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Common Fixed Point Theorem of Compatible Mappings of Type (K) and Property (E.A.) in Fuzzy 2 - Metric Space

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Abstract

In this paper we prove a common fixed point theorem in fuzzy 2- metric space on six self-mappings using the concept of compatible of type (K) and Property (E.A.).

Keywords: *Compatible of type (K), Fixed point, Fuzzy-2 metric space, Property (E.A.).*

1 Introduction

In 1965, L.A. Zadeh [8] introduced the concept of fuzzy sets which became active field of research for many researchers. In 1975, Kramosil and Michalek [5] came in front with the concept of Fuzzy metric space based on fuzzy sets which were further modified by George and Veermani [2] with the help of t-norms. Many authors did good work and are still doing in proving fixed point theorems in Fuzzy metric space. Singh and Chauhan [4] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces. Manandhar al. [6] introduced the concept of compatible maps of type (k) in Fuzzy metric space and proved fixed point theorems. Recently, many authors [1, 7, 9, 3] have also studied the fixed point theory in the fuzzy 2-metric spaces.

2 Preliminaries

Definition 2.1: *[7] A binary operation* $* : [0,1] \times [0,1] \times [0,1] \to [0,1]$ *is called a continuous t-norm if* $([0,1],*)$ *is an abelian topological monodies with unit 1 such that* $a_1 * b_1 * c_1 \geq a_2 * b_2 * c_2$ *whenever* $a_1 \geq a_2$, $b_1 \geq b_2$, $c_1 \geq c_2$ *for all* a_1, b_1, c_1, a_2, b_2 *and* c_2 *are in [0,1]*

Definition 2.2: *[1] The 3-tuple* $(X, M,*)$ *is called a fuzzy 2-metric space if X is an arbitrary set,* $*$ *is a continuous t-norm, and M is a fuzzy set in* $X^3 \times [0,\infty)$ *satisfying the following conditions.*

(1) M(x, y, a, 0) = 0.
(2) M(x, y, a, t) = 1, for all t > 0 if and only if at least two of them are equal.
(3) M(x, y, a, t) = M (y, a, x, t) = M(a, y, x, t). (Symmetric)
(4) M(x, y, a, r+s+t)≥M (x, y, z, r)∗M(x, z, a, s)∗M(z, y, a, t) for all x, y, z, a ∈ X and r, s, t > 0.
(5) M (x, y, a, .) : [0, ∞) → [0,1] is left continuous for all x, y, z, a ∈ X and t > 0.
(6) $\lim_{n\to\infty} M(x, y, a, t) = 1$ for all x, y, z, a ∈ X and t > 0.

Definition 2.3: *[9] Self- mappings S and T of a fuzzy 2- metric space (X, M, ∗) are said to be compatible if and only if M (*STx_n, TSx_n*, z, t) → 1* $\forall\, t > 0$ *whenever* $\{x_n\}$ *is a sequence in X such that* Tx_n, Sx_n *→p for some p in X as* $n \to \infty$.

Definition 2.4: *[1] A Fuzzy 2-metric space* $(X, M,*)$ *is said to be complete if every Cauchy sequence in X converges in X.*

Definition 2.5: *[7] Let* $(X, M,*)$ *be a fuzzy 2-metric space. A sequence* $\{x_n\}$ *in fuzzy 2-metric space X is said to be convergent to a point* $x \in X$, $lim_{n\to\infty} M(x_n, x, a, t) = 1$ *for all* $a \in X$, *and* $t > 0$.

Definition 2.6: *[7] A sequence* $\{x_n\}$ *in fuzzy 2-metric space X is called a Cauchy sequence, if* $lim_{n\to\infty} M(x_{n+p}, x_n, a, t) = 1$ *for all* $a \in X$, *and* $t, p > 0$.

Definition 2.7: *[7] A function M is continuous in a Fuzzy 2- metric space, if and only if whenever for all* $a > X$ *and t > 0.* $x_n \to x$, $y_n \to y$, *then*
$lim_{n\to\infty} M(x_n, y_n, a, t) = M(x, y, a, t)$ *for all* $a > X$ *and t > 0.*

Definition 2.8. *[6] The self maps A and S of a fuzzy metric space $(X, M,*)$ are said to be compatible of type (K) iff* $lim_{n\to\infty} M(AAx_n, Sx, t) = 1$ *and* $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = x$ *for some x in X and* $t > 0$.

