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Fuzzy Lindelof Closed Spaces and their Operations

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Abstract

In this paper, we investigate two new spaces an Lindelof closed space and a fuzzy Lindelof closed space, the main purpose of this work is to study the relation between fuzzy Lindelof closed and other spaces such as fuzzy Lindelof, fuzzy lightly compact, fuzzy semi regular, C_{II}*, fuzzy paracompact, fuzzy separable, and others. Also In this paper we show that a fuzzy topological space* X *is fuzzy Lindelof closed space if for every net* $\{K_n, n \in D\}$ *of fuzzy closed sets in* X *such that* $F-\limsup_D(K_n) = 0$*, there exists* $n_0 \in D$ *for which* $K_n = 0$*, for every* $n \in D, n \geq n_0$*. Also we show that* X *is fuzzy Lindelof closed space if for every* $\alpha \in I$*, for every net* $\{K_n, n \in D\}$ *of fuzzy closed sets in* X *such that* $F-\limsup_D(K_n) = 1-\alpha$ *and for every* $\varepsilon \in (0,\alpha]$*, there exists an element* $n_0 \in D$ *for which* $K_n \leq 1-\alpha+\varepsilon$*, for every* $n \in D, n \geq n_0$*.*

Keywords: *Fuzzy topological space, fuzzy compactness, fuzzy para compactness, separable space, fuzzy Lindelof space, fuzzy lightly compact.*

1 Introduction

The Theory of Fuzzy sets is introduced by Zadeh [17], and the fuzzy topology is defined by Chang [5]. Many mathematicians have tried to extend to fuzzy set theory the main notations of general topology see ([6]-[8], [13]-[15]) and others. In this paper we define an Lindelof closed space and a fuzzy Lindelof closed space, the main purpose of this work is to study the relation between fuzzy lindelof closed and other spaces such as fuzzy Lindelof, fuzzy lightly compact, fuzzy semi regular, C_{II}, fuzzy paracompact, fuzzy separable,and others. If μ is a fuzzy set in a fuzzy topological space (X,T) then the closure and the interior of μ will be as usual defined by $\overline{\mu} = \wedge\{\lambda : \lambda \geq \mu, 1-\lambda \in T\}$ and $\overset{o}{\mu} = \vee\{\lambda : \lambda \leq \mu, \lambda \in T\}$ respectively. A fuzzy set μ is called fuzzy regular open, if $\mu = (\overline{\mu})^o$ [3]. Let (X,T) be a fuzzy topological space, the fuzzy regular open sets in T form a base for a unique fuzzy topology T_0 called the fuzzy semi-regular topology on X associated with T. A fuzzy topology T is fuzzy semi-regular, if and only if $T = T_0$ [2]. In this paper we show that (X,T_0) is fuzzy Lindelof closed if (X,T) is fuzzy Lindelof . In another direction. Let D be non-empty set. Let $\geq$ be a semi-order on D. The pair $(D,\geq)$ is called a directed set, directed by $\geq$, if and only if for every pair $m,n \in D$, there exists a $p \in D$ such that $p \geq m$ and $p \geq n$. Let X be an ordinary set. Let χ be the collection of all the fuzzy points in X. The function $S : D \to \chi$. In other words, a fuzzy net is a pair $(S,\geq)$ such that S is a function $: D \to \chi$ and $\geq$ directs the domain of S. For $n \in D$, $S(n)$ is often denoted by S_n and hence a net S often denoted by $\{S_n, n \in D\}$. In this paper we show that a fuzzy topological space X is fuzzy Lindelof closed space if for every net $\{K_n, n \in D\}$ of fuzzy closed sets in X such that $F-\limsup_D (K_n) = 0$, there exists $n_0 \in D$ for which $K_n = 0$, for every $n \in D$, $n \geq n_0$. Also we show that X is fuzzy Lindelof closed space if for every $\alpha \in I$, for every net $\{K_n, n \in D\}$ of fuzzy closed sets in X such that $F-\limsup_D (K_n) = 1-\alpha$ and for every $\varepsilon \in (0,\alpha]$, there exists an element $n_0 \in D$ for which $K_n \leq 1-\alpha+\varepsilon$, for every $n \in D$, $n \geq n_0$.

