Gen. Math. Notes, Vol. 26, No. 1, January 2015, pp. 134-144
ISSN 2219-7184; Copyright © ICSRS Publication, 2015
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Marichev-Saigo-Maeda Fractional Integration Operators of Certain Special Functions

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(Received: 23-9-14 / Accepted: 21-11-14)

Abstract

The objective of this paper is to introduce generalized fractional calculus operators involving the Appell's function $F_3(.)$ to the product of the $\bar{\mathrm{H}}$-function and generalized polynomial set. Some special cases involving generalized hypergeometric function, Mittag-Leffler function, Riemann zeta function and generalized Wright Bessel function are presented to enhance the utility and importance of our main results.

Keywords: *Marichev-Saigo-Maeda fractional integral operators, $\bar{\mathrm{H}}$-function, Generalized Wright hypergeometric function, Mittag-Leffler function, Riemann zeta function, Generalized Wright Bessel function, Generalized polynomial set.*

1 Introduction

The fractional integral operators, involving various special functions with them, have found significant importance and applications in science and engineering.

During the last four decades fractional calculus has been applied to almost every field of science, engineering and mathematics. Many applications of fractional calculus can be found in fluid dynamics, stochastic dynamical system, non-linear control theory and astrophysics. A number of workers like Love [13], McBride [15], Kalla [6, 7], Kalla and Saxena [8, 9], Saigo [18, 19, 20], Saigo and Maeda [21], Kiryakova [11], etc. have studied in depth, the properties, applications and different extensions of various hypergeometric operators of fractional integration. A detailed account of such operators along with their properties and applications can be found in the research monographs by Miller and Ross [16], Kiryakova [11,12] and Debnath and Bhatta [25] etc. A useful generalized of the hypergeometric fractional integrals, including the Saigo operators [18] – [20], has been introduced by Marichev [14] (see details in Samko et al. [22, p.194, (10.47) and whole section 10.3] and later extended and studied by Saigo and Maeda [21, p.393, eqn. (4.12) and (4.13)] in terms of any complex order with Appell's function $F_3(.)$ in the kernel, as follows:

Let $\alpha,\alpha',\beta,\beta',\gamma\in C$ and $x>0$ then the generalized fractional calculus operators (the Marichev-Saigo-Maeda operators) involving the Appell's function [24] is defined as by the following equations:

$$(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}f)(x)=\frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x (x-t)^{\gamma-1}\,t^{-\alpha'}\,F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)\,dt$$
$$(R\,(\gamma)>0), \qquad \dots (1)$$

and

$$(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}f)(x)=\frac{x^{-\alpha'}}{\Gamma(\gamma)}\int_x^\infty (t-x)^{\gamma-1}\,t^{-\alpha}\,F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)\,dt$$
$$(R\,(\gamma)>0). \qquad \dots (2)$$

The left-hand side and right-hand sided generalized integration of the type (1) and (2) for a power function are given by:

$$(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\,x^{\rho-1})(x)=\Gamma\left[\begin{matrix}\rho,\rho+\gamma-\alpha-\alpha'-\beta,\rho+\beta'-\alpha'\\ \rho+\beta',\rho+\gamma-\alpha-\alpha',\rho+\gamma-\alpha'-\beta\end{matrix}\right]x^{\rho-\alpha-\alpha'+\gamma-1}, \qquad \dots (3)$$

where $\mathrm{Re}(\gamma)>0,\mathrm{Re}(\rho)>\max\{0,\mathrm{Re}(\alpha+\alpha'+\beta-\gamma),\mathrm{Re}(\alpha'-\beta')\}$ and

$$(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}\,x^{\rho-1})(x)=\Gamma\left[\begin{matrix}1-\rho-\gamma+\alpha+\alpha',1-\rho+\alpha+\beta'-\gamma,1-\rho-\beta\\ 1-\rho,1-\rho+\alpha+\alpha'+\beta'-\gamma,1-\rho+\alpha-\beta\end{matrix}\right]x^{\rho-\alpha-\alpha'+\gamma-1}, \qquad \dots (4)$$

where $\mathrm{Re}(\gamma) > 0, \mathrm{Re}(\rho) < 1 + \min\{\mathrm{Re}(-\beta), \mathrm{Re}(\alpha+\alpha'-\gamma), \mathrm{Re}(\alpha+\beta'-\gamma)\}$ the symbol occurring in (3) and (4) is given by

$$\Gamma\begin{bmatrix} a, b, c \\ d, e, f \end{bmatrix} = \frac{\Gamma(a)\,\Gamma(b)\Gamma(c)}{\Gamma(d)\Gamma(e)\Gamma(f)}.$$