Definition 2.9: *[3] Tow pairs of self mappings (A, S) and (B, T) defined on a fuzzy metric space (X, M, ∗) are said to share the common property (E. A) if there exist a sequence* $\{x_n\}$ *and* $\{y_n\}$ *in X such that*

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = lim_{n\to\infty} By_n = lim_{n\to\infty} Ty_n = z \text{ for some } z > X.$$

Definition 2.10: *[9] Self- maps S and T of a fuzzy 2- metric space $(X, M,*)$ are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points that is if Sp = Tp for some p ∈ X then STp = TSp.*

Lemma: *[9] M(x, y, z, .) is non –decreasing for all x, y, z ∈ X.*

Lemma: *[9] Let (X, M, ∗) be a fuzzy 2-metric space. If there exists k ∈ (0, 1) such that M(x,y,z,kt)≥M(x,y,z,t) for all x,y,z∈X with z ≠ x, z ≠ y and t > 0, then x = y.*

3 Main Result

Theorem 3.1 : *Let (X, M, ∗) be a complete Fuzzy 2-metric space and A, B, P, Q, S and T be a self mapping of X satisfying the following condition:*

(i) P(X) ⊂ BT(X) and Q(X) ⊂ SA(X)
(ii) SA and BT are continuous.
(iii) (P, SA) and(Q, BT) compatible of type of (K)
(iv)

$$[1 + aM(SAx, Px, a\,kt)] * M(Px, Qy, a, kt) \geq a[M(Px, SAx, a, kt) * M(BTy, Qy, a, kt) * M(BTy, Px, a, kt)] + M(BTy, SAx, a, t) * M(Px, SAx, a, \propto t) * M(Qy, BTy, a, (2-\propto)t) * M(Qy, SAx, a, \propto t) * M(Px, BTy, a, (2-\propto)t)$$

For all $x, y, a \in X$, $\propto \in (0,2)$, $a \geq 0$ and t>0
(v) (P,SA) and (BT, Q) are commute,

Then A, B, P, Q, S and T have a unique common fixed point.

Proof: Since P(X) ⊂ BT(X) and Q(X) ⊂ SA(X), so for any $x_0 \epsilon X$, there exists $x_1 \epsilon X$ such that $Px_0 = BTx_1$ and for this x_1 ,there exists $x_2 \epsilon X$ such that $BTx_1 = SAx_2$. Inductively, we define a sequences$\{y_n\}$ in X such that

$y_{2n+1} = Px_{2n} = BTx_{2n+1}$ and $y_{2n+2} = Qx_{2n+1} = SAx_{2n+2}$ for all n=1,2,3...

Putting $x = x_{2n}$ and $y = x_{2n+1}$ with ∝=1 Form (iv), we get

$$[1 + aM(SAx_{2n}, Px_{2n}, a, kt)] * M(Px_{2n}, Qx_{2n+1}, a, kt) \geq a[M(Px_{2n}, SAx_{2n}, a, kt) * M(BTx_{2n+1}, Qx_{2n+1}, a, kt) * M(BTx_{2n+1}, Px_{2n}, a, kt)]$$

$$+M(BTx_{2n+1}, SAx_{2n}, a, t) * M(Px_{2n}, SAx_{2n}, a, t) * M(Qx_{2n+1}, BTx_{2n+1}, a, t)$$

$$* M(Qx_{2n+1}, SAx_{2n}, a, t) * M(Px_{2n}, BTx_{2n+1}, a, t)$$

$$[1 + aM(y_{2n}, y_{2n+1}, a, kt)] * M(y_{2n+1}, y_{2n+2}, a, kt)$$

$$\geq a[M(y_{2n+1}, y_{2n}, a, kt) * M(y_{2n+1}, y_{2n+2}, a, kt) * M(y_{2n+1}, y_{2n+1}, a, kt)]$$

$$+M(y_{2n+1}, y_{2n}, a, t) * M(y_{2n+1}, y_{2n}, a, t) * M(y_{2n+2}, y_{2n+1}, a, t)$$