2 Definitions and Notations

The following definitions have been used to obtain the results and properties developed in this paper.

Definition 2.1: [4] *A fuzzy topological space* (X,δ) *is said to be fuzzy lightly compact if for all* $\{\mu_i\}_{i\in\Omega} \subseteq \delta$ *with* $\sup\{\mu_i\} = 1$, *there exists an* $n_0 \in N$ *such that* $\sup\{\overline{\mu_i}\}_{i=1}^{n_0} = 1$.

Definition 2.2: [11] *Let* λ *be a fuzzy set in a fuzzy topological space* (X,δ). λ *is said to be fuzzy compact in the Lowen's sense if for all family of fuzzy open sets cover* $\{\lambda_l \mid l \in L\}$ *such that* $\lambda \le \vee\{\lambda_l \mid l \in L\}$ *and for all* $\varepsilon > 0$ *there exist a finite subfamily* $\{\lambda_l \mid l \in L^*\}$ *such that* $\lambda - \varepsilon \le \vee\{\lambda_l \mid l \in L^*\}$.

Definition 2.3: [10] *A fuzzy topological space* (X,δ) *is called fuzzy compact if and only if for every family* Ψ *of fuzzy open sets of* X *and for every* $\alpha \in I$ *such that* $\vee\{U : U \in \Psi\} \ge \alpha$ *and for every* $\varepsilon \in (0,\alpha]$ *there exists a finite subfamily* Ψ^* *of* Ψ *such that* $\vee\{U : U \in \Psi^*\} \ge \alpha - \varepsilon$.

Definition 2.4: [1] *Let* λ *be a fuzzy set in a fuzzy topological space* (X,δ). λ *is said to be fuzzy paracompact if for every open cover in the sense of Lowen* H *of* λ *and for every* $\varepsilon \in (0,\alpha]$, *there exists an open refinement* D *of* H *which is both locally finite in* λ *and cover of* $\lambda - \varepsilon$ *in the sense of Lowen.*

Definition 2.5: [12] *Let* (X,τ) *be a topological space and* $w(\tau)$ *be the set of all semi continuous function from* (X,τ) *to the unit interval* $I = [0,1]$ *equipped with the usual topology, then* $(X, w(\tau))$ *is called induced fuzzy topological space by* (X,τ).

Definition 2.6: [1] *A fuzzy topological space* (X,δ) *is said to be fuzzy Lindelof, if for each family* $H \subset \delta$ *and for each* $\alpha \in I$ *such that* $\bigvee_{h\in H} h \ge \alpha$, *there exists for each* $\varepsilon \in (0,\alpha]$ *a countable subset* H^* *of* H *such that* $\bigvee_{h\in H^*} h \ge \alpha - \varepsilon$.

Definition 2.7: [16] *A fuzzy topological space* (X,δ) *is said to be separable iff there exist a countable sequence of fuzzy points* $\{p_i, i = 1,2,...\}$ *such that for every member* $\lambda \ne 0$ *of* δ *there exist a* p_i *such that* $p_i \in \lambda$.

Definition 2.8: [16] *A fuzzy topological space* (X,T) *is said to be* C_{II}, *if there exists a countable base for* T.

Theorem 2.9: [11] *Let* $\{A_n : n \in N\}$ *be a net of fuzzy closed sets in* Y *such that* $A_{n_1} \le A_{n_2}$ *if and only if* $n_2 \le n_1$. *Then* $F - \lim_N \sup(A_n) = \wedge\{A_n : n \in N\}$.

3 Fuzzy Lindelof Closed Spaces and Notations

In this section we define the concept of fuzzy Lindelof closed space and we discuss the relation between fuzzy Lindelof closed and the other fuzzy spaces.

