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Decompositions of Some Types of Soft Sets and Soft Continuity via Soft Ideals

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Abstract

In this paper, we extend the notions of γ-operation introduced in [12], by using the soft ideal notions. We introduce the notions of $\tilde{I}$-open soft sets, pre-$\tilde{I}$-open soft sets, α-$\tilde{I}$-open soft sets, semi-$\tilde{I}$-open soft sets and β-$\tilde{I}$-open soft sets to soft topological spaces. We study the relations between these different types of subsets of soft topological spaces with soft ideal. We also introduce the concepts of $\tilde{I}$-continuous soft, pre-$\tilde{I}$-continuous soft, α-$\tilde{I}$-continuous soft, semi-$\tilde{I}$-continuous soft and β-$\tilde{I}$-continuous soft functions and discuss their properties in detail.

Keywords: *Soft set, Soft topological space, Soft interior, Soft closure, Open soft, Closed soft, γ-operation,$\tilde{I}$-open soft sets, Pre-$\tilde{I}$-open soft sets, α-$\tilde{I}$-open soft sets, Semi-$\tilde{I}$-open soft sets, β-$\tilde{I}$-open soft sets.*

1 Introduction

The concept of soft sets was first introduced by Molodtsov [23] in 1999 as a general mathematical tool for dealing with uncertain objects. In [23, 24], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on.
After presentation of the operations of soft sets [20], the properties and applications of soft set theory have been studied increasingly [4, 16, 24, 26]. In recent years, many interesting applications of soft set theory have been expanded by

embedding the ideas of fuzzy sets [1, 3, 5, 8, 18, 19, 20, 21, 24, 25, 27, 32]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [9].
Recently, in 2011, Shabir and Naz [28] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. Hussain and Ahmad [11] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Min in [22] investigate some properties of these soft separation axioms mentioned in [28]. Banu and Halis in [7] studied some properties of soft Hausdorff space. Kandil et al.[12] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notions of γ-operation. The notion of soft ideal is initiated for the first time by Kandil et al.[13]. They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \tau, E, \tilde{I})$. In this paper we extend these different types of subsets to soft topological spaces with soft ideal.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 *[23] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \phi$ i.e*
$F_A = \{F(e) : e \in A \subseteq E,\ F : A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2 *[20] Let $F_A,\ G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if*

(1) *$A \subseteq B$, and*

(2) $F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3 *[20] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4 *[4] The complement of a soft set (F, A), denoted by $(F, A)'$, is defined by $(F, A)' = (F', A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$, $\forall\, e \in A$ and F' is called the soft complement function of F. Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.*

Definition 2.5 *[28] The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.*

Definition 2.6 *[28] Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.7 *[20] A soft set (F, A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).*

Definition 2.8 *[20] A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.*

Definition 2.9 *[20] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), \ e \in A - B, \\ G(e), \ e \in B - A, \\ F(e) \cup G(e), \ e \in A \cap B \end{cases} .$$

Definition 2.10 *[20] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X with the same set of parameter E. We denote the family of these soft sets by $SS(X)_E$.*

Definition 2.11 *[33] Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.*

(1) *The union of L is the soft set (H, E), where $H(e) = \bigcup_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i\in I}(F_i, E) = (H, E)$.*

(2) *The intersection of L is the soft set (M, E), where $M(e) = \bigcap_{i\in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i\in I}(F_i, E) = (M, E)$.*

Definition 2.12 *[28] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

(1) *$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,*

(2) *the union of any number of soft sets in τ belongs to τ,*

(3) *the intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.13 *[28] Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft closure of (F, A), denoted by $cl(F, A)$ is the intersection of all closed soft super sets of (F, A). Clearly $cl(F, A)$ is the smallest closed soft set over X which contains (F, A) i.e*
$cl(F, A) = \tilde{\cap}\{(H, C) : (H, C) \text{ is closed soft set and } (F, A)\tilde{\subseteq}(H, C)\})$.

Definition 2.14 *[33] Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft interior of (G, B), denoted by $int(G, B)$ is the union of all open soft subsets of (G, B). Clearly $int(G, B)$ is the largest open soft set over X which contained in (G, B) i.e*
$int(G, B) = \tilde{\cup}\{(H, C) : (H, C) \text{ is an open soft set and } (H, C)\tilde{\subseteq}(G, B)\})$.

Definition 2.15 *[33] The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.*

Proposition 2.16 *[29] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.*

Definition 2.17 *[33] A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood (briefly: nbd) of the soft point $x_e \tilde{\in} X_E$ if there exists an open soft set (H, E) such that $x_e \tilde{\in} (H, E) \tilde{\subseteq} (G, E)$.*
A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft (F, E) if there exists an open soft set (H, E) such that $(F, E) \tilde{\in} (H, E) \tilde{\subseteq} (G, E)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$.

Theorem 2.18 *[30] Let (X, τ, E) be a soft topological space. For any soft point x_e, $x_e \tilde{\in} cl(F, A)$ if and only if each soft neighborhood of x_e intersects (F, A).*

Definition 2.19 *[2] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a mapping. Then;*

(1) *If $(F, A) \in SS(X)_A$. Then the image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that*
$$f_{pu}(F)(b) = \begin{cases} \cup_{x \in p^{-1}(b) \cap A}\ u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.

(2) *Let $(G, B) \in SS(Y)_B$. The inverse image of (G, B) under f_{pu}, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that*
$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition 2.20 *[33] Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

(1) *The function f_{pu} is called continuous soft (cts-soft) if $f_{pu}^{-1}(G, B) \in \tau_1\ \forall\ (G, B) \in \tau_2$.*

(2) *The function f_{pu} is called open soft if $f_{pu}(G, A) \in \tau_2 \forall\ (G, A) \in \tau_1$.*

Definition 2.21 *[10]. A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions*

(1) *$A \in I$ and $B \in I \Rightarrow A \cup B \in I$,*

(2) *$A \in I$ and $B \subseteq A \Rightarrow B \in I$,*
i.e. I is closed under finite unions and subsets.

Definition 2.22 *[13] Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if*

(1) *$(F, E) \in \tilde{I}$ and $(G, E) \in \tilde{I} \Rightarrow (F, E) \tilde{\cup} (G, E) \in \tilde{I}$,*

(2) *$(F, E) \in \tilde{I}$ and $(G, E) \tilde{\subseteq} (F, E) \Rightarrow (G, E) \in \tilde{I}$,*
i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Definition 2.23 *[13] Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then*

$$(F,E)^*(\tilde{I},\tau)\ (or F_E^*) = \tilde{\cup}\{x_e \in \varepsilon : O_{x_e}\tilde{\cap}(F,E)\tilde{\notin}\tilde{I}\ \forall\ O_{x_e} \in \tau\}$$

is called the soft local function of (F, E) with respect to $\tilde{I}$ and τ, where O_{x_e} is a τ-open soft set containing x_e.

