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Reliability Analysis and Mathematical Modeling of Washing System in Paper Industry

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Abstract

The objective of every industrial manager is that industry should be in an operative state for a long period of time. Keeping in mind the above objective the present work is done. In the present paper washing system which is an important system of paper industry have been discussed in detail. It involving many unit operations and processes Reliability, long run availability and M.T.T.F. have been studied. Reliability of the system can be analyzed by forming differential equations with the help of mnemonic rule and the transition diagram of the process. These differential equations can be solved using well known integrating factor technique. Recursive method has been used to calculate the long run availability of the process. The effects of failure and repair rates of different sub systems on long run availability have been studied through tables and graphs.

Keywords: *Reliability, Long run availability, Differential equation, M.T.T.F., Process industry, Mnemonic rule etc.*

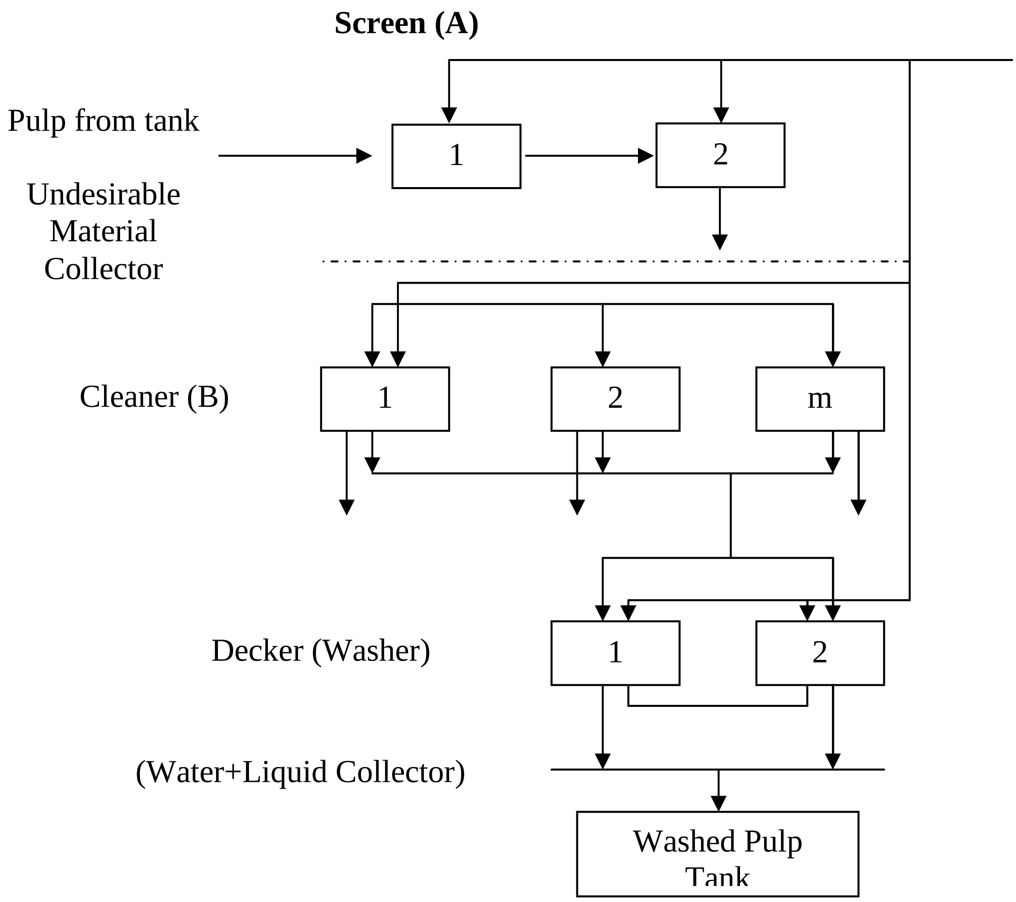
1 Introduction

Reliability Analysis can benefit the industry in terms of higher productivity and lower maintenance cost. This can also help the management to understand the effect of increasing/decreasing the repair rate of a particular component or subsystem on the overall system. These issues along with mathematical model are discussed in this study regarding washing subsystem, a subunit of paper industry. Several researchers for the last many years have discussed the various facts of reliability technology of the subsystems or systems in process industries at various level and a number of research papers have been published in this direction including Singh.J.(1982,84) Kumar.D.(1989,91) Mahajan.P. and Singh.J. (1996), Singh and Gupta (2004), Gupta Pawan and Jai Singh(2005), Singh T.P. and Satyavati (2007) and some other researchers applied reliability technology to various industrial systems obtaining important results.
