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QUASI SPECTRAL DATA CONSTRUCTION IN TWO POINTS IN PARTIALLY NONHOMOGENEOUS TURBULENCE

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Abstract. In this work, we study the numerical solution of the equations of correlations -or moment of order two - associated with the Navier-Stokes equations. We treat the spectral transformations of these equations by admitting directions of homogeneities; the problem of finding suitable initial conditions within the framework of the numerical resolution and the writing in two points is dealt with. We provide a new method of construction of these initial conditions in an intermediate space between physical space and spectral space (quasi spectral space). This original method departs from the formalism known in the homogeneous case and takes into account the presence of the walls. It is all the more interesting as the experimental data never give enough point of calculation making it possible to obtain these quantities in a quasi spectral space.

1. Introduction

When one is interested in nonhomogeneous turbulence, the Navier-Stokes equations (1.1) and the models based on a description in only one point such as turbulent viscosity [1], "the $k-\varepsilon$" [8] or other models, provide only one partial description of all of the phenomena associated with these flows. One thus expects the development of new writings in two (or more) points (i.e. descriptions taking into account the interaction between the various structures of turbulence).

$$\frac{\partial}{\partial t}V_i + V_j\frac{\partial}{\partial x_j}V_i - \eta\Delta V_i + \frac{\partial}{\partial x_i}P = 0 \tag{1.1}$$

where V_i is the velocity component along x_i, P is the pressure, and Δ is the Laplacian

The models in two points are based on the resolution of the equations of the correlations [2], $Q_{ij}(x,x')$, given by

$$Q_{ij}(x,x') = \langle v_i(x)v_j(x')\rangle \tag{1.2}$$

where $\langle\cdot\rangle$ stands for the statistical average and where v_i $(i=1,2,3)$ are the velocity fluctuations and are defined by $V_i = \langle V_i\rangle + v_i$ One must then solve the

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tensor of Reynolds (1.3) in which each term is described by a nonlinear equation containing other terms of the tensor.

$$\begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix} \tag{1.3}$$

Moreover, to be able to carry out a complete analysis, we must simultaneously determine turbulence in physical and spectral spaces. Obviously, development of such models is not easy and the numerical resolution of the new equations obtained starting from the equations of Navier-stokes is even more complicated; we must deal with problems in terms of mathematical formalism, numerical processing and physical modelling [3, 4, 5, 7].

A priority, for solving correlations equation, is fixed in the search for formulations being able to lower the constraints of calculation and storage.

Besides, The complexity of the double correlation's tensor and the non linearity make very difficult the control of the evolution of any inaccuracy allowed at the beginning of calculations.

Our study led us to the quasi spectral formulation. The one-dimensional Fourier transforms are employed according only to the homogeneity directions.The partial equations with the derivative are the simplest and allow exploiting symmetries which are presented by the flow geometry (non homogeneous partial turbulence). The space quasi spectral contains at the same time components of the wave number - according to one or several homogeneity directions - and of the Cartesian coordinates (on a nonhomogeneous directions).

Besides the physical interest, calculations in the quasi spectral domain present two other advantages:

- It makes it possible to write Green's functions in an exact analytical form.
- It allows optimization of the problem of the partial derivative while making it possible to take into account various degrees of homogeneity of the nonhomogenous flow (partially nonhomogenous turbulence).

As mentioned previously, the production of the data in a quasi-spectral space for the physical quantities of references constitutes a problem because of the lack of points of calculation: for reasons of wake of probe, for example, the experiments never provide enough points of calculation to evaluate these quantities in the quasi spectral space.

It is thus important, due to the complexity of the equations and the non linearity, to determine suitable initial conditions, and to set up processes which, on the basis of homogeneous quantities, take into account the presence of walls and produce physically acceptable nonhomogeneous quantities. We propose in what follows the processes that we built for this purpose and discuss their advantages and disadvantages.

2. Nonhomogeneous data construction

The problem of the influence of the starting data always arises when one wants to judge the quality of a physical model. A suitable representation of these data is of primary importance. In this paragraph we provide techniques which make it possible to build these data in a quasi-spectral space.

Let x_2 be a direction of non homogeneity transverse to the wall. $N(n_1, n_2, n_3)$ and $M(m_1, m_2, m_3)$ be two points of the wall by which the transversal line passes, $X(x_1, x_2, x_3)$ an internal point belonging to $[N, M]$ (see figure 1)

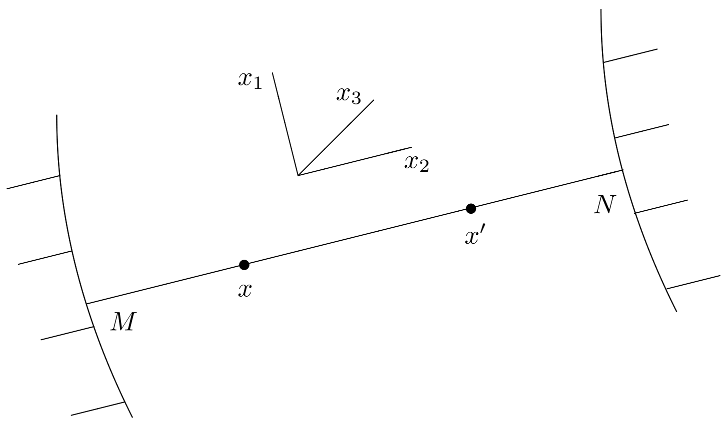

FIGURE 1. One direction of non homogeneity (x_2)

Let us consider the characteristic scales of the flow where ϵ is dissipation, u's are velocities, l is the length, and $\vec{K}(k_1, k_2, k_3)$ is the wave number with K its module.