Theorem 2.24 *[13] Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then the soft closure operator $cl^* : SS(X)_E \to SS(X)_E$ defined by:*

$$cl^*(F,E) = (F,E)\tilde{\cup}(F,E)^*. \tag{1}$$

satisfies Kuratwski's axioms.

Definition 2.25 *[14] A soft set $F_E \in SS(X, E)$ is called supra generalized closed soft with respect to a soft ideal $\tilde{I}$ (supra-$\tilde{I}g$-closed soft) in a supra soft topological space (X, μ, E) if $cl^s F_E \setminus G_E \in \tilde{I}$ whenever $F_E\tilde{\subseteq}G_B$ and $G_E \in \mu$.*

3 Subsets of Soft Topological Spaces via Soft Ideal

In this section we extend some special subsets of a soft topological space (X, τ, E) mentioned in [12] in a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$. In any soft topological space with soft ideal $(X, \tau, E, \tilde{I})$ we introduce an operator γ generalizing these previous kinds of open soft sets.

Definition 3.1 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is called $\tilde{I}$-open soft if $(F, E)\tilde{\subseteq}int((F, E)^*(\tilde{I}, \tau))$. We denote the set of all $\tilde{I}$-open soft sets by $\tilde{I}OS(X, \tau, E, \tilde{I})$, or when there can be no confusion by $\tilde{I}OS(X)$ and the set of all $\tilde{I}$-closed soft sets by $\tilde{I}CS(X, \tau, E, \tilde{I})$, or $\tilde{I}CS(X)$.*

Definition 3.2 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(X, \tau^*, E, \tilde{I})$ be its $\star$-soft topological space. A mapping $\gamma : SS(X)_E \to SS(X)_E$ is said to be an operation on $OS(X)$ if $F_E\tilde{\subseteq}\gamma(F_E)\ \forall\ F_E \in OS(X)$. The collection of all γ-open soft sets is denoted by $OS(\gamma) = \{F_E : F_E\tilde{\subseteq}\gamma(F_E), F_E \in SS(X)_E\}$. Also, the complement of γ-open soft set is called γ-closed soft set, i.e*
$CS(\gamma) = \{F'_E : F_E\ is\ a\ \gamma - open\ soft\ set, F_E \in SS(X)_E\}$ is the family of all γ-closed soft sets.

Definition 3.3 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Different cases of γ-operations on $SS(X)_E$ are as follows:*

(1) *If $\gamma = int(cl^*)$, then γ is called pre-$\tilde{I}$-open soft operator. We denote the set of all pre-$\tilde{I}$-open soft sets by $P\tilde{I}OS(X, \tau, E, \tilde{I})$, or $P\tilde{I}OS(X)$ and the set of all pre-I-closed soft sets by $P\tilde{I}CS(X, \tau, E, \tilde{I})$, or $PICS(X)$.*

(2) *If $\gamma = int(cl^*(int))$, then γ is called α-$\tilde{I}$-open soft operator. We denote the set of all α-$\tilde{I}$-open soft sets by $\alpha\tilde{I}OS(X, \tau, E, \tilde{I})$, or $\alpha\tilde{I}OS(X)$ and the set of all α-$\tilde{I}$-closed soft sets by $\alpha\tilde{I}CS(X, \tau, E, \tilde{I})$, or $\alpha\tilde{I}CS(X)$.*

(3) *If $\gamma = cl^*(int)$, then γ is called semi-$\tilde{I}$-open soft operator. We denote the set of all semi-$\tilde{I}$-open soft sets by $S\tilde{I}OS(X, \tau, E, \tilde{I})$, or $S\tilde{I}OS(X)$ and the set of all semi-$\tilde{I}$-closed soft sets by $SICS(X, \tau, E, \tilde{I})$, or $S\tilde{I}CS(X)$.*

(4) *If $\gamma = cl(int(cl^*))$, then γ is called β-$\tilde{I}$-open soft operator. We denote the set of all β-$\tilde{I}$-open soft sets by $\beta\tilde{I}OS(X, \tau, E, \tilde{I})$, or $\beta\tilde{I}OS(X)$ and the set of all β-$\tilde{I}$-closed soft sets by $\beta\tilde{I}CS(X, \tau, E, \tilde{I})$, or $\beta\tilde{I}CS(X)$.*

Theorem 3.4 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $\gamma : SS(X)_E \to SS(X)_E$ be an operation on $OS(X)$.*
If $\gamma \in \{int(cl^*), int(cl^*(int)), cl^*(int), cl(int(cl^*))\}$. Then

(1) *Arbitrary soft union of γ-open soft sets is γ-open soft.*

(2) *Arbitrary soft intersection of γ-closed soft sets is γ-closed soft.*

Proof.

(1) We give the proof for the case of pre-open soft operator i.e $\gamma = int(cl^*)$. Let $\{F_{jE} : j \in J\} \subseteq P\tilde{I}OS(X)$. Then $\forall\ j \in J$, $F_{jE} \tilde{\subseteq} int(cl^*(F_{jE}))$. It follows that $\tilde{\bigcup}_j F_{jE} \tilde{\subseteq} \tilde{\bigcup}_j int(cl^*(F_{jE}))$
$\tilde{\subseteq} int(\tilde{\bigcup}_j cl^*(F_{jE})) = int(\tilde{\bigcup}_j(((F_{jE})^* \tilde{\cup} (F_{jE}))) = int(\tilde{\bigcup}_j (F_{jE})^* \tilde{\cup} (\tilde{\bigcup}_j (F_{jE}))) =$
$int((\tilde{\bigcup}_j (F_{jE}))^*$
$\tilde{\cup} (\tilde{\bigcup}_j (F_{jE}))) = int(cl^*(\tilde{\bigcup}_j (F_{jE}))$ from [[13], Theorem 3.2]. Hence $\tilde{\bigcup}_j F_{jE} \in P\tilde{I}OS(X)\ \forall\ j \in J$. The rest of the proof is similar.

(2) Immediate.

