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Corrigendum to “Topological entropy for impulsive differential equations” [*Electron. J. Qual. Theory Differ. Equ.* 2020, No. 68, 1–15]

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Abstract. The aim of this corrigendum is two-fold: (i) to indicate the incorrect parts in two propositions of our recent paper with the same title, (ii) to state the correct statements.

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1 Incorrect propositions, their consequences and corrections

The vector impulsive differential equation under our consideration in [1] takes the form

$$\begin{cases} x' = F(t,x),\ t \neq t_j := j\omega,\ \text{for some given } \omega > 0, \\ x(t_j^+) = I(x(t_j^-)),\ j \in \mathbb{Z}, \end{cases} \tag{1.1}$$

where $F\colon \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is the Carathéodory mapping such that $F(t,x) \equiv F(t+\omega,x)$, equation $x' = F(t,x)$ satisfies a uniqueness condition and a global existence of all its solutions on $(-\infty,\infty)$. Let, furthermore, $I\colon \mathbb{R}^n \to \mathbb{R}^n$ be a compact continuous impulsive mapping such that $K_0 := \overline{I(\mathbb{R}^n)}$ and $I(K_0) = K_0$.

Unfortunately, there is a gap in the second part of the proof of the following proposition.

Proposition 1.1 (cf. [1, Proposition 3.1]). *Let $T_\omega\colon \mathbb{R}^n \to \mathbb{R}^n$ be the associated Poincaré translation operator along the trajectories of $x' = F(t,x)$, such that $K_1 := T_\omega(K_0)$ and $K_0 \subset K_1$. Then the equality*

$$h\left(I\big|_{K_1} \circ T_\omega\big|_{K_0}\right) = h\left(I\big|_{K_0}\right) \tag{1.2}$$

holds for the topological entropies h of the maps $I\big|_{K_1} \circ T_\omega\big|_{K_0}\colon K_0 \to K_0$ and $I\big|_{K_0}\colon K_0 \to K_0$.

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Since the equality (1.2) was used in the proof of the first main theorem (see [1, Theorem 3.5]), this theorem can be corrected in the simplest way, when assuming (1.2) or, more generally the inequality

$$h\left(T_\omega|_{K_0} \circ I|_{K_1}\right) \geq h\left(I|_{K_0}\right), \tag{1.3}$$

explicitly. Then the following correction has rather a character of a proposition.

Theorem 1.2. *The vector impulsive differential equation* (1.1) *exhibits under* (1.3) *chaos in the sense of a positive topological entropy of the composition* $I|_{K_1} \circ T_\omega|_{K_0}$*, i.e.* $h(I|_{K_1} \circ T_\omega|_{K_0}) > 0$*, provided* $I(K_0) = K_0$ *and* $K_0 \subset K_1$*, where* $K_0 := \overline{I(\mathbb{R}^n)}$ *and* $K_1 := T_\omega(K_0)$*, jointly with* $h(I|_{K_0}) > 0$.

Despite this gap, all the related illustrative examples (see [1, Examples 3.7–3.9]) can be shown to be correct, when verifying (1.3), by means of e.g. a slightly generalized version of [2, Proposition 3.2].

The same type of a gap is in the proposition for the problem (1.1) considered, under the natural additional assumptions

$$F(t, \dots, x_j, \dots) \equiv F(t, \dots, x_{j+1}, \dots), \quad j = 1, \dots, n, \tag{1.4}$$

and

$$I(\dots, x_j, \dots) \equiv I(\dots, x_{j+1}, \dots) \ (\operatorname{mod} 1), \quad j = 1, \dots, n, \tag{1.5}$$

on the torus $\mathbb{R}^n/\mathbb{Z}^n$ (see [1, Proposition 4.1]). Quite analogously, the second main theorem (see [1, Theorem 4.3]) can be corrected by the additional technical assumption

$$h\left((\tau \circ T_\omega) \circ (\tau \circ I)\right) \geq h(\tau \circ I), \tag{1.6}$$

where $\tau\colon \mathbb{R}^n \to \mathbb{R}^n/\mathbb{Z}^n$ denotes the natural projection.

Since on tori, we have to our disposal the Ivanov inequality for the lower estimate of topological entropy in terms of the asymptotic Nielsen numbers (see [4] and cf. [1, Proposition 2.7]), the third main theorem in [1, Theorem 4.6] remains valid, even without verifying (1.6), in the following way.

Theorem 1.3. *Consider, under the above assumptions and* (1.4), (1.5), *the vector impulsive differential equation* (1.1) *on* $\mathbb{R}^n/\mathbb{Z}^n$*. Assume that the impulsive mapping* $(\tau \circ I)\colon \mathbb{R}^n/\mathbb{Z}^n \to \mathbb{R}^n/\mathbb{Z}^n$ *is homotopic to a continuous map* $f\colon \mathbb{R}^n/\mathbb{Z}^n \to \mathbb{R}^n/\mathbb{Z}^n$ *such that* $N^\infty(f) > 1$*, i.e.*

$$\limsup_{m\to\infty} |\lambda(f^m)|^{\frac{1}{m}} > 1,$$

where $\lambda(f^m)$ *stands for the Lefschetz number of the m-th iterate of* f*.*

Then

$$\begin{aligned} h\left((\tau \circ I) \circ (\tau \circ T_\omega)\right) &\geq \limsup_{m\to\infty} \frac{1}{m} \log N\Big(\left((\tau \circ I) \circ (\tau \circ T_\omega)\right)^m\Big) \\ &= \limsup_{m\to\infty} \frac{1}{m} \log N\left((\tau \circ I)^m\right) = \limsup_{m\to\infty} \frac{1}{m} \log N\left(f^m\right) > 0 \end{aligned}$$

holds, where $N(f^m)$ *denotes the Nielsen number of the m-th iterate of* f*, and subsequently equation* (1.1) *exhibits on* $\mathbb{R}^n/\mathbb{Z}^n$ *chaos in the sense of a positive topological entropy of the composition* $(\tau \circ I) \circ (\tau \circ T_\omega)$.

That is also why that all the related illustrative examples (see [1, Examples 4.5, 4.7, 4.9]) remain on this basis correct.

2 Concluding remarks

To verify the inequalities (1.3) and (1.6) is not an easy task (see e.g. [3]). We will try to affirm them at least in some particular cases elsewhere. In $\mathbb{R}$, the most promising way seems to be via the statements along the lines of [2, Proposition 3.2].

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