Paper making process is a very complex process industry involving many unit operations and processes. It is a process industry in which we cannot bypass intermediate sub process. It consists of six subsystems: feeding, pulp preparation, washing, bleaching, screening and paper production. Washing system is an important sub system of paper industry. It consist of many units. During operation the complex subsystem may fail partially or completely due to fault in any unit. The failed unit are repaired or replaced as soon as possible to keep the system available for good output. We discuss in detail the availability of system under some assumptions, taking constant failure and repair times for various equipments of the system. The differential equations associated with the system are solved recursively. Expressions for various parameters of the system as reliability function, availability function followed by some special cases are discussed. The effects of failure and repair rates on Long run availability of different subsystems are studied by tables.

2 Washing System Description

It consists of the following subsystems:-
The screen (A) having two units in series. It is used to remove oversize uncooked and odd shape fibers from the pulp through straining. Failure of one unit causes complete failure of the subsystem.
The cleaner (B) having m units in parallel. The failure of any one unit reduces the efficiency of the subsystem which downs the quality of the paper and hence reduces the profit. The failed unit can be repaired by unskilled workers.
The Decker (D) having one main unit and a standby. Failure of both units causes complete failure of the subsystem.

The system description is shown in Block diagram 1.



(Block Diagram 1)

3 Assumptions

(a) The repaired/ replaced units are as good as new performance wise. Units are repaired or replaced upon failure only.

(b) Each subsystem is provided separate repair facility.

(c) Mean failure and repair rates of the units are constant over time unless otherwise stated about any change for equal time interval and are statistically independent.

(d) The probability of more than one subsystem failure / repair during the interval $(t, t+\Delta t)$ is zero. (e) The standby units (if any) are of same nature.

(f) The subsystem B failed through reduced state.