Remark 3.5 *A finite soft intersection of pre-$\tilde{I}$-open (resp. semi-$\tilde{I}$-open,β-$\tilde{I}$-open) soft sets need not to be pre-$\tilde{I}$-open (resp. semi-$\tilde{I}$-open,β-$\tilde{I}$-open) soft, as it can be seen from the following examples.*

Examples 3.6 **(1)** *Let* $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ *and* $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E)$ *are soft sets over* X *defined by* $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2, h_3\}$ *and* $F_3(e) = \{h_1, h_2, h_3\}$. *Then* (X, τ, E) *is a soft topological space over* X. *Let* $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, *where* $(I_1, E), (I_2, E)$, (I_3, E) *are soft sets over* X *defined by* $I_1(e) = \{h_1\}$, $I_2(e) = \{h_4\}$ *and* $I_3(e) = \{h_1, h_4\}$. *Hence the soft sets* (G, E) *and* (H, E) *which defined by* $G(e) = \{h_1, h_2, h_4\}$, $H(e) = \{h_1, h_3, h_4\}$, *are pre-*$\tilde{I}$*-open soft sets of* $(X, \tau, E, \tilde{I})$, *but their soft intersection* $(G, E)\tilde{\cap}(H, E) = (M, E)$, *where* $M(e) = \{h_1, h_4\}$, *is not pre-*$\tilde{I}$*-open soft set.*

(2) *Let* $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ *and* $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ *are soft sets over* X *defined by* $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2\}$, $F_3(e) = \{h_1, h_2\}$, $F_4(e) = \{h_2, h_3\}$, *and* $F_5(e) = \{h_1, h_2, h_3\}$. *Then* (X, τ, E) *is a soft topological space over* X. *Let* $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, *where* $(I_1, E), (I_2, E)$, (I_3, E) *are soft sets over* X *defined by* $I_1(e) = \{h_1\}$, $I_2(e) = \{h_2\}$ *and* $I_3(e) = \{h_1, h_2\}$. *Hence the soft sets* (G, E) *and* (H, E) *which defined by* $G(e) = \{h_1, h_2\}$, $H(e) = \{h_2, h_3\}$, *are semi-*$\tilde{I}$*-open soft sets of* $(X, \tau, E, \tilde{I})$, *but their soft intersection* $(G, E)\tilde{\cap}(H, E) = (M, E)$, *where* $M(e) = \{h_2\}$, *is not semi-*$\tilde{I}$*-open soft set.*

(3) *Let* $X = \{h_1, h_2, h_3\}$, $E = \{e\}$ *and* $\tau = \{\tilde{X}, \tilde{\phi}, (F, E)\}$ *where* (F, E) *is a soft set over* X *defined by* $F(e) = \{h_1, h_2\}$. *Let* $\tilde{I} = \{\tilde{\phi}, (G, E)\}$ *be soft ideal over* X *where* (G, E) *is a soft set over* X *defined by* $G(e) = \{h_3\}$. *Then the soft sets* (K, E) *and* (H, E) *which defined by* $K(e) = \{h_1, h_3\}$ *and* $H(e) = \{h_2, h_3\}$ *are* β*-*$\tilde{I}$*-open soft sets of* $(X, \tau, E, \tilde{I})$, *but their soft intersection* $(K, E)\tilde{\cap}(H, E) = (M, E)$, *where* $M(e) = \{h_3\}$, *is not a* β*-*$\tilde{I}$*-open soft set.*

Proposition 3.7 *Let* $(X, \tau, E, \tilde{I})$ *be a soft topological space with soft ideal and* $(F, E) \in SS(X)_E$. *Then we have:*

(1) *If* $\tilde{I} = \{\tilde{\phi}\}$, *then* (F, E) *is pre-*$\tilde{I}$*- (resp. semi-*$\tilde{I}$*-,* α*-*$\tilde{I}$*- and* β*-*$\tilde{I}$*-) open soft* $\Leftrightarrow$ *it is pre- (resp. semi-,* α*- and* β*-) open soft.*

(2) *If* $\tilde{I} = SS(X)_E$, *then* (F, E) *is pre-*$\tilde{I}$*- (resp. semi-*$\tilde{I}$*-,* α*-*$\tilde{I}$*- and* β*-*$\tilde{I}$*-) open soft* $\Leftrightarrow$ *it is* τ*-open soft.*

Proof. Immediate.

Definition 3.8 *Let* $(X, \tau, E, \tilde{I})$ *be a soft topological space with soft ideal and and* $(F, E) \in SS(X)_E$. *Then,*

(1) *x_e is called an γ-soft interior point of (F,E) if $\exists$ $(G,E) \in OS(\gamma)$ such that $x_e \in (G,E)\tilde{\subseteq}(F,E)$, the set of all γ-interior soft points of (F,E) is called the γ-soft interior of (F,E) and is denoted by $\gamma - Sint(F,E)$ consequently, $\gamma - Sint(F,E) = \tilde{\bigcup}\{(G,E) : (G,E)\tilde{\subseteq}(F,E),\ (G,E) \in OS(\gamma)\}$.*

(2) *x_e is called a*

γ-cluster soft point of (F,E) if $(F,E)\tilde{\cap}(H,E) \neq \tilde{\phi}$ $\forall$ $(H,E) \in OS(\gamma)$. The set of all γ-cluster soft points of (F,E) is called γ-soft closure of (F,E) and is denoted by $\gamma - Scl(F,E)$ consequently, $\gamma - Scl(F,E) = \tilde{\bigcap}\{(H,E) : (H,E) \in CS(\gamma),\ (F,E)\tilde{\subseteq}(H,E)\}$.

Theorem 3.9 *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal, $\gamma : SS(X)_E \rightarrow SS(X)_E$ be one of the operations in Definition 3.10 and $(F,E),(G,E) \in SS(X)_E$. Then the following properties are satisfied for the γ S-interior operators, denoted by $\gamma - Sint$.*

(1) *$\gamma - Sint(\tilde{X}) = \tilde{X}$ and $\gamma - Sint(\tilde{\phi})) = \tilde{\phi}$.*

(2) *$\gamma - Sint(F,E)\tilde{\subseteq}(F,E)$.*

(3) *$\gamma - Sint(F,E)$ is the largest γ-open soft set contained in (F,E).*

(4) *if $(F,E)\tilde{\subseteq}(G,E)$, then $\gamma - Sint(F,E)\tilde{\subseteq}\gamma - Sint(G,E)$.*

(5) *$\gamma - Sint(\gamma - Sint(F,E)) = \gamma - Sint(F,E)$.*

(6) *$\gamma - Sint(F,E)\tilde{\cup}\gamma - Sint(G,E)\tilde{\subseteq}\gamma - Sint[(F,E)\tilde{\cup}(G,E)]$.*

(7) *$\gamma - Sint[(F,E)\tilde{\cap}(G,E)]\tilde{\subseteq}\gamma - Sint(F,E)\tilde{\cap}\gamma - Sint(G,E)$.*

Proof. Immediate.