$\overline{\mathrm{H}}$ -Function

The $\overline{\mathrm{H}}$-function, which is a generalization of the Fox H-function was introduced by Inayat-Hussain [1, 2] and studied by Buschman and Srivastava [26] and others, is defined and represented in the following manner:

$$\overline{H}^{M,N}_{P,Q}[z] = \overline{H}^{M,N}_{P,Q}\left[z \,\middle|\, \begin{matrix} (e_j, E_j; a_j)_{1,N}, (e_j, E_j)_{N+1,P} \\ (f_j, F_j)_{1,M}, (f_j, F_j; b_j)_{M+1,Q} \end{matrix} \right]$$

$$= \frac{1}{2\pi i}\int_L \theta(s)\, z^s\, ds, \qquad \dots (5)$$

where

$$\theta(s) = \frac{\prod_{j=1}^{M} \Gamma(f_j - F_j s) \prod_{j=1}^{N} \{\Gamma(1 - e_j + E_j s)\}^{a_j}}{\prod_{j=M+1}^{Q} \{\Gamma(1 - f_j + F_j s)\}^{b_j} \prod_{j=N+1}^{P} \Gamma(e_j - E_j s)}, \qquad \dots (6)$$

and the contour L is the line from c – i∞ to c +i∞, suitably intended to keep poles of $\Gamma(f_j - F_j s), j = 1,2,\dots,M$ to the right of the path and the singularities of $\{\Gamma(1 - e_j + E_j s)\}^{a_j}, j = 1,2,\dots,N$ to the left of the path. The following sufficient conditions for the absolute convergence of the defining integral for the $\overline{\mathrm{H}}$-function given by (5) have been given by Buschman and Srivastava [26]

$$T = \sum_{j=1}^{M} F_j + \sum_{j=1}^{N} |a_j E_j| - \sum_{j=M+1}^{Q} |b_j F_j| - \sum_{j=N+1}^{P} E_j > 0, \qquad \dots (7)$$

and

$$|\arg(z)| < \frac{1}{2}\pi\, T. \qquad \dots (8)$$

A Generalized Polynomial Set

Raizada has introduced and studied a generalized polynomial set and is defined by the following Rodrigues type formula [27, p.64, eq. (2.1.8)]:

$$S_n^{A,B,\tau}[x;r,h,q,A',B',m,k,\ell]=(A'x+B')^{-A}(1-\tau x^r)^{-\frac{B}{\tau}}$$

$$.\ T_{k,\ell}^{m+n}\left[(A'x+B')^{A+qn}(1-\tau x^r)^{\frac{B}{\tau}+hn}\right], \qquad \ldots (9)$$

where the differential operator $T_{k,\ell}$ being defined as

$$T_{k,\ell}=x^{\ell}\left(k+\ x\ \frac{d}{dx}\right). \qquad \ldots (10)$$

The explicit form of this generalized polynomial set [27, p.71, eq. (2.3.4)] is given by

$$S_n^{A,B,\tau}[x\ ;r,h,q,A',B',m,k,\ell]\ =\ B'^{qn}\ x^{\ell(m+n)}(1-\tau x^r)^{hn}\ \ell^{(m+n)}$$

$$.\sum_{p=0}^{m+n}\sum_{e=0}^{p}\sum_{\delta=0}^{m+n}\sum_{i=0}^{\delta}\frac{(-1)^{\delta}(-\delta)_i(A)_{\delta}(-p)_e}{p!\ \delta!i!\ e!}\frac{(-A-qn)_i}{(1-A-\delta)!}\left(-\frac{B}{\tau}-hn\right)_p$$

$$.\left(\frac{i+k+re}{\ell}\right)_{m+n}\left(\frac{-\tau x^r}{1-\tau x^r}\right)^p\left(\frac{A'x}{B'}\right)^i. \qquad \ldots (11)$$