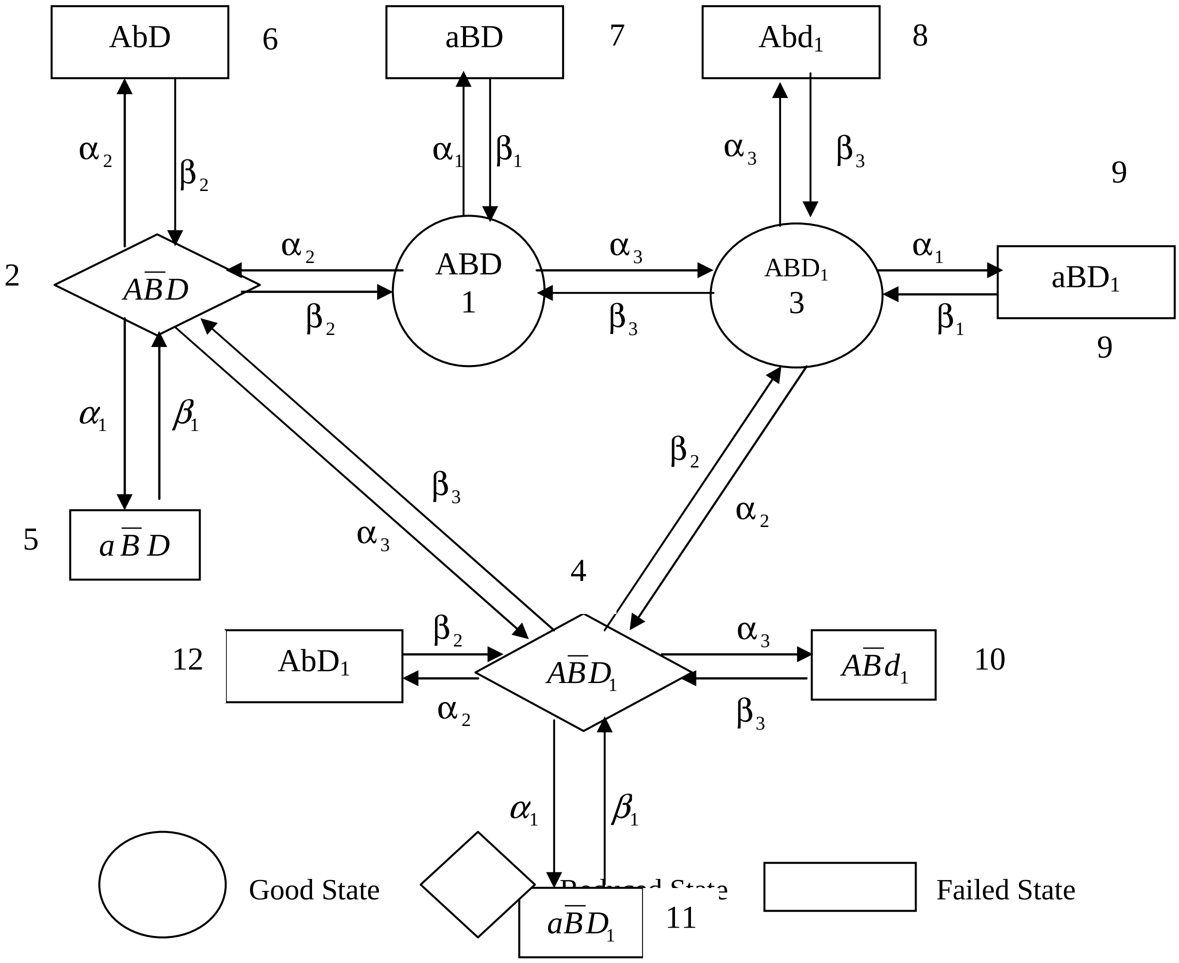
4 Notations

a,b,d_1	:	denote the failed state of the system.
$\overline{B}$	:	denote the reduced states of the system.
D_1	:	denote the failure of one unit of D.
α_i	:	denotes the respective failure rate of the unit.

β_i : denotes the respective repair rate of the unit.

$p_i(t)$: State probability at time t

Taking the above assumptions and notations, we obtain the transition diagram shown in fig.2



State Transition Diagram – 2

5 Mathematical Formulation

Probability considerations give the following differential difference equations.

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i\right]p_1(t)=\beta_1\,p_7(t)+\beta_2\,p_2(t)+\beta_3\,p_3(t) \quad \text{........................ (1)}$$

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i+\beta_2\right]p_2(t)=\alpha_2\,p_1(t)+\beta_1 p_5(t)+\beta_2\,p_6(t)+\beta_3\,p_4(t) \text{...... (2)}$$

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i+\beta_3\right]p_3(t)=\alpha_3\,p_1(t)+\beta_1 p_9(t)+\beta_2 p_4(t)+\beta_3\,p_8(t) \text{............ (3)}$$

$$\left[\frac{d}{dt}+\sum_{2}^{3}\beta_i+\sum_{1}^{3}\alpha_i\right]p_4(t)=\alpha_2 p_3(t)+\alpha_3\, p_2(t)+\beta_3 p_{10}(t)+\beta_1\, p_{11}(t)+\beta_2\, p_{12}(t) \quad \text{……...(4)}$$

