text{-}\Lambda_p(\cup\{x : x \in P^c\}$ [by Lemma 3.2] $= (1,2)^*$-$\Lambda_p(P^c)$. This shows that $P^c = (1,2)^*$-$\Lambda_p(P^c)$ and by Remark 3.6, $P = (1,2)^*$-$V_p(P)$. (5) $\Rightarrow$ (6) By (5) $(1,2)^*$-PO$(X, \tau_1, \tau_2) \subseteq \tau_{1,2}^{V_p}$. First we show that $\tau_{1,2}^{\Lambda_p} \subseteq \tau_{1,2}^{V_p}$. Let A be any $(1,2)^*$-Λ_p-set ($(1,2)^*$-pre-Λ-set) of (X, τ_1, τ_2). Then $A = (1,2)^*$-$\Lambda_p(A) = \cap\{G : A \subseteq G, G \in (1,2)^*\text{-PO}(X, \tau_1, \tau_2)\}$. Since $(1,2)^*$-PO$(X, \tau_1, \tau_2) \subseteq \tau_{1,2}^{V_p}$, by Proposition 3.7 we have $A \in \tau_{1,2}^{V_p}$ and $\tau_{1,2}^{\Lambda_p} \subseteq \tau_{1,2}^{V_p}$. Next, let $A \in \tau_{1,2}^{V_p}$. Then $X - A \in \tau_{1,2}^{\Lambda_p} \subseteq \tau_{1,2}^{V_p}$. Therefore $A \in \tau_{1,2}^{\Lambda_p}$ and $\tau_{1,2}^{V_p} \subseteq \tau_{1,2}^{\Lambda_p}$. Hence (X, τ_1, τ_2) is a $(1,2)^*$-T^{V_P}-space. (6) $\Rightarrow$ (7) Let $V \in \tau_{1,2}^{\Lambda_p}$ and $x \in V$. Since (X, τ_1, τ_2) is a $(1,2)^*$-T^{V_P}-space, $V \in \tau_{1,2}^{V_p}$ and $V^c \in \tau_{1,2}^{\Lambda_p}$. Since $\{x\} \cap V^c = \phi$, $\tau_{1,2}^{\Lambda_p}$-cl$(\{x\}) \cap V^c = \phi$ and $\tau_{1,2}^{\Lambda_p}$-cl$(\{x\}) \subseteq V$. Hence $(X, \tau_{1,2}^{\Lambda_p})$ is R_0. (7) $\Rightarrow$ (1) Let $V \in (1,2)^*$-PO(X, τ_1, τ_2) and $x \in V$. Since $(1,2)^*$-PO$(X, \tau_1, \tau_2) \subseteq \tau_{1,2}^{\Lambda_p}$, by (7), $\tau_{1,2}^{\Lambda_p}$-cl$(\{x\} \subseteq V$. Since $\tau_{1,2}^{\Lambda_p}$-cl$(\{x\} \in \tau_{1,2}^{V_p}$, $\tau_{1,2}^{\Lambda_p}$-cl$(\{x\} = \cup\{F : F \subseteq \tau_{1,2}^{\Lambda_p}$-cl$(\{x\})$ *and* $F \in (1,2)^*$-PC$(X, \tau_1, \tau_2)\}$ and $x \in \tau_{1,2}^{\Lambda_p}$-cl$(\{x\}$. Therefore for some $F \in (1,2)^*$-PC(X, τ_1, τ_2), $x \in F$ and hence $(1,2)^*$-pcl$(\{x\}) \subseteq F \subseteq \tau_{1,2}^{\Lambda_p}$-cl$(\{x\} \subseteq V$. This shows that (X, τ_1, τ_2) is $(1,2)^*$-pre-R_0.