Definition 3.1: *A topological space* (X,T) *is said to be Lindelof closed space if and only if for each open cover* $\{A_l : l \in L\}$ *of* X *, i.e* $X = \bigcup_{l \in L} A_l$ *, there exists a countable subfamily* $\{A_l : l \in L^*\}$ *of* $\{A_l : l \in L\}$ *whose closures cover* X *.*

Definition 3.2: *A fuzzy topological space* (X,T) *is said to be fuzzy Lindelof closed space if and only if for every family* Ψ *of fuzzy open sets of* X *and for every* $\alpha \in I$ *such that* $\vee\{U : U \in \Psi\} \geq \alpha$ *and for every* $\varepsilon \in (0,\alpha]$ *there exists a countable subfamily* Ψ^* *of* Ψ *whose closures such that* $\vee\{\overline{U} : U \in \Psi^*\} \geq \alpha - \varepsilon$ *. In this connection one can prove the following results easily.*

Proposition 3.3: *A space* (X,T_0) *is fuzzy Lindelof closed, if* (X,T) *is fuzzy Lindelof closed.*

Proposition 3.4: *A fuzzy semi-regular space* (X,T) *is fuzzy Lindelof closed* $\Leftrightarrow$ (X,T_0) *is fuzzy Lindelof closed.*

Proposition 3.5: *A space* (X,T) *is fuzzy Lindelof closed, if* (X,T) *is fuzzy compact.*

Proposition 3.6: *A space* (X,T) *is fuzzy Lindelof closed, if* (X,T) *is fuzzy Lindelof.*

The converse of Proposition (3.6) is not necessarily true as the following example shows.

Example 3.7: Let $X = R$ be the set of all real numbers, and $C \subseteq R - \{0\}$. Define

$\lambda_1 : R \to [0,1]$ as $\lambda_1(x) = 0$ for all $x \in X$,

$$\lambda_2 : R \to [0,1] \text{ as } \lambda_2(x) = \begin{cases} 1 \text{ if } x = 0 \\ 0 \text{ if } x \neq 0 \end{cases}$$

$$\lambda_C : R \to [0,1] \text{ as } \lambda_C(x) = \begin{cases} 1 \text{ if } x \in C \cup \{0\} \\ 0 \text{ if } x \notin C \cup \{0\} \end{cases}$$

Let $\delta = \{\lambda_1, \lambda_2, \lambda_C \mid C \subseteq R - \{0\}\}$. Then (X,δ) is fuzzy topological space. For each $0 \neq \lambda \in \delta$, we have $\overline{\lambda} = 1$, then for any family Ψ of fuzzy open sets of X such that $\vee\{U : U \in \Psi\} \geq 1$, there exists $0 \neq U^* \in \Psi \Rightarrow \overline{U^*} = 1$. Then (X,δ) is fuzzy Lindelof closed. However, (X,δ) is not fuzzy Lindelof space. Let $\Psi = \{\lambda_{\{r\}} \mid r \in R - \{0\}\}$ be a family of fuzzy open sets of X . Given $\varepsilon > 0$, and let

$r=\varepsilon$. Then at $x=\varepsilon \Rightarrow \lambda_{\{\varepsilon\}}(x)=1>1-\varepsilon$. So $\bigvee_{r\in R-\{0\}} \lambda_{\{r\}}=1 \; >1-\varepsilon$. Then Ψ is fuzzy open cover of X . But if L is a countable subset of $R-\{0\}$, then we can find $x'\in R-\{0\}\subset X$ and $x'\notin L$. It follows that at $x=x'$ we have $\lambda_{\{r\}}(x)=0$ for all $r\in L$, and certainly $\bigvee_{r\in L}\lambda_{\{r\}}<1$. Then Ψ is cover of X by members of δ has not countable sub-cover of X .