$$\left[\frac{d}{dt}+\beta_1\right]p_5(t)=\alpha_1 p_2(t) \text{...................... (5)}$$

$$\left[\frac{d}{dt}+\beta_2\right]p_6(t)=\alpha_2\, p_2(t) \text{.................... (6)}$$

$$\left[\frac{d}{dt}+\beta_1\right]p_7(t)=\alpha_1 p_1(t) \text{...................... (7)}$$

$$\left[\frac{d}{dt}+\beta_3\right]p_8(t)=\alpha_3 p_3(t) \text{...................... (8)}$$

$$\left[\frac{d}{dt}+\beta_1\right]p_9(t)=\alpha_1 p_3(t) \text{...................... (9)}$$

$$\left[\frac{d}{dt}+\beta_3\right]p_{10}(t)=\alpha_3\, p_4(t) \text{..................... (10)}$$

$$\left[\frac{d}{dt}+\beta_1\right]p_{11}(t)=\alpha_1 p_4(t) \text{......................... (11)}$$

$$\left[\frac{d}{dt}+\beta_2\right]p_{12}(t)=\alpha_2\, p_4(t) \text{...................... (12)}$$

Initial condition {$p_i(o) = 1$ if i = 1

= 0 , otherwise}.

Solution of the equations (1) to (12)

$$p_1(t)=e^{-\sum_1^3\alpha_i}\left[\int\{\beta_1 p_7(t)+\beta_2 p_2(t)+\beta_3 p_3(t)\}e^{\sum_1^3\alpha_i}\,dt\right]+e^{-\sum_1^3\alpha_i}$$

$$p_2(t)=e^{-\lambda_1,t}\left[\int\{\alpha_2\, p_1(t)+\beta_1\, p_5(t)+\beta_2\, p_6(t)+\beta_3\, p_4(t)\}e^{\lambda_1 t}\,dt\right]$$

$$p_3(t)=e^{-\lambda_2 t}\left[\int\{\alpha_3\, p_1(t)+\beta_1 p_9(t)+\beta_2\, p_4(t)+\beta_3\, p_8(t)\}e^{\lambda_2 t}\,dt\right]$$

$$p_4(t)=e^{-\lambda_3 t}\left[\int\{\alpha_2\, p_3(t)+\alpha_3\, p_2(t)+\beta_3 p_{10}(t)+\beta_1\, p_{11}(t)+\beta_2\, p_{12}(t)\}e^{\lambda_3 t}\,dt\right]$$

$$p_5(t)=e^{-\beta_1 t}\left[\int\{\alpha_1\, p_2(t)\}e^{\beta_1 t}\,dt\right]$$

$$p_6(t)=e^{-\beta_2 t}\left[\int\{\alpha_2\, p_2(t)\}e^{\beta_2 t}dt\right]$$

$$p_7(t)=e^{-\beta_1 t}\left[\int\alpha_1 p_1(t)e^{\beta_1 t}\,dt\right]$$

$$p_8(t)=e^{-\beta_3 t}\left[\int\alpha_3 p_3(t)e^{\beta_3 t}\,dt\right]$$

$$p_9(t)=e^{-\beta_1 t}\left[\int\alpha_1 p_3(t)e^{\beta_1 t}\,dt\right]$$

$$p_{10}(t)=e^{-\beta_3 t}\left[\int\alpha_3\, p_4(t)e^{\beta_3 t}\,dt\right]$$

$$p_{11}(t)=e^{-\beta_1 t}\left[\int\alpha_1 p_4(t)e^{\beta_1 t}\,dt\right]$$

$$p_{12}(t) = e^{-\beta_2 t}\left[\int \alpha_2 p_4(t) e^{\beta_2 t}\, dt\right]$$

Where $\lambda_1 = \sum_1^3 \alpha_i + \beta_2$ $\lambda_2 = \sum_1^3 \alpha_i + \beta_3$ $\lambda_3 = \sum_2^3 \beta_i + \sum_1^3 \alpha_i$