It is to be noted that the polynomial set defined by (9) is very general in nature and it unifies and extends a number of classical polynomials introduced and studied by various research workers such as Chatterjea [30], Gould-Hopper [28], Singh and Srivastava [29] etc., some of the special cases of (9) are given by Raizada in a tabular form [27]. We shall require the following explicit form of (9), which will be obtained by taking $A'=1, B'=0$ and let $\tau\to 0$ in (9) and use the well known confluence principle

$$\left[\underset{|b|\to\infty}{\text{limit}}=(b)_n\left(\frac{x}{b}\right)^n=x^n\right], \qquad \ldots (12)$$

We arrive at the following polynomial set

$$S_n^{A,B,0}[x]=S_n^{A,B,0}[x;r,q,1,0,m,k,\ell]=x^{qn+\ell(m+n)}\ \ell^{m+n}\sum_{p=0}^{m+n}\sum_{e=0}^{p}\frac{(-p)_e}{p!e!}$$

$$. \left(\frac{A+qn+k+re}{\ell}\right)_{m+n} (Bx^{r})^{p}. \qquad \ldots (13)$$

2 Main Results

This section starts with the assumption of two theorems on the product of the $\overline{H}$-function and the generalized polynomial set associated with Saigo-Maeda fractional integral operators (1) and (2). These theorems can be used to establish image formulas for the $\overline{H}$-function in terms of the various special functions.

Theorem 2.1: *Let* $\alpha, \alpha', \beta, \beta', \gamma, \rho, \in C,\ x > 0,\ T > 0$ and $|\arg(z)| < \frac{1}{2}\pi T$ *be such that*

$$\mathrm{Re}(\gamma) > 0, \mathrm{Re}[\rho + \lambda qn + \lambda \ell(m+n) + \lambda rp] > \max[0, \mathrm{Re}(\alpha + \alpha' + \beta - \gamma, \mathrm{Re}(\alpha' - \beta')],$$

then there holds the formula

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho-1} S_n^{A,B,0}[z^{\lambda}; r, q, 1, 0, m, k, \ell] \overline{H}_{P,Q}^{M,N}\left[z^{\xi}\left|\begin{matrix}(e_j, E_j; a_j)_{1,N}, (e_j, E_j)_{N+1,P}\\(f_j, F_j)_{1,M}, (f_j, F_j; b_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{R-\alpha-\alpha'+\gamma-1}\, \ell^{m+n} \sum_{p=0}^{m+n} \sum_{e=0}^{p} \frac{(-p)_e}{p!e!} \left(\frac{A+qn+k+re}{\ell}\right)_{m+n} (B)^{p}$$

$$. \overline{H}_{P+3,Q+3}^{M,N+3}\left[x^{\xi}\left|\begin{matrix}(1-R,\xi;1),(1\text{-}R-\gamma+\alpha+\alpha'+\beta,\xi;1),(1-R-\beta'+\alpha',\xi;1),(e_j,E_j;a_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;b_j)_{M+1,Q},(1-R-\beta',\xi;1),(1-R-\gamma+\alpha+\alpha',\xi;1),(1-R-\gamma+\alpha'+\beta,\xi;1)\end{matrix}\right.\right], \qquad \ldots (14)$$

where

$$R = \rho + \lambda qn + \lambda \ell(m+n) + \lambda rp \,\cdot$$

Proof: In order to prove (14), first, we express the generalized polynomial set occurring on the left-hand side of (14), in the series form given by (13), and replace the $\overline{H}$-function in well-known Mellin-Barnes contour integral with the help of (5), and also using (1), then inter changing the order of summations and integration, which is permissible under the conditions stated with the Theorem 2.1, it takes the following form after a little simplification:

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho-1} S_n^{A,B,0}[z^{\lambda}; r, q, 1, 0, m, k, \ell] \overline{H}_{P,Q}^{M,N}\left[z^{\xi}\left|\begin{matrix}(e_j, E_j; a_j)_{1,N}, (e_j, E_j)_{N+1,P}\\(f_j, F_j)_{1,M}, (f_j, F_j; b_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= \ell^{m+n} \sum_{p=0}^{m+n} \sum_{e=0}^{p} \frac{(-p)_e}{p!e!} \left(\frac{A+qn+k+re}{\ell}\right)_{m+n}$$

$$.(B)^p \frac{1}{2\pi i}\int_L \theta(s)\{I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp+\xi s-1}\}(x)\, ds. \qquad \dots (15)$$

Finally, applying the known result (3), with ρ replaced by $\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp+\xi s$, we arrive at the desired result (14).

Theorem 2.2: *Let* $\alpha,\alpha',\beta,\beta',\gamma,\rho,\in C$ and $x>0, T>0$ and $|\arg(z)|<\frac{1}{2}\pi T$, *be such that* $\mathrm{Re}(\gamma)>0, \mathrm{Re}(\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp-\xi s))<1+\min[\mathrm{Re}(\alpha+\alpha'-\gamma),$ $\mathrm{Re}(\alpha+\beta'-\gamma), \mathrm{Re}(-\beta)]$, *then there holds the formula:*

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho-1} S_n^{A,B,0}[z^\lambda;r,q,1,0,m,k,\ell]\,\overline{H}_{P,Q}^{M,N}\left[\frac{1}{z^\xi}\middle|\begin{matrix}(e_j,E_j;a_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;b_j)_{M+1,Q}\end{matrix}\right]\right)(x)$$

$$=x^{R-\alpha-\alpha'+\gamma-1}\,\ell^{m+n}\sum_{p=0}^{m+n}\sum_{e=0}^{p}\frac{(-p)_e}{p!e!}\left(\frac{A+qn+k+re}{\ell}\right)_{m+n}(B)^p$$

$$.\ \overline{H}_{P+3,Q+3}^{M,N+3}\left[\frac{1}{x^\xi}\middle|\begin{matrix}(R+\gamma-\alpha-\alpha',\xi;1),(R-\alpha-\beta'+\gamma,\xi;1),(R+\beta,\xi;1),(e_j,E_j;a_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;b_j)_{M+1,Q},(R,\xi;1),(R-\alpha-\alpha'-\beta'+\gamma,\xi;1),(R-\alpha+\beta,\xi;1)\end{matrix}\right] \qquad \dots (16)$$

where

$$R=\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp\ .$$

Proof: By using the definitions (2), (5) and (13), and changing the order of summations and integration, which is permissible under the conditions stated with the Theorem 2.2, we get

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho-1} S_n^{A,B,0}[z^\lambda;r,q,1,0,m,k,\ell]\,\overline{H}_{P,Q}^{M,N}\left[\frac{1}{x^\xi}\middle|\begin{matrix}(e_j,E_j;a_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;b_j)_{M+1,Q}\end{matrix}\right]\right)(x)$$

$$=\ \ell^{m+n}\sum_{p=0}^{m+n}\sum_{e=0}^{p}\frac{(-p)_e}{p!e!}\left(\frac{A+qn+k+re}{\ell}\right)_{m+n}$$

$$.(B)^p \frac{1}{2\pi i}\int_L \theta(s)\{I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma} z^{\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp-\xi s-1}\}(x)\, ds. \qquad \dots (17)$$

Finally, applying the known result (4), with ρ replaced by $\rho+\lambda qn+\lambda\ell(m+n)+\lambda rp-\xi s$, we arrive at the desired result (16).

3 Special Cases

In view of the importance and usefulness of the theorems discussed in the last section, we mention some interesting special cases, which indicate manifold generality of the results obtained in this article.