$$* M(y_{2n+2}, y_{2n}, a, t) * M(y_{2n+1}, y_{2n+1}, a, t)$$

$$M(y_{2n+1}, y_{2n+2}, a, kt) \geq M(y_{2n+1}, y_{2n}, a, t) * M(y_{2n+1}, y_{2n}, a, t) *$$

$$M(y_{2n+2}, y_{2n+1}, a, t) * M(y_{2n+2}, y_{2n+1}, a, t) * M(y_{2n+1}, y_{2n}, a, t)$$

$$M(y_{2n+1}, y_{2n+2}, a, kt) \geq M(y_{2n+1}, y_{2n}, a, t) * M(y_{2n+2}, y_{2n+1}, a, t)$$

Similarly, we also have

$$M(y_{2n+2}, y_{2n+3}, a, kt) \geq M(y_{2n+2}, y_{2n+1}, a, t) * M(y_{2n+3}, y_{2n+2}, a, t)$$

In general for m = 1, 2, 3.....

$$M(y_{m+1}, y_{m+2}, a, kt) \geq M(y_{m+1}, y_m, a, t) * M(y_{m+2}, y_{m+1}, a, t)$$

Consequently, it follows that for m = 1, 2, 3.... and p = 1, 2, 3......

$$M(y_{m+1}, y_{m+2}, a, kt) \geq M(y_{m+1}, y_m, a, t) * M\left(y_{m+2}, y_{m+1}, a, \frac{t}{k^p}\right)$$

We have $M(y_{m+1}, y_{m+2}, a, kt) \geq M\left(y_{m+1}, y_m, a, \frac{t}{k}\right)$

$$\geq M\left(y_m, y_{m-1}, a, \frac{t}{k^2}\right) \geq \cdots \geq M\left(y_2, y_1, a, \frac{t}{k^n}\right) \to \infty$$

As n→∞, so $M(y_{m+1}, y_m, a, t) \to 1$ for any t>0. For each ε>0 and each t>0,

we can choose $m_0 \in N$ such that $M(y_{m+1}, y_m, a, t) > 1 - \varepsilon$ for all $m > m_0$ for $m_1, m_0 \in N$. Then $M(y_{m+1}, y_{m+2}, a, kt) \geq M(y_m, y_{m+1}, a, t)$

Hence by lemma $\{y_n\}$ is a Cauchy sequence in X. Since X is complete then $\{y_n\}$ converges to some point $z \in X$, and so that $\{Px_{2n}\}$, $\{BTx_{2n+1}\}$, $\{Qx_{2n+1}\}$ and $\{SAx_{2n+2}\}$ also converges to z. Since (P, SA) and (Q, BT) are compatible of type (K), we have

$PPx_{2n} \to SAz$, $(SA)SAx_{2n} \to Pz$, $QQx_{2n+1} \to BTz$ and $(BT)BTx_{2n+1} \to Qz$

Putting $x = Px_{2n}$ and $y = Qx_{2n+1}$ with ∝=1 Form (iv), we get

$$[1 + aM(SA(Px_{2n}), P(Px_{2n}), a, kt)] * M(P(Px_{2n}), Q(Qx_{2n+1}), a, kt)$$

$$\geq a[* M(BT(Qx_{2n+1}), Q(Qx_{2n+1}), a, kt) * M(BT(Qx_{2n+1}), P(Px_{2n}), a, kt)]$$

$$+M(BT(Qx_{2n+1}), SA(Px_{2n}), a, t) * M(P(Px_{2n}), SA(Px_{2n}), a, t)$$

$$* M(Q(Qx_{2n+1}), BT(Qx_{2n+1}), a, t) * M(Q(Qx_{2n+1}), SA(Px_{2n}), a, t)$$

$$* M(P(Px_{2n}), BT(Qx_{2n+1}), a, t)$$

Letting n→∞, we have

$$[1 + aM(SAz, SAz, a, kt)] * M(SAz, BTz, a, kt) \geq a[M(SAz, SAz, a, kt)$$

$$* M(BTz, BTz, a, kt) * M(BTz, SAz, a, kt)] + M(BTz, SAz, a, t)$$

$$* M(SAz, SAz, a, t) * M(BTz, BTz, a, t) * M(BTz, SAz, a, t) * M(SAz, BTz, a, t)$$

$$M(SAz, BTz, a, kt) \geq M(BTz, SAz, a, t) * M(BTz, SAz, a, t) * M(SAz, BTz, a, t)$$