Theorem 3.8: *Every fuzzy lightly compact space* (X,δ) *is fuzzy Lindelof closed.*

Proof: Let $\{\lambda_l\}_{l\in L}\subseteq\delta$ be a fuzzy open cover of X , $\vee\{\lambda_l\}_{l\in L}=\sup\{\lambda_l : l\in L\}=1$ then there exists $J\subseteq L$ such that $\lambda_{lj}\in\{\lambda_l\}_{l\in L}$ with $\sup\{\lambda_{lj}\}=1$ for $j\in J$. But (X,δ) is fuzzy lightly compact and $\{\lambda_{lj}\}_{j\in J}\subseteq\delta$ with $\sup\{\lambda_{lj}\}=1$. Therefore $\{\lambda_{lj}\}_{j\in J}$ has a finite subfamily $\{\lambda_{lj}{}^{k} : k=1,2,...,n\}$ whose closures such that $\sup\{\overline{\lambda_{lj}{}^{k}}\}_{k=1}^{n}=1$, but each finite family is a countable, then for each family $\{\lambda_l\}_{l\in L}\subseteq\delta$ with $\vee\{\lambda_l\}_{l\in L}=1$, there exists a countable subfamily of $\{\lambda_l\}_{l\in L}$ whose closures cover of X . Then (X,δ) is fuzzy Lindelof closed.

Note the above example a fuzzy topological space (X,δ) is fuzzy lightly compact space and fuzzy lindelof closed. But it is not fuzzy lindelof.

Theorem 3.9: *If* $(X,w(\delta))$ *is fuzzy Lindelof closed, then* (X,δ) *is Linelof closed.*

Proof: Let $\{A_j : j\in J\}$ be an open cover of (X,δ), i.e $\bigcup_{j\in J}A_j=X$. Then $\bigvee_{j\in J}\chi_{A_j}=\sup\{\chi_{A_j}\,;\, j\in J\}=1$ and $\{\chi_{A_j}\,;\, j\in J\}$ is an open Lowen's cover of $(X,w(\delta))$. By the assumption of the fuzzy lindelof closed of $(X,w(\delta))$, choose $\varepsilon>0$, then there exists a countable open Lowen's sub-cover $\{\chi_{A_{jk}}\,;\, k\in N\}$ such that $\bigvee_{k\in N}\overline{\chi}_{A_{jk}}\geq 1-\varepsilon$. Since ε is arbitrary then $X=\bigcup_{k\in N}\overline{A}_{jk}$.

Theorem 3.10: *Let* f *be a* $F-$*continuous surjection map between fuzzy topological spaces* (X,δ) *and* (Y,π). *If* (X,δ) *is fuzzy Lindelof closed then* (Y,π) *is verifies the same property.*

Proof: Let α be a constant fuzzy set in Y. Let $K=\{\lambda_j\}_{j\in J}\subset\pi$ such that $\bigvee_{j\in J}\{\lambda_j\}\geq\alpha$, and let $\varepsilon\in(0,\alpha]$, since f is $F-$continuous, then $M=\{f^{-1}(\lambda_j);\, j\in J\}$ is an $L-$cover of the constant fuzzy set β in X such that $f(\beta)=\alpha$. Since X is fuzzy Lindelof closed, then there exists a fuzzy open countable subset M^* of M such that $\bigvee_{\gamma\in M^*}\{\overline{\gamma}\}\geq\beta-\varepsilon$. Then $K^*=\{f(\gamma);\gamma\in M^*\}$ is a fuzzy open countable subfamily of K such that $\bigvee_{\lambda_j\in K^*}\{\overline{\lambda_j}\}\geq\alpha-\varepsilon$. Then (Y,π) is fuzzy Lindelof closed.

Theorem 3.11: *If a fuzzy topological space* (X,δ) *is* C_{II}, *then it is also fuzzy Lindelof closed space.*

Proof: Let $\{\lambda_j\}_{j\in J}\subset\delta$ such that $\bigvee_{j\in J}\{\lambda_j\}\geq\alpha$ for each $\alpha\in I$. Since (X,δ) is C_{II} then there exists a countable subfamily $\psi=\{\psi_l, l=1,2,...\}$ of δ such that $\lambda_j=\overset{t}{\underset{k=1}{\vee}}\{\psi_{jk}\}$, where t my be infinitely. Let $\psi^*=\{\psi_{jk}\}$, $j\in J$, $k=1,2,...,t$, ψ^* is countable because it is subfamily of ψ. Let $x\in X$ and since $\bigvee_{j\in J}\{\lambda_j\}\geq\alpha$, there exist $m\in J$ such that $\lambda_m(x)\geq\alpha$ and we have $\lambda_m=\overset{t}{\underset{k=1}{\vee}}\{\psi_{jk}\}$ which implies that $\overset{t}{\underset{k=1}{\vee}}\{\psi_{jk}\}\geq\alpha$ and then ψ^* is open cover. Let $\varepsilon\in(0,1]$, then $\overset{t}{\underset{k=1}{\vee}}\{\overline{\psi_{jk}}\}\geq\overset{t}{\underset{k=1}{\vee}}\{\psi_{jk}\}$ $\geq\alpha-\varepsilon$. Finally (X,δ) is fuzzy Lindelof closed.