Theorem 3.10 *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal, $\gamma : SS(X)_E \rightarrow SS(X)_E$ be one of the operations in Definition 3.10 and $(F,E),(G,E) \in SS(X)_E$. Then the following properties are satisfied for the γ-soft closure operators, denoted by $\gamma - Scl$.*

(1) *$\gamma - Scl(\tilde{X}) = \tilde{X}$ and $\gamma - Scl(\tilde{\phi}) = \tilde{\phi}$.*

(2) *$(F,E)\tilde{\subseteq}\gamma - Scl(F,E)$.*

(3) *$\gamma - Scl(F,E)$ is the smallest γ-closed soft set contains (F,E).*

(4) *if $(F,E)\tilde{\subseteq}(G,E)$, then $\gamma - Scl(F,E)\tilde{\subseteq}\gamma - Scl(G,E)$.*

(5) *$\gamma - Scl(\gamma - Scl(F,E)) = \gamma - Scl(F,E)$.*

(6) $\gamma - Scl(F,E)\tilde{\cup}\gamma - Scl(G,E)\tilde{\subseteq}\gamma - Scl[(F,E)\tilde{\cup}(G,E)]$.

(7) $\gamma - Scl[(F,E)\tilde{\cap}(G,E)]\tilde{\subseteq}\gamma - Scl(F,E)\tilde{\cap}\gamma - Scl(G,E)$.

Proof. Immediate.

Remark 3.11 *Note that the family of all γ-open soft sets on a soft topological space with soft ideal $(X,\tau,E,\tilde{I})$ forms a supra soft topology, which is a collection of soft sets contains $\tilde{X},\tilde{\phi}$ and closed under arbitrary soft union.*

4 Relations between Subsets of Soft Topological Spaces via Soft Ideal

In this section we introduce the relations between some special subsets of a soft topological space with soft ideal $(X,\tau,E,\tilde{I})$.

Theorem 4.1 *Every $\tilde{I}$-open soft set is pre-$\tilde{I}$-open soft.*

Proof. Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $(F,E)\in \tilde{I}OS(X)$. Then $(F,E)\tilde{\subseteq}int((F,E)^*(\tilde{I},\tau))\tilde{\subseteq}int((F,E)\cup(F,E)^*(\tilde{I},\tau)) = int(cl^*(F,E))$. Hence $(F,E)\in P\tilde{I}OS(X)$.

The following example show that the converse of Theorem 4.1 is not true in general.

Example 4.2 *Let $X=\{h_1,h_2,h_3,h_4\}$, $E=\{e\}$, $\tau=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$, where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$ are soft sets over X defined by $F_1(e)=\{h_1\}$, $F_2(e)=\{h_2\}$, $F_3(e)=\{h_1,h_2\}$, $F_4(e)=\{h_1,h_2,h_4\}$ and $\tilde{I}=\{\tilde{\phi},(I_1,E),(I_2,E),(I_3,E)\}$ be a soft ideal over X where $(I_1,E),(I_2,E),(I_3,E)$ are soft sets over X defined by $I_1(e)=\{h_1\}$, $I_2(e)=\{h_3\}$ and $I_3(e)=\{h_1,h_3\}$. Then the soft set (F_3,E) is pre-$\tilde{I}$-open soft but it is not $\tilde{I}$-open soft.*

Theorem 4.3 *Every pre-$\tilde{I}$-open soft set is pre-open soft.*

Proof. Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $(F,E)\in P\tilde{I}OS(X)$. Then $(F,E)\tilde{\subseteq}int(cl^*(F,E)) = int((F,E)\cup(F,E)^*(\tilde{I},\tau))\tilde{\subseteq}int((F,E)\cup cl(F,E)) = int(cl(F,E))$. Hence $(F,E)\in POS(X)$.

The following example show that the converse of Theorem 4.3 is not true in general.

Example 4.4 *Let $X=\{h_1,h_2,h_3\}$, $E=\{e\}$, $\tau=\{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$, where $(F_1,E),(F_2,E)$ are soft sets over X defined by $F_1(e)=\{h_1\}$ and $F_2(e)=\{h_2,h_3\}$ and $\tilde{I}=\{\tilde{\phi},(I_1,E),(I_2,E)(I_3,E)\}$ be a soft ideal over X where (I_1,E), $(I_2,E),(I_3,E)$ are soft sets over X defined by $I_1(e)=\{h_1\}$, $I_2(e)=\{h_3\}$ and $I_3(e)=\{h_1,h_3\}$. Then the soft set (H,E), where (H,E) is a soft set over X defined by $H(e)=\{h_1,h_3\}$, is pre-open soft but it is not pre-$\tilde{I}$-open soft.*

Theorem 4.5 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in P\tilde{I}OS(X)$. Then we have:*

(1) $cl(int(cl^*(F, E))) = cl(F, E)$,

(2) $cl^*(int(cl^*(F, E))) = cl^*(F, E)$.

Proof.

(1) Let $(F, E) \in P\tilde{I}OS(X)$, then $(F, E)\tilde{\subseteq} int\,(cl^*(F, E))$, hence $cl(F, E)\tilde{\subseteq} cl(int\,(cl^*(F, E)))\tilde{\subseteq} cl(int(cl(F, E)))\tilde{\subseteq} cl(F, E)$. Thus, $cl(int(cl^*(F, E))) = cl(F, E)$.

(2) Follows from (1).

Theorem 4.6 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E), (H, E) \in SS(X)_E$. Then the following properties hold;*

(1) *$(F, E) \in S\tilde{I}OS(X)$ if and only if $cl^*(F, E) = cl^*(int(F, E))$.*

(2) *$(F, E) \in S\tilde{I}OS(X)$ if and only if there exists $(G, E) \in \tau$ such that $(G, E)\tilde{\subseteq}(F, E)\tilde{\subseteq} cl^*(G, E)$.*

(3) *If $(F, E) \in S\tilde{I}OS(X)$ and $(F, E)\tilde{\subseteq}(H, E)\tilde{\subseteq} cl^*(F, E)$, then $(H, E) \in S\tilde{I}OS(X)$.*

Proof. Obvious.

Theorem 4.7 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal, $\gamma : SS(X)_E \to SS(X)_E$ be one of the operations in Definition 3.10 (we give an example for the pre-$\tilde{I}$-open soft operator i.e ($\gamma = int(cl^*)$)) and $(F, E) \in SS(X)_E$. Then the following hold:*

(1) $P\tilde{I}S(int(\tilde{X} - (F, E))) = \tilde{X} - P\tilde{I}S(cl(F, E))$.

(2) $P\tilde{I}S(cl(\tilde{X} - (F, E))) = \tilde{X} - P\tilde{I}S(int(F, E))$.

Proof.