Which implies that $M(SAz, BTz, a, kt) \geq M(BTz, SAz, a, t)$

Therefore by lemma, we have, SAz = BTz. (3.1)

Putting x = z and $y = Qx_{2n+1}$ with ∝=1 Form (iv), we get

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Q(Qx_{2n+1}), a, kt) \geq a[M(Pz, SAz, a, kt) *$$

$$M(BT(Qx_{2n+1}), Q(Qx_{2n+1}), a, kt) * M(BT(Qx_{2n+1}), Pz, a, kt)] +$$

$$M(BT(Qx_{2n+1}), SAz, a, t) * M(Pz, SAz, a, t) * M(Q(Qx_{2n+1}), BT(Qx_{2n+1}), a, t)$$

$$* M(Q(Qx_{2n+1}), SAz, a, t) * M(Pz, BT(Qx_{2n+1}), a, t)$$

Letting n→∞, we have

$$[1 + aM(BTz, Pz, a, kt)] * M(Pz, BTz, a, kt) \geq a[M(Pz, BTz, a, kt) *$$

$$M(BTz, BTz, a, kt) * M(BTz, Pz, a, kt)] + M(BTz, BTz, a, t)$$

$$* M(Pz, BTz, a, t) * M(BTz, BTz, a, t) * M(BTz, BTz, a, t) * M(Pz, BTz, a, t)$$

Which implies that $M(Pz, BTz, a, kt) \geq M(Pz, BTz, a, t)$.

Therefore by lemma, we have Pz = BTz (3.2)

Putting x = z and y = z, using (3.1), (3.2) with ∝=1 Form (iv), we get

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Qz, a, kt) \geq a[M(Pz, SAz, a, kt) *$$
$$M(BTz, Qz, a, kt) * M(BTz, Pz, a, kt)] + M(BTz, SAz, a, t) * M(Pz, SAz, a, t)$$
$$* M(Qz, BTz, a, t) * M(Qz, SAz, a, t) * M(Pz, BTz, a, t)$$

$$[1 + aM(Pz, Pz, a, kt)] * M(Pz, Qz, a, kt) \geq a[M(Pz, Pz, a, kt) *$$
$$M(Pz, Qz, a, kt) * M(Pz, Pz, a, kt)] + M(Pz, Pz, a, t) * M(Pz, Pz, a, t)$$
$$* M(Qz, Pz, a, t) * M(Qz, Pz, a, t) * M(Pz, Pz, a, t)$$

Which implies that $M(Pz, Qz, a, kt) \geq M(Pz, Qz, a, t)$.

Therefore by lemma, we have Pz = Qz (3.3)

Therefore form (3.1), (3.2) and (3.3), we have SAz = BTz = Pz = Qz (3.4)

Now we show that Qz = z. Putting $x = x_{2n}$ and y = z with ∝=1 Form (iv), we get

$$[1 + aM(SAx_{2n}, Px_{2n}, a, kt)] * M(Px_{2n}, Qz, a, kt) \geq a[M(Px_{2n}, SAx_{2n}, a, kt)$$
$$* M(BTz, Qz, a, kt) * M(BTz, Px_{2n}, a, kt)] + M(BTz, SAx_{2n}, a, t) *$$
$$M(Px_{2n}, SAx_{2n}, a, t) * M(Qz, BTz, a, t) * M(Qz, SAx_{2n}, a, t) * M(Px_{2n}, BTz, a, t)$$

Letting n→∞, we have

$$[1 + aM(z, z, a, kt)] * M(z, Qz, a, kt)$$
$$\geq a[M(z, z, a, kt) * M(Qz, Qz, a, kt) * M(Qz, z, a, kt)]$$
$$+M(Qz, z, a, t) * M(z, z, a, t) * M(Qz, Qz, a, t) * M(Qz, z, a, t) * M(z, Qz, a, t)$$

Which implies that $M(z, Qz, a, kt) \geq M(z, Qz, a, t)$.