(i) If we reduce the generalized polynomial set to unity and the $\overline{H}$-function to the generalized Wright hypergeometric function ${}_P\overline{\Psi}_Q$ [5], for this replacing

$$\overline{H}^{M,N}_{P,Q}\left[z^{\xi}\left|\begin{matrix}(e_j,E_j;a_j),(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;b_j)_{M+1,Q}\end{matrix}\right.\right]\text{by } \overline{H}^{1,P}_{P,Q+1}\left[-z^{\xi}\left|\begin{matrix}(1-e_j,E_j;a_j)_{1,P}\\(0,1),(1-f_j,F_j;b_j)_{1,Q}\end{matrix}\right.\right],$$

in equation (14), under the conditions stated for the Theorem 2.1, we arrive at the following interesting result:

$$\left(I^{\alpha,\alpha',\beta,\beta',\gamma}_{0,+}\, z^{\rho-1}\,\overline{H}^{1,P}_{P,Q}\left[-z^{\xi}\left|\begin{matrix}(1-e_j,E_j;a_j)_{1,P}\\(0,1),(1-f_j,F_j;b_j)_{1,Q}\end{matrix}\right.\right]\right)(x)$$

$$=x^{\rho-\alpha-\alpha'+\gamma-1}\;{}_{P+3}\overline{\Psi}_{Q+3}\left[x^{\xi}\left|\begin{matrix}(\rho,\xi;1),(\rho+\gamma-\alpha-\alpha'-\beta,\xi;1),(\rho+\beta'-\alpha',\xi;1)\\(\rho+\beta',\xi;1),(\rho+\gamma-\alpha-\alpha',\xi;1),(\rho+\gamma-\alpha'+\beta,\xi;1)\end{matrix}\right.\right]. \qquad \ldots (18)$$

Further, on setting $E_j = 1$ ($j = 1,\ldots,P$) and $F_j = 1$ ($j = 1,\ldots,Q$) in (18), then in similar way the $\overline{H}$-function reduces to the generalized hypergeometric function ${}_P\overline{F}_Q$ [5].

Similarly, we can find the result from the Theorem 2.2.

(ii) If we reduce $S^{A,B,0}_n$ polynomial to unity, the $\overline{H}$-function to Mittag-Leffler function, defined in the monograph by Erdélyi et al. [4] in (14), under the conditions stated for the Theorem 2.1, we obtain the following result

$$\left(I^{\alpha,\alpha',\beta,\beta',\gamma}_{0,+}\, z^{\rho-1}\,E_{\lambda',\sigma}(z^{\xi})\right)(x)=x^{\rho-\alpha-\alpha'+\gamma-1}$$

$$.\ \overline{H}^{1,4}_{4,5}\left[-x^{\xi}\left|\begin{matrix}(0,\xi;1),(1-\rho,\xi;1),(1-\rho-\gamma+\alpha+\alpha'+\beta,\xi;1),(1-\rho-\beta'+\alpha',1;1)\\(0,1),(1-\rho-\beta',\xi;1),(1-\rho-\gamma+\alpha+\alpha',\xi;1),(1-\rho-\gamma+\alpha'+\beta,1;1),(1-\sigma,\lambda';1)\end{matrix}\right.\right]. \qquad \ldots (19)$$

Similarly, we can find the result from the Theorem 2.2.

(iii) Now, we reduce the $\bar{H}$-function to the generalized Riemann zeta function and $S_n^{A,B,0}$ polynomial to unity in (14), under the conditions stated for the Theorem 2.1, where the generalized Riemann zeta function [3, p.27, 1.11, eq. (1), 5] is given by

$$\varphi(z,p',\eta) = \sum_{s=0}^{\infty} \frac{1}{(\eta+s)^{p'}} z^s, \qquad \dots (20)$$

In view of the definition of gamma function [3, 1.1(5)]

$$\Gamma(z) = s^z \int_0^{\infty} e^{-st} t^{z-1} dt, \qquad \mathrm{Re}(z) > 0, \qquad \dots (21)$$

Since for Re(s) > 0 and Re (v) > 0, we have from (21)

$$(v+n)^{-s}\,\Gamma(s) = \int_0^{\infty} e^{-(v+n)t}\, t^{s-1}\, dt, \qquad \dots (22)$$