(1) Let $x_e \tilde{\notin} P\tilde{I}S(cl(F, E))$. Then $\exists\ (G, E) \in P\tilde{I}OS(\tilde{X}, x_e)$ such that $(G, E)\tilde{\cap}(F, E) = \tilde{\phi}$, hence $x_e \tilde{\in} P\tilde{I}S(int(\tilde{X} - (F, E)))$. Thus, $\tilde{X} - P\tilde{I}S(cl((F, E)))\tilde{\subseteq} P\tilde{I}S(int(\tilde{X} - (F, E)))$. It follows that $\tilde{X} - P\tilde{I}S(cl(F, E))\tilde{\subseteq} P\tilde{I}S(int(\tilde{X} - (F, E)))$. Now, let $x_e \tilde{\in} P\tilde{I}S(int(\tilde{X} - (F, E)))$. Since $P\tilde{I}S(int(\tilde{X} - (F, E)))\tilde{\cap}(F, E) = \tilde{\phi}$, so $x_e \tilde{\notin} P\tilde{I}S(cl(F, E))$. It follows that $x_e \tilde{\in} \tilde{X} - P\tilde{I}S(cl(F, E))$. Therefore, $P\tilde{I}S(int(\tilde{X} - (F, E)))\tilde{\subseteq} \tilde{X} - P\tilde{I}S(cl(F, E))$. Thus, $P\tilde{I}S(int(\tilde{X} - (F, E)))\tilde{\subseteq} \tilde{X} - P\tilde{I}S(cl(F, E))$. This completes the proof.

(2) By a similar way.

Theorem 4.8 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal, $(F, E) \in \tau$ and $(G, E) \in P\tilde{I}OS(X, \tau, E, \tilde{I})$. Then $(F, E)\tilde{\cap}(G, E)\tilde{\in}P\tilde{I}OS(X, \tau, E, \tilde{I})$.*

Proof. Let $(F, E) \in \tau$ and $(G, E) \in P\tilde{I}OS(X, \tau, E, \tilde{I})$. Then $(F, E)\tilde{\cap}(G, E)\tilde{\subseteq}(F, E)\tilde{\cap}int(cl^*(G, E)) = (F, E)\tilde{\cap}int((G, E)\tilde{\cup}(G, E)^*) = int[((F, E)\tilde{\cap}(G, E))\tilde{\cup}((F, E)\tilde{\cap}(G, E)^*)]\tilde{\subseteq}int[((F, E)\tilde{\cap}(G, E))\tilde{\cup}((F, E)\tilde{\cap}(G, E)^*)] = int(cl^*(F, E)\tilde{\cap}(G, E))$ from [[13], Theorem 3.2]. Thus, $(F, E)\tilde{\cap}(G, E) \in P\tilde{I}OS(X, \tau, E, \tilde{I})$.

Theorem 4.9 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then $(F, E) \in P\tilde{I}CS(X) \Leftrightarrow cl(int^*(F, E)) \subseteq (F, E)$.*

Proof. Let $(F, E) \in P\tilde{I}CS(X)$, then $\tilde{X} - (F, E)$ is pre-$\tilde{I}$-open soft. This means that, $\tilde{X} - (F, E)\tilde{\subseteq}int(cl^*(\tilde{X} - (F, E))) = \tilde{X} - (cl(int^*(F, E)))$. Therefore, $cl(int^*(F, E))\tilde{\subseteq}(F, E)$. Conversely, let $cl(int^*(F, E))\tilde{\subseteq}(F, E)$. Then $\tilde{X} - (F, E)\tilde{\subseteq}int(cl^*(\tilde{X} - (F, E)))$. Hence $\tilde{X} - (F, E)$ is pre-$\tilde{I}$-open soft. Therefore, (F, E) is pre-$\tilde{I}$-closed soft.

Corollary 4.10 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. If $(F, E) \in P\tilde{I}CS(X)$. Then $cl^*(int(F, E)) \subseteq (F, E)$.*

Proof. Let $(F, E) \in P\tilde{I}CS(X)$, then $cl(int^*(F, E))\tilde{\subseteq}(F, E)$ from Theorem 4.9. Hence $cl^*int(F, E))\tilde{\subseteq}cl(int^*(F, E))\tilde{\subseteq}(F, E)$. Therefore, $cl^*(int(F, E))\tilde{\subseteq}(F, E)$.

Theorem 4.11 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is $\alpha-\tilde{I}$-closed soft set $\Leftrightarrow cl(int^*(cl(F, E)))\tilde{\subseteq}(F, E)$.*

Proof. Let $(F, E) \in \alpha\tilde{I}CS(X)$, then $\tilde{X} - (F, E)$ is α-$\tilde{I}$-open soft. This means that, $\tilde{X} - (F, E)\tilde{\subseteq}int(cl^*(int(\tilde{X} - (F, E))))$. Therefore, $cl(int^*(cl(F, E)))\tilde{\subseteq}(F, E)$. Conversely, let $cl(int^*(cl(F, E)))\tilde{\subseteq}(F, E)$. Then $(\tilde{X} - (F, E)\tilde{\subseteq}int(cl^*(int(\tilde{X} - (F, E)))$. Hence $\tilde{X} - (F, E)$ is α-$\tilde{I}$-open soft. Therefore, (F, E) is α-$\tilde{I}$-closed soft.

Corollary 4.12 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. If (F, E) is α-$\tilde{I}$-closed soft set. Then $cl^*(int(cl^*(F, E)))\tilde{\subseteq}(F, E)$.*

Proof. It is obvious from Theorem 4.11.

Theorem 4.13 . *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is semi-$\tilde{I}$-closed soft set $\Leftrightarrow int^*(cl(F, E))\tilde{\subseteq}(F, E)$.*

Proof. Immediate.

Corollary 4.14 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. If (F, E) is semi-$\tilde{I}$-closed soft set. Then $int(cl^*(F, E)) \tilde{\subseteq} (F, E)$.*

Proof. It is obvious from Theorem 4.13.

Theorem 4.15 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. If (F, E) is semi-$\tilde{I}$-closed soft set. Then,*

(1) *If (F, E) is pre-$\tilde{I}$-open soft and semi-$\tilde{\tilde{I}}$-open soft, then (F, E) is α-$\tilde{I}$-open soft set.*

(2) *If (F, E) is pre-$\tilde{I}$-closed soft and semi-$\tilde{I}$-closed soft, then (F, E) is α-$\tilde{I}$-closed soft set.*

Proof.

(1) Let $(F, E) \in P\tilde{I}OS(X)$ and $(F, E) \in S\tilde{I}OS(X)$. Then $(F, E) \tilde{\subseteq} int(cl^*(F, E))$ and $(F, E) \tilde{\subseteq} cl^*(int(F, E))$. It follows that $(F, E) \tilde{\subseteq} int(cl^*(cl^*(int(F, E))))$. Thus, $(F, E) \tilde{\subseteq} int$ $(cl^*(int(F, E)))$. Hence (F, E) is α-$\tilde{I}$-open soft set.