Therefore by lemma, we have z = Qz.

Hence by (3.4), we have SAz = BTz = Pz = Qz = z (3.5)

Now to prove Az = z, putting x = Az, y = z with ∝ = 1 in (iv), we obtain
$$[1 + aM(SA(Az), P(Az), a, kt)] * M(P(Az), Qz, a, kt)$$

$$\geq a[M(P(Az), SA(Az), a, kt) * M(BTz, Qz, a, kt) * M(BTz, P(Az), a, kt)]$$

$$+M(BTz, SA(Az), a, t) * M(P(Az), SA(Az), a, t) * M(Qz, BTz, a, t)$$

$$* M(Qz, SA(Az), a, t) * M(P(Az), BTz, a, t)$$

$$[1 + aM(Az, Az, a, kt)] * M(Az, z, a, kt)$$

$$\geq a[M(Az, Az, a, kt) * M(z, z, a, kt) * M(z, Az, a, kt)]$$

$$+M(z, Az, a, t) * M(Az, Az, a, t) * M(z, z, a, t) * M(z, Az, a, t) * M(Az, z, a, t)$$

Which implies that $M(Az, z, a, kt) \geq M(Az, z, a, t)$.

Therefore by lemma, we have z = Az. Since SAz = z which implies that Sz = z. Again, Now to prove Tz = z, putting x = z, y = Tz with $\propto = 1$ in (iv), we obtain $[1 + aM(SAz, Pz, a, kt)] * M(Pz, Q(Tz), a, kt)$

$$\geq a[M(Pz, SAz, a, kt) * M(BT(Tz), Q(Tz), a, kt) * M(BT(Tz), Pz, a, kt)]$$

$$+M(BT(Tz), SAz, a, t) * M(Pz, SAz, a, t) * M(Q(Tz), BT(Tz), a, t) *$$

$$M(Q(Tz), SAz, a, t) * M(Pz, BT(Tz), a, t)$$

$$[1 + aM(z, z, a, kt)] * M(z, Tz, a, kt)$$

$$\geq a[M(z, z, a, kt) * M(Tz, Tz, a, kt) * M(Tz, z, a, kt)]$$

$$+M(Tz, z, a, t) * M(z, z, a, t) * M(Tz, Tz, a, t) * M(Tz, z, a, t) * M(z, Tz, a, t)$$

Which implies that $M(z, Tz, a, kt) \geq M(Tz, z, a, t)$.

Therefore by lemma, we have z = Tz. Since BTz = z which implies that Bz = z.

Thus combining all the above result, we have Az = Bz = Pz = z = Qz = Sz = Tz,

Hence z is common fixed point of A, B, P, Q, S and T.

Uniqueness: let u be an another common fixed point of A, B, P, Q, S and T. putting x = z, y = u with $\propto = 1$ in (iv), we obtain

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Qu, a, kt) \geq a[M(Pz, SAz, a, kt) *$$

$$M(BTu, Qu, a, kt) * M(BTu, Pz, a, kt)] + M(BTu, SAz, a, t) *$$

$$M(Pz, SAz, a, t) * M(Qu, BTu, a, t) * M(Qu, SAz, a, t) * M(Pz, BTu, a, t)$$

$[1 + aM(z,z,a,kt)] * M(z,u,a,kt)$

$\geq a[M(z,z,a,kt) * M(u,u,a,kt) * M(u,z,a,kt)]$

$+M(u,z,a,t) * M(z,z,a,t) * M(u,u,a,t) * M(u,z,a,t) * M(z,u,a,t)$

Which implies that $M(z,u,a,kt) \geq M(u,z,a,t)$.

Therefore by lemma, we have z = u.

Hence z is unique common fixed point of A, B, P, Q, S and T.