Corollary 5.5 *(X, τ_1, τ_2) is a $(1,2)^*$-pre-R_0 bitopological space if and only if $(X, \tau_{1,2}^{\Lambda_p})$ is an R_0-space.*

Definition 5.6 *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be*

(1) *$(1,2)^*$-Λ_p-continuous if $f^{-1}(V)$ is a $(1,2)^*$-pre-Λ-set ($(1,2)^*$-Λ_p-set) in (X, τ_1, τ_2) for every $\sigma_{1,2}$-open set V of (Y, σ_1, σ_2).*

(2) *$(1,2)^*$-pre-Λ_p-continuous if $f^{-1}(V)$ is a $(1,2)^*$-Λ_p-set in (X, τ_1, τ_2) for every $(1,2)^*$-preopen set V of (Y, σ_1, σ_2).*

(3) *$(1,2)^*$-Λ_p-irresolute if $f^{-1}(B)$ is a $(1,2)^*$-Λ_p-set in (X, τ_1, τ_2) for every $(1,2)^*$-Λ_p-set B of (Y, σ_1, σ_2).*

(4) *$(1,2)^*$-pre-Λ_p-open if $f(A)$ is a $(1,2)^*$-Λ_p-set in (Y, σ_1, σ_2) for every $(1,2)^*$-Λ_p-set A in (X, τ_1, τ_2).*

Definition 5.7 *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called a $(1,2)^*$-Λ_p-homeomorphism if f is a $(1,2)^*$-Λ_p-irresolute, $(1,2)^*$-pre-Λ_p-open and bijective.*

Theorem 5.8 *Let f be a function from (X, τ_1, τ_2) to (Y, σ_1, σ_2).*

(1) *If f is a $(1,2)^*$-Λ_p-irresolute injection and (Y, σ_1, σ_2) is a $(1,2)^*$-T^{V_P}-space, then (X, τ_1, τ_2) is a $(1,2)^*$-T^{V_P}-space.*

(2) *If f is a $(1,2)^*$-pre-Λ_p-open surjection and (X,τ_1,τ_2) is a $(1,2)^*$-T^{V_P}-space then (Y,σ_1,σ_2) is a $(1,2)^*$-T^{V_P}-space.*

(3) *If f is a $(1,2)^*$-Λ_p-homeomorphism then (X,τ_1,τ_2) is a $(1,2)^*$-T^{V_P}-space if and only if (Y,σ_1,σ_2) is a $(1,2)^*$-T^{V_P}-space.*

Proof (1) Since (Y,σ_1,σ_2) is a $(1,2)^*$-T^{V_P}-space, $(Y,\sigma_{1,2}^{\Lambda_p})$ is discrete by Theorem 5.4. Then $\{f(x)\}\in\sigma_{1,2}^{\Lambda_p}$ for every $x\in X$. Since f is $(1,2)^*$-Λ_p-irresolute, $f^{-1}(\{f(x)\})\in\tau_{1,2}^{\Lambda_p}$ for every $x\in X$. This implies $\{x\}\in\tau_{1,2}^{\Lambda_p}$ for every $x\in X$, since f is injective. Therefore $(X,\tau_{1,2}^{\Lambda_p})$ is discrete and by Theorem 5.4, (X,τ_1,τ_2) is a $(1,2)^*$-T^{V_P}-space. (2) let $y\in Y$. $\{f^{-1}(y)\}\neq\phi$ since f is surjective. Since $(X,\tau_{1,2}^{\Lambda_p})$ is discrete , $\{f^{-1}(y)\}\in\tau_{1,2}^{\Lambda_p}$ for every $y\in Y$. Since f is $(1,2)^*$-Λ_p-preopen, $f(\{f^{-1}(y)\})\in\sigma_{1,2}^{\Lambda_p}$ for every $y\in Y$. This implies $\{y\}\in\sigma_{1,2}^{\Lambda_p}$ for every $y\in Y$ or $(Y,\sigma_{1,2}^{\Lambda_p})$ is discrete. Hence (Y,σ_1,σ_2) is a $(1,2)^*$-T^{V_P}-space. (3) Follows from (1) and (2).

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