(2) By a similar way.

Theorem 4.16 *In a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$, the following statements hold,*

(1) *every open (resp. closed) soft set is α-$\tilde{I}$-open (resp. α-$\tilde{I}$-closed) soft set.*

(2) *every open (resp. closed) soft set is semi-$\tilde{I}$-open (resp. semi-$\tilde{I}$-closed) soft set.*

(3) *every α-$\tilde{I}$-open (resp. α-I-closed) soft set is semi-$\tilde{I}$-open (resp. semi-$\tilde{I}$-closed) soft set.*

(4) *every α-$\tilde{I}$-open (resp. α-I-closed) soft set is pre-$\tilde{I}$-open (resp. pre-$\tilde{I}$-closed) soft set.*

Proof. We prove the assertion in the case of open soft set, the other case is similar.

(1) Let $(F, E) \in OS(X)$. Then $int(F, E) = (F, E)$. Since $(F, E) \tilde{\subseteq} cl^*(F, E)$, then $(F, E) \tilde{\subseteq} int(cl^*(F, E))$. It follows that $(F, E) \tilde{\subseteq} int(cl^*(int(F, E)))$. Therefore, $(F, E) \in \alpha\tilde{I}OS(X)$.

(2) Let $(F, E) \in OS(X)$. Then $int(F, E) = (F, E)$. Since $(F, E)\tilde{\subseteq}cl^*(F, E)$, then $(F, E)\tilde{\subseteq}cl^*(int(F, E))$. Hence $(F, E) \in S\tilde{I}OS(X)$.

(3) Let $(F, E) \in \alpha\tilde{I}OS(X)$. Then $(F, E)\tilde{\subseteq}int(cl^*(int(F, E)))\tilde{\subseteq}cl^*(int(F, E))$. Hence $(F, E) \in S\tilde{I}OS(X)$.

(4) Let $(F, E) \in \alpha\tilde{I}OS(X)$. Then $(F, E)\tilde{\subseteq}int(cl^*(int(F, E)))\tilde{\subseteq}int(cl^*(F, E))$. Hence $(F, E) \in P\tilde{I}OS(X)$.

Remark 4.17 *The converse of Theorem 4.16 is not true in general as shown by the following examples.*

Examples 4.18 **(1)** *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined by $F_1(e) = \{h_2, h_3\}$ and $F_2(e) = \{h_1, h_2, h_3\}$. Then (X, τ, E) is a soft topological space over X. Let $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, where $(I_1, E), (I_2, E)$, (I_3, E) are soft sets over X defined by $I_1(e) = \{h_1\}$, $I_2(e) = \{h_4\}$ and $I_3(e) = \{h_1, h_4\}$. Hence the soft set (G, E), which defined by $G(e) = \{h_2, h_3, h_4\}$, is α-$\tilde{I}$-open soft set but it is not open soft set.*

(2) *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined by $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2, h_3\}$ and $F_3(e) = \{h_1, h_2, h_3\}$. Then (X, τ, E) is a soft topological space over X. Let $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by $I_1(e) = \{h_1\}$, $I_2(e) = \{h_4\}$ and $I_3(e) = \{h_1, h_4\}$. Hence the soft set (G, E), which defined by $G(e) = \{h_2, h_3, h_4\}$, is semi-$\tilde{I}$-open soft set but it is not open soft set.*

(3) *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined by $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2, h_3\}$ and $F_3(e) = \{h_1, h_2, h_3\}$. Then (X, τ, E) is a soft topological space over X. Let $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by $I_1(e) = \{h_1\}$, $I_2(e) = \{h_4\}$ and $I_3(e) = \{h_1, h_4\}$. Hence the soft set (G, E), which defined by $G(e) = \{h_2, h_3, h_4\}$, is semi-$\tilde{I}$-open soft set but it is not α-open soft set.*

(4) *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined by $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2, h_3\}$ and $F_3(e) = \{h_1, h_2, h_3\}$. Then (X, τ, E) is a soft topological space over X. Let $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by $I_1(e) = \{h_1\}$, $I_2(e) = \{h_4\}$ and $I_3(e) = \{h_1, h_4\}$. Hence the soft set (G, E), which* *defined by $G(e) = \{h_1, h_2, h_4\}$, is pre-$\tilde{I}$-open soft sets of $(X, \tau, E, \tilde{I})$, but it is not α-$\tilde{I}$-open soft set.*

Theorem 4.19 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is semi-$\tilde{I}$-closed soft set $\Leftrightarrow$ $int^*(cl(F, E)) = int^*(F, E)$.*

Proof.
Let $(F, E) \in S\tilde{I}CS(X)$, then $int^*cl(F, E)\tilde{\subseteq}(F, E)$ from Theorem 4.13. Then $int^*cl(F, E)\tilde{\subseteq}int^*(F, E)$. But clearly $int^*(F, E)\tilde{\subseteq}int^*cl(F, E)$, hence int^* $cl(F, E) = int^*(F, E)$. Conversely, if $int^*cl(F, E) = int^*(F, E)$, then (F, E) is semi-$\tilde{I}$-closed soft. This completes the proof.

Corollary 4.20 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is semi-$\tilde{I}$-closed $\Leftrightarrow$ $(F, E) = (F, E)\tilde{\cup}int^*(cl$ $(F, E))$.*

Proof. It is obvious from Theorem 4.19.

Theorem 4.21 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in \alpha\tilde{I}OS(X)$. Then $(F, E)\tilde{\cap}(G, E) \in S\tilde{I}OS(X)$ $\forall(G, E) \in S\tilde{I}OS(X)$.*

Proof. Let $(F, E) \in \alpha\tilde{I}OS(X)$ and $(G, E) \in S\tilde{I}OS(X)$. Then $(F, E)\tilde{\subseteq}int(cl^*$ $(int(F, E)))$ and $(G, E)\tilde{\subseteq}cl^*(int(G, E))$. It follows that $(F, E)\tilde{\cap}(G, E)\tilde{\subseteq}(int(cl^*$ $(int(F, E))))\tilde{\cap}(cl^*(int(G, E)))\tilde{\subseteq}cl^*((int(cl^*(int(F, E))))\tilde{\cap}(int(G, E)))\tilde{\subseteq}cl^*((cl^*$ $(int(F, E))))\tilde{\cap}(int(G, E)))\tilde{\subseteq}cl^*$ $(cl^*$ $(int(F, E)))$ $\tilde{\cap}(int(G, E)) = cl^*((int(F, E))$ $\tilde{\cap}(int(G, E)))\tilde{\subseteq}cl^*(int((F, E)\tilde{\cap}(G, E)))$. Thus, $(F, E)\tilde{\cap}(G, E) \in S\tilde{I}OS(X)$ $\forall$ $(G, E) \in S\tilde{I}OS(X)$.