Corollary: *Let (X, M, ∗) be a complete Fuzzy 2-metric space and A, B, P and Q be a self mapping of X satisfying the following condition:*

(i) $P(X) \subset B(X)$ and $Q(X) \subset A(X)$
(ii) A and B are continuous
(iii) (P, A) and(Q, B) compatible of type of (K)
(iv)$[1 + aM(Ax,Px,a,kt)] * M(Px,Qy,a,kt) \geq$
$a[M(Px,Ax,a,kt) * M(By,Qy,a,kt) * M(By,Px,a,kt)] + M(By,Ax,a,t) *$
$M(Px,Ax,a,\propto t) * M(Qy,By,a,(2-\propto)t) * M(Qy,Ax,a,\propto t)$
$* M(Px,By,a,(2-\propto)t)$
For all $x, y \in X, \propto \in (0,2), a \geq 0$ and t>0
(v) (P, A) and (B, Q) are commute,
Then A, B, P and Q have a unique common fixed point.

Example: Let X = [4, 20] with the metric d defined by d(x, y) = $|x-y|$ define M(x, y, t) = $\frac{t}{d(x,y)}$ for all x, y ∈ X, t>0 clearly (X, M, *) is a complete fuzzy metric space define A, B, P, Q, S and T : X→Y as follows Px = 2 if x≤6, Px = 6 if x>6, Qx = 4 if x≤6 and Qx = 6 if x>10 and Sax, BTx = x for all x ∈ X. The A, B, P, Q, S and T satisfy all the conditions of the above theorem and have a unique common fixed point x = 4.

Theorem 3.2: *Let (X, M, ∗) be a Fuzzy 2-metric space and A, B, P, Q, S and T be a self mapping of X satisfying the following condition:*

(i) $P(X) \subset BT(X)$ and $Q(X) \subset SA(X)$
(ii) (P, SA) and(Q, BT) weakly compatible.
(iii)$[1 + aM(SAx,Px,a,kt)] * M(Px,Qy,a,kt) \geq$
$a[M(Px,SAx,a,kt) * M(BTy,Qy,a,kt) * M(BTy,Px,a,kt)]$
$+M(BTy,SAx,a,t) * M(Px,SAx,a,\propto t) * M(Qy,BTy,a,(2-\propto)t) *$
$M(Qy,SAx,a,\propto t) * M(Px,BTy,a,(2-\propto)t)$

For all $x, y \in X, \propto \in (0,2), a \geq 0$ and t>0

(iv) The pair (P, SA) and (BT, Q) are commute.

(v) The pair (P, SA) and (BT, Q) satisfy E.A. Property.
(vi) One of SA(X) or BT(X) is closed subset of X

Then A, B, P, Q, S and T have a unique common fixed point.

Proof: We assume that the pair (Q, BT) satisfy the E.A. property. Then there exists a sequence $\{x_n\}$ in X such tha$\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} BTx_n = z$t. for some $z \in X$. Since $Q(X) \subset SA(X)$, there exists a sequence $\{y_n\}$ in X such that $Qx_n = SAy_n$. Hence $\lim_{n\to\infty} SAy_n = z$. Also P(X) ⊂ BT(X), there exists a sequence $\{y'_n\}$ in X such that $Py'_n = BTx_n$. Hence $\lim_{n\to\infty} Py'_n = z$. Suppose that SA(X) is a closed subset of X. Then z = SAu for some $u \in X$. Subsequently, we have $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} BTx_n = \lim_{n\to\infty} Py'_n = \lim_{n\to\infty} SAy_n = z = SAu$. For some $u \in X$. Now, To prove that Pu = SAu. From (3) putting x = u and y = x_n with $\propto = 1$.

$$[1 + aM(SAu, Pu, a, kt)] * M(Pu, Qx_n, a, kt) \geq a[M(Pu, SAu, a, kt) * M(BTx_n, Qx_n, a, kt) * M(BTx_n, Pu, a, kt)] + M(BTx_n, SAu, a, t) * M(Pu, SAu, a, t) * M(Qx_n, BTx_n, a, t) * M(Qx_n, SAu, a, t) * M(Pu, BTx_n, a, t)$$

Letting n→∞, we have

$$[1 + aM(z, Pu, a, kt)] * M(Pu, z, a, kt) \geq a[M(Pu, z, a, kt) * M(z, z, a, kt) * M(z, Pu, a, kt)] + M(z, z, a, t) * M(Pu, z, a, t) * M(z, z, a, t) * M(z, z, a, t) * M(Pu, z, a, t)$$

Which implies that $M(Pu, z, a, kt) \geq M(Pu, z, a, t)$.