By the definition of the generalized Riemann zeta function given by (20), and using the above equation leads to the result

$$\phi(z,p',\eta) = \sum_{s=0}^{\infty} \frac{1}{(\eta+s)^{p'}} z^s = \bar{H}_{2,2}^{1,2}\left[-z \,\middle|\, \begin{matrix} (0,1;1),(1-\eta,1;p') \\ (0,1),(-\eta,1;p') \end{matrix} \right]. \qquad \dots (23)$$

Then we arrive at the following interesting result after a little simplification

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\, z^{\rho-1}\, \varphi(z^{\xi},p',\eta)\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1}$$

$$.\ \bar{H}_{5,5}^{1,5}\left[-x^{\xi} \,\middle|\, \begin{matrix} (0,1;1),(1-\eta,1;p'),(1-\rho,\xi;1),(1-\rho-\beta'+\alpha',\xi;1),(1-\rho-\gamma+\alpha+\alpha'+\beta,\xi;1) \\ (0,1),(-\eta,1;p'),(1-\rho-\beta',\xi;1),(1-\rho-\gamma+\alpha+\alpha',\xi;1),(1-\rho-\gamma+\alpha'+\beta,\xi;1) \end{matrix} \right]. \qquad \dots (24)$$

Similarly, we can find the result from the Theorem 2.2.

(iv) Further, on reducing the $\bar{H}$-function to the generalized Wright Bessel function $\bar{J}_g^{\delta,\mu}(z)$ (see [5]) and $S_n^{A,B,0}$ polynomial set to unity in (14),under the conditions stated for the Theorem 2.1, we obtain the following result:

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\, z^{\rho-1}\, \bar{J}_g^{\delta,\mu}(z^{\xi})\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1}$$

$$.\ \bar{H}_{3,5}^{1,3}\left[x^{\xi} \,\middle|\, \begin{matrix} (1-\rho,\xi;1),(1-\rho-\gamma+\alpha+\alpha'+\beta,\xi;1),(1-\rho-\beta'+\alpha',\xi;1) \\ (0,1),(1-\rho-\beta',\xi;1),(1-\rho-\gamma+\alpha+\alpha',\xi;1),(1-\rho-\gamma+\alpha'+\beta,\xi;1),(-g,\delta;\mu) \end{matrix} \right]. \qquad \dots (25)$$

Similarly, we can find the result from the Theorem 2.2.

4 Conclusion

We conclude this investigation by remarking that many other properties of the $\overline{H}$-function and generalized polynomial set $S_n^{A,B,0}$ as well as the associated generalized fractional calculus operators $I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}$ and $I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}$ can be derived by applying the methods and techniques which we have discussed. Secondly, by suitably specializing the various parameters in the generalized polynomial set $S_n^{A,B,0}$ it reducing in terms of Gould-Hopper polynomial. Thus, the results presented in this paper would at once yield a very large number of results involving a large variety of special functions occurring in the problems of science and engineering.

Acknowledgement

The authors are very grateful to Professor H.M. Srivastava, University of Victoria, Canada for his kind helps and valuable suggestions in the preparation of this paper.

References

[1] A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals-I: Transformation and reduction formulae, *J. Phys. A.: Math. Gen.*, 20(1987), 4109-4117.

[2] A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals- II: A generalization of H-function, *J. Phys. A.: Math. Gen.*, 20(1987), 4119-4128.

[3] A. Erdélyi et al., *Higher Transcendental Functions (Vol. 1)*, McGraw-Hill, New York, (1953).

[4] A. Erdélyi et al., *Higher Transcendental Functions (Vol. 3)*, McGraw-Hill, New York, (1955).

[5] K.C. Gupta and R.C. Soni, On a basic formula involving the product of $\overline{H}$-function and Fox H-function, *J. Raj. Acad. Phy. Sci.*, 4(3) (2005), 157-164.

[6] S.L. Kalla, Integral operators involving Fox's H-function I, *Acta Mexicana Cienc. Tecn.*, 3(1969), 117-122.

[7] S.L. Kalla, Integral operators involving Fox's H-function II, *Acta Mexicana Cienc. Tecn.*, 7(1969), 72-79.