Theorem 4.22 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and $(F, E) \in SS(X)_E$. Then (F, E) is β-$\tilde{I}$-closed soft set $\Leftrightarrow$ $int(cl(int^*(F, E)))\tilde{\subseteq}$ (F, E).*

Proof. Immediate.

Corollary 4.23 *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and (F, E) is β-$\tilde{I}$-closed soft set. Then $int(cl^*(int(F, E)) \subseteq (F, E)$.*

Proof. It is obvious from Theorem 4.22.

Theorem 4.24 *In a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$, the following statements hold,*

(1) *every pre-$\tilde{I}$-open (resp. pre-$\tilde{I}$-closed) soft set is β-$\tilde{I}$-open (resp. β-$\tilde{I}$-closed) soft set.*

(2) *every semi-$\tilde{I}$-open (resp. semi-$\tilde{I}$-closed) soft set is β-$\tilde{I}$-open (resp. β-$\tilde{I}$-closed) soft set.*

Proof. Immediate.

Remark 4.25 *The converse of Theorem 4.24 is not true in general as shown by the following examples.*

Examples 4.26 **(1)** *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over X defined by $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2\}$, $F_3(e) = \{h_1, h_2\}$, $F_4(e) = \{h_1, h_2, h_4\}$ and $\tilde{I} = \{\tilde{\phi}, (I_1, E), (I_2, E), (I_3, E)\}$ be a soft ideal over X, where $(I_1, E), (I_2, E), (I_3, E)$ are soft sets over X defined by $I_1(e) = \{h_1\}$, $I_2(e_1) = \{h_3\}$ and $I_3(e_1) = \{h_1, h_3\}$. Then the soft set (G, E), defined by $G(e) = \{h_1, h_4\}$, is β-$\tilde{I}$-open soft but it is not pre-$\tilde{I}$-open soft.*

(2) *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$, where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over X defined by $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2\}$, $F_3(e) = \{h_1, h_2\}$, $F_4(e) = \{h_1, h_2, h_4\}$ and $\tilde{I} = \{\tilde{\phi}, (I, E)$ be a soft ideal over X, where (I, E) is a soft set over X defined by $I(e) = \{h_3\}$. Then the soft set (F_1, E) is β-$\tilde{I}$-open soft but it is not semi-$\tilde{I}$-open soft.*

Proposition 4.27 *From Theorems 4.1, 4.16 and Theorem 4.24 we have the following implications for a soft topological space with soft ideal $(X, \tau, E, \tilde{I})$.*

$$\begin{tikzcd}
OS(X) \arrow[r, Rightarrow] & \alpha\tilde{I}OS(X) \arrow[r, Rightarrow] \arrow[dr] & S\tilde{I}OS(X) \arrow[r, Rightarrow] & \beta\tilde{I}OS(X) \arrow[r, Rightarrow] & \beta OS(X) \\
& \tilde{I}OS(X) \arrow[r, Rightarrow] & P\tilde{I}OS(X) \arrow[r, Rightarrow] \arrow[ur] & POS(X) \arrow[ur] &
\end{tikzcd}$$

5 Decompositions of Some Forms of Soft Continuity in Soft Topological Spaces via Soft Ideal

Definition 5.1 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $u : X_1 \to X_2$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then*

(1) *The function f_{pu} is called $\tilde{I}$-continuous soft ($\tilde{I}$-cts soft) if $f_{pu}^{-1}(G, B) \in \tilde{I}OS(X_1)\ \forall\ (G, B) \in \tau_2$.*

(2) *The function f_{pu} is called a pre-$\tilde{I}$-continuous soft function (Pre-$\tilde{I}$-cts soft) if $f_{pu}^{-1}(G,B) \in P\tilde{I}OS(X_1)\ \forall\ (G,B) \in \tau_2$.*

(3) *The function f_{pu} is called an α-$\tilde{I}$-continuous soft function (α-$\tilde{I}$-cts soft) if $f_{pu}^{-1}(G,B) \in \alpha\tilde{I}OS(X_1)\forall\ (G,B) \in \tau_2$.*

(4) *The function f_{pu} is called semi-$\tilde{I}$-continuous soft function (semi-$\tilde{I}$-cts soft) if $f_{pu}^{-1}(G,B) \in S\tilde{I}OS(X_1)\forall\ (G,B) \in \tau_2$.*

(5) *The function f_{pu} is called β-$\tilde{I}$-continuous soft function (β-$\tilde{I}$-cts soft) if $f_{pu}^{-1}(G,B) \in \beta\tilde{I}OS(X_1)\forall\ (G,B) \in \tau_2$.*

Theorem 5.2 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $u : X_1 \to X_2$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then every $\tilde{I}$-continuous soft function is pre-$\tilde{I}$-continuous soft function.*

Proof.
It is obvious from Theorem 4.1.

Theorem 5.3 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $u : X_1 \to X_2$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. If f_{pu} is $\tilde{I}$-continuous soft function , then it is pre-$\tilde{I}$-continuous soft.*

Proof.
It is obvious from Theorem 4.3.

Proposition 5.4 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $u : X_1 \to X_2$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then we have:*

(1) *If $\tilde{I} = \{\tilde{\phi}\}$, then f_{pu} is pre-$\tilde{I}$-continuous soft (resp. semi-$\tilde{I}$-continuous soft, α-$\tilde{I}$-continuous soft and β-$\tilde{I}$-continuous soft) function $\Leftrightarrow$ it is pre-continuous soft (resp. semi-continuous soft, α-continuous soft and β-continuous soft).*

(2) *If $I = SS(X)_E$, then f_{pu} is pre-$\tilde{I}$-continuous soft (resp. semi-$\tilde{I}$-continuous soft, α-$\tilde{I}$-continuous soft and $\tilde{I}$-continuous soft) function $\Leftrightarrow$ it is continuous soft.*

Proof. Immediate.

Theorem 5.5 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $u : X_1 \to X_2$ and $p : A \to B$ be a mappings. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then for the classes, pre-$\tilde{I}$-continuous soft (resp. α-$\tilde{I}$-continuous soft, semi-$\tilde{I}$-continuous soft and β-$\tilde{I}$-continuous soft) functions the following are equivalent (we give an example for the the class of pre-$\tilde{I}$-continuous soft functions).*

(1) f_{pu} *is pre-$\tilde{I}$-continuous soft function.*

(2) $f_{pu}^{-1}(H, B) \in P\tilde{I}CS(X_1)\ \forall\ (H, B) \in CS(X_2)$.