All the probabilities are in terms of $p_1(t)$ which is terms given by (1)
The reliability of the subsystem is given by

$$R(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t)$$

6 Long Run Availability

Since the management is interested in long run availability of the system, we find the probabilities as follows, taking $\frac{d}{dt} \to o \quad as\ t \to \infty,\ \ p_i(t) \to p_i$

$$\sum_1^3 \alpha_i\ p_1 = \beta_1 p_7 + \beta_2 p_2 + \beta_3 p_3 \text{ ....} \ (13)$$

$$\left(\sum_1^3 \alpha_i + \beta_2\right) p_2 = \alpha_2 p_1 + \beta_1 p_5 + \beta_2 p_6 + \beta_3 p_4 \text{ ....} \ (14)$$

$$\left(\sum_1^3 \alpha_1 + \beta_3\right) p_3 = \alpha_3 p_1 + \beta_1 p_9 + \beta_2 p_4 + \beta_3 p_8 \text{ ....} \ (15)$$

$$\left(\sum_2^3 \beta_i + \sum_1^3 \alpha_i\right) p_4 = \alpha_2 p_3 + \alpha_3 p_2 + \beta_3 p_{10} + \beta_1 p_{11} + \beta_2 p_{12} \text{ ....} \ (16)$$

$$\beta_1 p_5 = \alpha_1 p_2 \text{ ....} \ (17) \qquad \beta_2 p_6 = \alpha_2\ p_2 \text{ ....} \ (18)$$

$$\beta_1 p_7 = \alpha_1 p_1 \text{ ....} \ (19) \qquad \beta_3 p_8 = \alpha_3 p_3 \text{ ....} \ (20)$$

$$\beta_1 p_9 = \alpha_1 p_3 \text{ ....} \ (21) \qquad \beta_3 p_{10} = \alpha_3 p_4 \text{ ....} \ (22)$$

$$\beta_1 p_{11} = \alpha_1 p_4 \text{ ....} \ (23) \qquad \beta_2 p_{12} = \alpha_2 p_4 \text{ ....} \ (24)$$

Initial condition p_1= 1, otherwise zero.
These equations are solved recursively in terms of p_1. Now p_1 is evaluated using normalizing condition $\sum_{i=1}^{12} p_i = 1$

The long run availability of the system is obtained as follows :

$A_3 = P_1 + P_2 + P_3 + P_4$

7 Non – Repairable Case

If system is non repairable, then putting $\beta_j = 0,$ for all j, we have

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i\right]p_1(t)=0 \quad \text{....................(25)}$$

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i\right]p_2(t)=\alpha_2 p_1(t) \text{....................(26)}$$

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i\right]p_3(t)=\alpha_3 p_1(t) \text{.................... (27)}$$

$$\left[\frac{d}{dt}+\sum_{1}^{3}\alpha_i\right]p_4(t)=\alpha_2 p_3(t)+\alpha_3 p_2(t) \text{.................... (28)}$$

$$\frac{d}{dt}p_5(t)=\alpha_1 p_2(t) \text{.................... (29)}$$

$$\frac{d}{dt}p_6(t)=\alpha_2 p_2(t) \text{.................... (30)}$$

$$\frac{d}{dt}p_7(t)=\alpha_1 p_1(t) \text{.................... (31)}$$

$$\frac{d}{dt}p_8(t)=\alpha_3 p_3(t) \text{.................... (32)}$$

$$\frac{d}{dt}p_9(t)=\alpha_1 p_3(t) \text{.................... (33)}$$

$$\frac{d}{dt}p_{10}(t)=\alpha_3 p_4(t) \text{.................... (34)}$$

$$\frac{d}{dt}p_{11}(t)=\alpha_1 p_4(t) \text{.................... (35)}$$

$$\frac{d}{dt}p_{12}(t)=\alpha_2 p_4(t) \text{.................... (36)}$$

Initial condition is $p_1(0)=1$, otherwise zero.
All the above probabilities are in terms of $p_1(t)$ which is given by solving equation (25)