Theorem 3.12: *Let* (X,δ) *be a fuzzy paracompact separable topological space, then* (X,δ) *is fuzzy Lindelof closed.*

Proof: Let $H=\{\lambda_j\}_{j\in J}\subset\delta$ be a family such that $\bigvee_{j\in J}\{\lambda_j\}\geq\alpha$ for each $\alpha\in I$. Let H^* be a $\delta-$open refinement of H which is locally finite and $\vee_{h\in H^*}h\geq\alpha-\varepsilon$, $\varepsilon\in(0,\alpha]$. X has countable sequence of fuzzy points $\{p_i, i=1,2,...\}$ such that for every $h\neq 0$ there exists a $p_i\in h$. Then the family $\{h;h\in H^*\}$ is at most countable, otherwise since each h contains at least one p_i. This implies that there would be some p_n contained in uncountable many $h\in H$ which would contradiction locally finite. Choose for each $h_i\in H^*$ an element $\lambda_i\in H$ such that $h_i<\lambda_i\leq\overline{\lambda_i}$. Then there exists an open countable H' subset of H such that $\bigvee_{\lambda\in H'}\{\overline{\lambda}\}\geq\alpha-\varepsilon$. Therefore (X,δ) is fuzzy Lindelof closed.

Theorem 3.13: *Let* (X,δ) *be a* C_{II} *fuzzy topological space is, then the continuous image of fuzzy paracompact is fuzzy Lindelof closed.*

Proof: Let be $f:(X,\delta)\to(Y,\pi)$ $F-$continuous and let β be a fuzzy paracompact subset of X. For every subspace (W,δ_W) of C_{II} fuzzy topological space (X,δ) is C_{II}, since by assumption δ has a countable base $L=\{L_i\}, i=1,2,...,$ then $\{W\wedge L_i\}\subset\delta_W$, $i=1,2,..,$ be a countable base for δ_W such that (W,δ_W) subspace of (X,δ), then (W,δ_W) is C_{II} and by (Theorem 3.4, [16]) we have (W,δ_W) is separable. So β is separable. Then by Theorem (3.12) β is Lindelof closed fuzzy, and finally $f(\beta)$ is Lindelof closed fuzzy by Theorem (3.10).

Theorem 3.14: *A fuzzy topological space X is fuzzy Lindelof closed space if for every net $\{K_n, n\in D\}$ of fuzzy closed sets in X such that $F-\lim\sup_D(K_n)=0$, there exists $n_0\in D$ for which $K_n=0$, for every $n\in D$, $n\geq n_0$.*

Proof: Suppose that the fuzzy topological satisfies the condition of the theorem. We prove that X is fuzzy Lindelof closed. Let π be an open cover of fuzzy sets of X. Let D be the set of all finite subsets of π directed by inclusion and let $\{K_n, n\in D\}$ be a net of fuzzy closed sets in X such that $K_n^c=\vee\{A, A\in n\}$. Obviously $K_{n_1}\subseteq K_{n_2}$ if $n_2\leq n_1$. Hence by Theorem (2.9) it follows that $F-\lim\sup_D(K_n)=\wedge\{K_n, n\in D\}$. Also, we have:

$$\wedge\{K_n, n\in D\}=(\vee\{K_n^c : n\in D\})^c=(\vee\{A:A\in\pi\})^c=0$$

Thus $F-\lim\sup_D(K_n)=0$. By assumption there exists an element $n_0\in D$ for which $K_n=0$ for every $n\in D$, $n\geq n_0$. By the above we have $1=K_n^c=$ $\vee\{A, A\in n_0\}$. But $\vee\{A, A\in n_0\}\leq\vee\{\overline{A}, A\in n_0\}$, and any finite family is a countable family. Therefore the fuzzy topological space X is fuzzy Lindelof closed.