(3) $f_{pu}(P\tilde{I} - Scl(G, A)) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G, A))\ \forall\ (G, A) \in SS(X_1)_A$. *0*

(4) $P\tilde{I} - Scl(f_{pu}^{-1}(H, B)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H, B))\ \forall\ (H, B) \in SS(X_2)_B$.

(5) $f_{pu}^{-1}(int_{\tau_2}(H, B)) \tilde{\subseteq} P\tilde{I} - Sint(f_{pu}^{-1}(H, B))\ \forall\ (H, B) \in SS(X_2)_B$.

Proof.

(1) $\Rightarrow$ (2) Let (H, B) be a closed soft set over X_2. Then $(H, B)' \in \tau_2$ and $f_{pu}^{-1}(H, B)' \in P\tilde{I}OS(X_1)$ from Definition 5.1. Since $f_{pu}^{-1}(H, B)' = (f_{pu}^{-1}(H, B))'$ from [[33], $Theorem$3.14]. Thus, $f_{pu}^{-1}(H, B) \in P\tilde{I}CS(X_1)$.

(2) $\Rightarrow$ (3) Let $(G, A) \in SS(X_1)_A$. Since $(G, A) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(G, A)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A))) \in P\tilde{I}CS(X_1)$ from (2) and [[33], $Theorem$3.14]. Then $(G, A) \tilde{\subseteq} P\tilde{I}S(cl(G, A)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A)))$. Hence $f_{pu}(P\tilde{I}S(cl(G, A))) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(G, A)))) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G, A)))$ from [[33], $Theorem$3.14]. Thus, $f_{pu}(P\tilde{I}S(cl(G, A))) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(G, A))$.

(3) $\Rightarrow$ (4) Let $(H, B) \in SS(X_2)_B$ and $(G, A) = f_{pu}^{-1}(H, B)$. Then $f_{pu}(P\tilde{I} - Scl f_{pu}^{-1}(H, B)) \tilde{\subseteq} cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H, B)))$ From (3). Hence $P\tilde{I} - Scl(f_{pu}^{-1}(H, B)) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(P\tilde{I} - Scl(f_{pu}^{-1}(H, B)))) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H, B)))) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H, B))$ from [[33], $Theorem$3.14]. Thus, $P\tilde{I} - Scl(f_{pu}^{-1}(H, B)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H, B))$.

(4) $\Rightarrow$ (2) Let (H, B) be a closed soft set over X_2. Then $P\tilde{I} - Scl(f_{pu}^{-1}(H, B)) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H, B)) = f_{pu}^{-1}(H, B) = f_{pu}^{-1}(H, B)\ \forall\ (H, B) \in SS(X_2)_B$ from (4), but clearly $f_{pu}^{-1}(H, B) \tilde{\subseteq} P\tilde{I} - Scl(f_{pu}^{-1}(H, B))$. This means that, $f_{pu}^{-1}(H, B) = P\tilde{I} - Scl(f_{pu}^{-1}(H, B)) \in PCS(X_1)$.

(1) $\Rightarrow$ (5) Let $(H, B) \in SS(X_2)_B$. Then $f_{pu}^{-1}(int_{\tau_2}(H, B)) \in P\tilde{I}OS(X_1)$ from (1). Hence $f_{pu}^{-1}(int_{\tau_2}(H, B)) = P\tilde{I} - Sint(f_{pu}^{-1} int_{\tau_2}(H, B)) \tilde{\subseteq} P\tilde{I} - Sint(f_{pu}^{-1}(H, B))$. Thus, $f_{pu}^{-1}(int_{\tau_2}(H, B)) \tilde{\subseteq} P\tilde{I} - Sint(f_{pu}^{-1}(H, B))$.

(5) $\Rightarrow$ (1) Let (H,B) be an open soft set over X_2. Then $int_{\tau_2}(H,B) = (H,B)$ and $f_{pu}^{-1}(int_{\tau_2}(H,B)) = f_{pu}^{-1}((H,B))\tilde{\subseteq} P\tilde{I} - Sint(f_{pu}^{-1}(H,B))$ from (5). But we have $P\tilde{I} - Sint(f_{pu}^{-1}(H,B)) = f_{pu}^{-1}(H,B) \in P\tilde{I}OS(X_1)$. Thus, f_{pu} is pre-$\tilde{I}$-continuous soft function.

Theorem 5.6 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then f_{pu} is α-$\tilde{I}$-continuous soft function if and only if it is a pre-$\tilde{I}$-continuous and semi-$\tilde{I}$-continuous soft function .*

Proof.

It is obvious from Theorem 4.15.

Theorem 5.7 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then*

(1) *every continuous soft function is α-$\tilde{I}$-continuous soft function.*

(2) *every continuous soft function is semi-$\tilde{I}$-continuous soft function.*

(3) *every α-$\tilde{I}$-continuous soft function is semi-$\tilde{I}$-continuous soft function.*

(4) *every α-$\tilde{I}$-continuous soft function is pre-$\tilde{I}$-continuous soft function.*

Proof. It is obvious from Theorem 4.16.

Theorem 5.8 *Let $(X_1, \tau_1, A, \tilde{I})$ be a soft topological space with soft ideal and (X_2, τ_2, B) be a soft topological space. Let $f_{pu} : SS(X_1)_A \to SS(X_2)_B$ be a function. Then*

(1) *Every pre-$\tilde{I}$-continuous soft function is β-$\tilde{I}$-continuous soft function.*

(2) *Every semi-$\tilde{I}$-continuous soft function is β-$\tilde{I}$-continuous soft function.*

Proof. It is obvious from Theorem 4.24.

6 Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied and improved the soft set theory, which is initiated by Molodtsov [23] and easily applied to many problems having uncertainties from social life. Kandil et al.[12] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notions of γ-operation. The notion of soft ideal is initiated for the first time by Kandil et al.[13]. In this paper we extend the notions of γ-operation, pre-$\tilde{I}$-open soft sets, $\tilde{I}$-open soft sets, α-$\tilde{I}$-open soft sets, semi-$\tilde{I}$-open soft sets and β-$\tilde{I}$-open soft sets to soft topological spaces with soft ideal. We study the relations between these different types of subsets of soft topological spaces with soft ideal. We also introduce the concepts of $\tilde{I}$-continuous soft, pre-$\tilde{I}$-continuous soft, α-$\tilde{I}$-continuous soft, semi-$\tilde{I}$-continuous soft and β-$\tilde{I}$-continuous soft functions and discuss their properties in detail. We notice that the family η of all γ-open soft sets on a soft topological space with soft ideal $(X, \tau, \tilde{I})$ forms a supra soft topology, i.e $\tilde{X}, \tilde{\phi} \in \eta$ and η is closed under arbitrary soft union. We see that this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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