$$p_1(t)=\exp\left(-\sum_{1}^{3}\alpha_i t\right)$$

8 Analysis of System

Using equation (13) to (24)

$$P_1=\left[\left(1+\frac{\alpha_2}{\beta_2}\right)\left(1+\frac{\alpha_3}{\beta_3}\right)\left(1+\frac{\alpha_1}{\beta_1}\right)+\left(\frac{\alpha_2}{\beta_2}\right)^2\left(1+\frac{\alpha_3}{\beta_3}\right)+\left(\frac{\alpha_3}{\beta_3}\right)^2\left(1+\frac{\alpha_2}{\beta_2}\right)\right]^{-1}$$

$$P_2=\frac{\alpha_2}{\beta_2}P_1 \qquad P_3=\frac{\alpha_3}{\beta_3}P_1 \qquad P_4=\frac{\alpha_2\alpha_3}{\beta_2\beta_3}P_1$$

$$P_5 = \frac{\alpha_1\alpha_2}{\beta_1\beta_2} P_1 \qquad P_6 = \left(\frac{\alpha_2}{\beta_2}\right)^2 P_1 \qquad P_7 = \frac{\alpha_1}{\beta_1} P_1$$

$$P_8 = \left(\frac{\alpha_3}{\beta_3}\right)^2 P_1 \qquad P_9 = \frac{\alpha_1\,\alpha_3}{\beta_1\,\beta_3} P_1 \qquad P_{10} = \left(\frac{\alpha_3}{\beta_3}\right)^2 \frac{\alpha_2}{\beta_2} P_1$$

$$P_{11} = \frac{\alpha_1\alpha_2\alpha_3}{\beta_1\beta_2\beta_3} P_1 \qquad P_{12} = \left(\frac{\alpha_2}{\beta_2}\right)^2 \frac{\alpha_3}{\beta_3} P_1$$

$$A = P_1 + P_2 + P_3 + P_4 = \left(1 + \frac{\alpha_2}{\beta_2}\right)\left(1 + \frac{\alpha_3}{\beta_3}\right) P_1$$

(a) Effect of failure rate (α_1) of screen and (α_2) of cleaner on long Run Availability

Taking $\alpha_3 = .02 \quad \beta_1 = .03 \quad \beta_2 = .04 \quad \beta_3 = .05$

Table1

$\alpha_1 \rightarrow$ $\alpha_2 \downarrow$	**.01**	**.02**	**.03**	**.04**	**.05**
.001	.690500	.561307	.472837	.408458	.359510
.002	.689671	.560748	.472441	.408164	.359290
.003	.688317	.559873	.471805	.407694	.358922
.004	.686494	.558647	.4709480	.407049	.358418
.005	.684226	.557173	.469886	.406261	.357802

(b) Effect of repair rate (β_1) of screen and (β_2) of cleaner on long run availability.

Taking $\alpha_1 = .01, \quad \alpha_2 = .001, \quad \alpha_3 = .02, \quad \beta_3 = .05$

Table 2

$\beta_1 \rightarrow$ $\beta_2 \downarrow$	**.01**	**.02**	**.03**	**.04**	**.05**
.01	.470948	.616	.686478	.728132	.755643

.02	.472441	.618557	.689655	.731707	.759494
.03	.472733	.619057	.690279	.732409	.760268
.04	.472837	.619235	.690499	.732657	.760517
.05	.472885	.619319	.690602	.732774	.760643

9 Concluding Remarks

1 The study of Table 1 shows that the failure rate α_1 of screen affects the availability more than failure rate α_2 of cleaner.

2 Study of Table II shows that the repair rate β_1 of screen affects the availability more than repair rate β_2 of cleaner.

Thus we can make an inference that Management should take more care about failure rate, repair rate of screen in order to improve availability of the system.

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