Therefore by lemma, we have Pu = z and hence Pu = SAu = z.

Since P(X) ⊂ BT(X), there exists a point v∈X such that Pu = z = BTv.

Now , we claim that BTv = Qv. From (3) putting x = u and y = v with $\propto = 1$,

we have

$$[1 + aM(SAu, Pu, a, kt)] * M(Pu, Qv, a, kt) \geq a[M(Pu, SAu, a, kt) * M(BTv, Qv, a, kt) * M(BTv, Pu, a, kt)] + M(BTv, SAu, a, t) * M(Pu, SAu, a, t) * M(Qv, BTv, a, t) * M(Qv, SAu, a, t) * M(Pu, BTv, a, t)$$

$$[1 + aM(z, z, a, kt)] * M(z, Qv, a, kt) \geq a[M(z, z, a, kt) * M(z, Qv, a, kt) * M(z, z, a, kt)] + M(z, z, a, t) * M(z, z, a, t) * M(Qv, z, a, t) * M(Qv, z, a, t) * M(z, z, a, t)$$

Which implies that $M(z, Qv, a, kt) \geq M(Qv, z, a, t)$.

Therefore by lemma, we have z = Qv. Hence we have BTv = Qv. Thus Pu = SAu = BTv = Qv = z. Since the pairs (P, SA) and (Q, BT) are weakly compatible points, respectively, we obtain Pz = P(SAu) = SA(Pu) = SAz and Qz = Q(BTv) = BT(Qv) = BTz. Now To prove that Pz = z. from (3) putting x = z and y = v with $\propto = 1$, we have

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Qv, a, kt) \geq a[M(Pz, SAz, a, kt) * M(BTv, Qv, a, kt) * M(BTv, Pz, a, kt)] + M(BTv, SAz, a, t) * M(Pu, SAz, a, t) * M(Qv, BTv, a, t) * M(Qv, SAz, a, t) * M(Pz, BTv, a, t)$$

$$[1 + aM(Pz, Pz, a, kt)] * M(Pz, z, a, kt) \geq a[M(Pz, Pz, a, kt) * M(z, z, a, kt) * M(z, Pz, a, kt)] + M(z, Pz, a, t) * M(Pz, Pz, a, t) * M(z, z, a, t) * M(z, Pz, a, t) * M(Pz, z, a, t)$$

Which implies that $M(Pz, z, a, kt) \geq M(Pz, z, a, t)$.

Therefore by lemma, we have z = Pz. Since Pz = SAz which implies that SAz = z.

Now to prove Qz = z, from (3) putting x = z and y = z with $\propto = 1$, we have

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Qz, a, kt) \geq a[M(Pz, SAz, a, kt) * M(BTz, Qz, a, kt) * M(BTz, Pz, a, kt)] + M(BTz, SAz, a, t) * M(Pz, SAz, a, t) * M(Qz, BTz, a, t) * M(Qz, SAz, a, t) * M(Pz, BTz, a, t)$$

$$[1 + aM(z, z, a, kt)] * M(z, Qz, a, kt) \geq a[M(z, z, a, kt) * M(Qz, Qz, a, kt) * M(Qz, z, a, kt)] + M(Qz, z, a, t) * M(z, z, a, t) * M(Qz, Qz, a, t) * M(Qz, z, a, t) * M(z, Qz, a, t)$$

Which implies that $M(z, Qz, a, kt) \geq M(z, Qz, a, t)$. Therefore by lemma, we have z = Qz. Since Qz = BTz. which implies that BTz = z.

Now to prove Az = z, from (3) putting x = Az and y = z with $\propto = 1$, we have

$$[1 + aM(SA(Az), P(Az), a, kt)] * M(P(Az), Qz, a, kt) \geq a[M(P(Az), SA(Az), a, kt) * M(BTz, Qz, a, kt) * M(BTz, P(Az), a, kt)] + M(BTz, SA(Az), a, t) * M(P(Az), SA(Az), a, t) * M(Qz, BTz, a, t) * M(Qz, SA(Az), a, t) * M(P(Az), BTz, a, t)$$

$$[1 + aM(Az, Az, a, kt)] * M(Az, z, a, kt) \geq a[M(Az, Az, a, kt) * M(z, z, a, kt) * M(z, Az, a, kt)] + M(z, Az, a, t) * M(Az, Az, a, t) * M(z, z, a, t) * M(z, Az, a, t) * M(Az, z, a, t)$$

Which implies that $M(Az, z, a, kt) \geq M(Az, z, a, t)$.