[8] S.L. Kalla and R.K. Saxena, Integral operators involving hypergeometric functions, *Math. Z.*, 108(1969), 231-234.

[9] S.L. Kalla and R.K. Saxena, Integral operators involving hypergeometric functions II, *Univ. Nac. Tucuman Rev. Ser.*, A24(1974), 31-36.

[10] S.D. Purohit, D.L. Suthar and S.L. Kalla, Merichev-Saigo-Maeda fractional integration operators of the Bessel functions, *Le Matematiche*, 67(2012), 21-32.

[11] V. Kiryakova, *Generalized Fractional Calculus and Applications*, Longman Scientific & Tech., Essex, (1994).

[12] V. Kiryakova, A brief story about the operators of the generalized fractional calculus, *Fract. Calc. Appl. Anal.*, 11(2008), 203-220.

[13] E.R. Love, Some integral equations involving hypergeometric functions, *Proc. Edin. Math. Soc.*, 15(1967), 169-198.

[14] O.I. Marichev, Volterra equation of Mellin convolution type with a Horn function in the kernel (In Russian), *Izv. ANBSSR Ser. Fiz.-Mat. Nauk*, 1(1974), 128-129.

[15] A.C. McBride, Fractional powers of a class of ordinary differential operators, *Proc. London Math. Soc. (III)*, 45(1982), 519-546.

[16] K.S. Miller and B. Ross, *An Introduction to the Fractional Calculus and Differential Equations*, A Wiley Intersciences Publilcation, John Wiley and Sons Inc., New York, (1993).

[17] A.P. Prudnikov, Yu. A. Brychkov and O.I. Marichev, *Integrals and Series: Special Functions (Vol. 1-5)*, Gordon and Breach, New York, (1986).

[18] M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, *Math. Rep. Kyushu Univ.*, 11(1978), 135-143.

[19] M. Saigo, A certain boundary value problem for the Euler-Darboux equation I, *Math. Japonica*, 24(1979), 377-385.

[20] M. Saigo, A certain boundary value problem for the Euler-Darboux equation II, *Math. Japonica*, 25(1980), 211-220.

[21] M. Saigo and N. Maeda, More generalization of fractional calculus, In *Transform Methods and Special Functions*, Verna, (1996), *Proc. 2nd Intern. Workshop*, P. Rusev, I. Dimovski and V. Kiryakova (Eds.), IMI-BAS, Sofia, (1998), 386-400.

[22] S. Samko, A. Kilbas and O. Marichev, *Fractional Integrals and Derivatives: Theory and Applications*, Gordon and Breach Sci. Publ., New York, (1993).

[23] R.K. Saxena and M. Saigo, Generalized fractional calculus of the H-function associated with the Appell function, *J. Frac. Calc.*, 19(2001), 89-104.

[24] H.M. Srivastava and P.W. Karlson, *Multiple Gaussian Hypergeometric Series*, Ellis Horwood Limited, New York, (1985).

[25] L. Debnath and D. Bhatta, *Integral Transforms and their Applications*, Chapman and Hall/CRC Press, Boca Raton FL, (2006).

[26] R.G. Buschman and H.M. Srivastava, The $\overline{\mathrm{H}}$-function associated with a certain class of Feynman integrals, *J. Phys. A: Math. Gen.*, 23(1990), 4707-4710.

[27] S.K. Raizada, A study of unified representation of special functions of mathematical physics and their use in statistical and boundary value problems, *Ph.D. Thesis*, Bundelkhand Univ. India, (1991).
[28] H.W. Gould and A.T. Hopper, Operational formulas connected with two generalizations of Hermite polynomials, *Duke Math. J.*, 29(1962), 51-63.
[29] R.P. Singh and K.N. Srivastava, A note on generalization of Leguerre and Humbert polynomials, *Ricerco (Napoli)*, 14(2) (1963), 11-21, Eratta, ibid, 15(1964), 63.
[30] S.K. Chatterjea, Quelques functions gnratrices des polynmes d'Hermite, du poiuntdevue del'a legebrede lie, *C.R. Acad. Sci. Paris Ser A-B*, 268(1969), A600-A602.