Theorem 3.15: *A fuzzy topological space X is fuzzy Lindelof closed space if for every $\alpha\in I$, for every net $\{K_n, n\in D\}$ of fuzzy closed sets in X such that $F-\lim\sup_D(K_n)=1-\alpha$ and for every $\varepsilon\in(0,\alpha]$, there exists an element $n_0\in D$ for which $K_n\leq 1-\alpha+\varepsilon$, for every $n\in D, n\geq n_0$.*

Proof: Suppose that the fuzzy topological satisfies the condition of the theorem. We prove that X is fuzzy Lindelof closed. Let $\alpha\in I$, $\Re$ be a family in δ such that $\vee\{U:U\in\Re\}\geq\alpha$ and $\varepsilon\in(0,\alpha]$. Let D be the set of all finite subset of $\Re$ directed by inclusion and let $\{K_n, n\in D\}$ be a net of fuzzy closed sets in X such that $K_n^c=\vee\{A, A\in n\}$. Obviously $K_{n_1}\leq K_{n_2}$ if $n_2\subseteq n_1$. Hence by (Theorem 2.3, [9]) it follows that $F-\lim\sup_D(K_n)=\wedge\{K_n, n\in D\}$. Also, we have:

$$\wedge\{K_n, n\in D\} = (\vee\{K_n^c : n\in D\})^c = (\vee\{A : A\in \Re\})^c$$

For the family $\Re$ we have $\vee\{A : A\in\Re\}\geq\alpha$. Thus $\wedge\{K_n, n\in D\}$ $=(\vee\{A: A\in\Re\})^c\leq 1-\alpha$. And therefore $F-\limsup_D(K_n)\leq 1-\alpha$. By assumption there exists an element $n_0\in D$ for which $K_n\leq 1-\alpha+\varepsilon$, for every $n\in D, n\geq n_0$. By the above we have: $K_{n_0}^c=\vee\{A: A\in n_0\}\ \geq\alpha-\varepsilon$, and therefore the family $\{A: A\in n_0\}$ is a countable finite subfamily of $\Re$ such that:

$$\vee\{\overline{A}: A\in n_0\}\geq\vee\{A: A\in n_0\}\geq\alpha-\varepsilon$$

Then the fuzzy topological space X is fuzzy Lindelof closed.

4 Concluding Remarks

In this paper, we have presented an Lindelof closed and a fuzzy Lindelof closed. In other words, these two spaces guide us to investigate new spaces. A topological space X is said to be a semi Lindelof closed if and only if for each semi open cover $\{A_l : l\in L\}$ of X, there exists a countable subfamily $\{A_l : l\in L^*\}$ of $\{A_l : l\in L\}$ whose closures cover X. Also, X is said to be a fuzzy semi Lindelof closed if and only if for every family ψ of fuzzy semi open sets of X and for every $\alpha\in I$ such that $\vee\{U : U\in\Psi\}\geq\alpha$ and for every $\varepsilon\in(0,1]$, there exists a countable sub- family Ψ^* of Ψ whose closures satisfy $\vee\{\overline{U} : U\in\Psi^*\}\geq\alpha-\varepsilon$. Then by present research in the current work, the following are interesting questions for future paper considered:

1- What is the possible relationships which considered between fuzzy semi Lindelof closed and each of the following fuzzy spaces: fuzzy Lindelof, fuzzy lightly compact, fuzzy semi regular, C_{II}, fuzzy paracompact, fuzzy separable, and others?

2- What is the possible new types of spaces can be obtained that coincided with the Lindelof closed space?

3- What is the possible new types of fuzzy spaces can be obtained that coincided with the fuzzy Lindelof closed space?

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