Therefore by lemma, we have Az = z. Since SAz = z which implies that Sz = z.

Now to prove Tz = z, from (3) putting x = z and y = Tz with $\propto = 1$, we have

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Q(Tz), a, kt) \geq a[M(Pz, SAz, a, kt) * M(BT(Tz), Q(Tz), a, kt) * M(BT(Tz), Pz, a, kt)] + M(BT(Tz), SAz, a, t) * M(Pz, SAz, a, t) * M(Q(Tz), BT(Tz), a, t) * M(Q(Tz), SAz, a, t) * M(Pz, BT(Tz), a, t)$$

$$[1 + aM(z, z, a, kt)] * M(z, Tz, a, kt) \geq a[M(z, z, a, kt) * M(Tz, Tz, a, kt) * M(Tz, z, a, kt)] + M(Tz, z, a, t) * M(z, z, a, t) * M(z, Tz, a, t) * M(Tz, z, a, t) * M(z, Tz, a, t)$$

Which implies that $M(z, Tz, a, kt) \geq M(z, Tz, a, t)$.

Therefore by lemma, we have z = Tz. Since z = BTz which implies that Bz = z.

Thus combining all the above result, we have Az = Bz = Pz = z = Qz = Sz = Tz

Hence z is common fixed point of A, B, P, Q, S and T.

Uniqueness: Let u be an another common fixed point of A, B, P, Q, S and T.

Putting x = z, y = u with $\propto = 1$ in (iv), we obtain

$$[1 + aM(SAz, Pz, a, kt)] * M(Pz, Qu, a, kt) \geq a[M(Pz, SAz, a, kt) * M(BTu, Qu, a, kt) * M(BTu, Pz, a, kt)] + M(BTu, SAz, a, t) * M(Pz, SAz, a, t) * M(Qu, BTu, a, t) * M(Qu, SAz, a, t) * M(Pz, BTu, a, t)$$

$$[1 + aM(z, z, a, kt)] * M(z, u, a, kt) \geq a[M(z, z, a, kt) * M(u, u, a, kt) * M(u, z, a, kt)] + M(u, z, a, t) * M(z, z, a, t) * M(u, u, a, t) * M(u, z, a, t) * M(z, u, a, t)$$

Which implies that $M(z, u, a, kt) \geq M(u, z, a, t)$.

Therefore by lemma, we have z = u.

Hence z is unique common fixed point of A, B, P, Q, S and T.

Corollary: Let (X, M, $*$) be a Fuzzy 2-metric space and P, Q, S and T be a self mapping of X satisfying the following condition:

(i) $P(X) \subset T(X)$ and $Q(X) \subset S(X)$
(ii) (P, S) and(Q, T) weakly compatible.
(iii) $[1 + aM(Sx, Px, a, kt)] * M(Px, Qy, a, kt) \geq a[M(Px, Sx, a, kt) * M(Ty, Qy, a, kt) * M(Ty, Px, a, kt)] + M(Ty, Sx, a, t) * M(Px, Sx, a, \propto t) * M(Qy, Ty, a, (2-\propto)t) * M(Qy, Sx, a, \propto t) * M(Px, Ty, a, (2-\propto)t)$
For all $x, y \in X$, $\propto \in (0,2)$, $a \geq 0$ and t>0

(iv) The pair (P, S) and (T, Q) are commute.
(v) The pair (P, S) and (T, Q) satisfy E.A. Property.
(vi) One of S(X) or T(X) is closed subset of X

Then P, Q, S and T have a unique common fixed point.

4 Conclusion

In this paper, we have presented common fixed point theorem for six self mappings in Fuzzy 2-metric spaces through concept of compatible of type (K) and Property (E.A.).

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