Let $E^h(K)$ be the homogeneous and isotropic energy spectrum associated on the scalesu, ϵ and l. $E^h(K)$ can come from a modelling of the type K-ϵ or turbulent viscosity (or different as we will take it further).

The homogeneous and isotropic spectral correlation Φ_{ij}^h associated to $E^h(K)$ is calculated from

$$\Phi_{ij}^h(k_1, k_2, k_3) = \frac{E^h(K)}{4\pi K^2}(\delta_{ij} - \frac{k_i k_j}{K^2}) \tag{2.1}$$

The passage to a quasi spectral space according to x_2 is done by applying the traditional Fourier transforms:

$$\hat{Q}_{ij}^h(k_1, r, k_3) = \hat{Q}_{ij}^h(k_1, x_2, x_2 + r, k_3) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi_{ij}^h(k_1, k_2, k_3)\exp(ik_2 r)dk_2$$

The problem now is to produce non homogeneous correlations $\hat{Q}_{ij}$. For this purpose, it is initially necessary to notice the following differences between the functions $\hat{Q}_{ij}^h$ and the nonhomogeneous correlations $\hat{Q}_{ij}$ according to x_2:

(i) The homogeneous correlations verify the condition:

$$\hat{Q}_{ij}^h(k_1, x_2, x_2 + r, k_3) = \hat{Q}_{ij}^h(k_1, x_2, x_2 - r, k_3)\,.$$

Which is significantly false in the nonhomogenous case especially close to the walls where the double correlations have a broad asymmetry.

(ii) Boundary condition at the frontiers

$$Q_{ij}^h(k_1, x_2, n_2, k_3) = Q_{ij}^h(k_1, x_2, m_2, k_3) = 0$$

(n_2 and m_2 coordinates of the transverse frontier points) are not respected.

To estimate the non homogeneous correlations Hamadiche [6] has proposed to take

$$\hat{Q}_{ij}(k_1, x_2, x_2 + r, k_3) = \frac{1}{2\pi} \exp(ik_2 r) \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Phi_{ij}^h(k_1, k_2, k_3) dk_2 \tag{2.2}$$

and to cancel these quantities at the wall. However this writing is symmetrical and does not generate contributions for the imaginary parts although these last-mentioned contain information about the signal obtained after Fourier transformation.

First model. To improve the preceding writing the first idea [4] consists in using the functions of cap of Lebesgue: $Lx_2()$ is linear on $[x_2, m_2]$ and $[n_2, x_2]$ respectively, $Lx_2(n_2) = 0$, $Lx_2(m_2) = 0$, $Lx_2(x_2) = 1$ (See figure2).

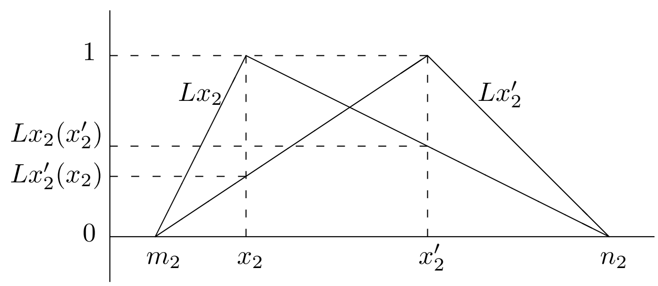

Figure 2. Lebesgue's functions

One defines then the non-homogenous correlation as follows:

$$\hat{Q}_{ij}(k_1, x_2, x'_2, k_3) = \hat{Q}^h_{ij}(k_1, x_2, x'_2, k_3)\ L_{x_2}(x'_2)$$

This interpretation has the advantage of making it possible to gradually take into account the presence of the walls. One can however check that it does not establish the fundamental equality

$$\hat{Q}_{ij}(k_1, x_2, x'_2, k_3) = \hat{Q}_{ji}(k_1, x'_2, x_2, k_3) \tag{2.3}$$

because $Lx_2(x'_2) \neq Lx'_2(x_2$ (see figure 2) which lead us to try to improve it.

Second model: Information by the First Most Distant Point (IFMDP). We develop another technique which has the advantage of observing all the conditions quoted previously. It consists in informing the points of the most distant wall initially and proceeding by intervals in the following way: To construct $\hat{Q}_{ij}(k_1, x_2, ., k_3)$ (for fixed values of x_2, k_1 and k_3)

1- One is interested initially in the interval $[x, M]$ for all the values of x_2, we inform only the half interval starting from x_2 and moving towards the most distant wall from the point of calculation (interval $[x_2, m_2]$) for all the values of i and j. See two distinct examples of functions $\hat{Q}_{ij}(k_1, x_2, ., k_3)$ and $\hat{Q}_{ji}(k_1, x'_2, ., k_3)$ in figure 3.

2- Then, the rest of the points in the interval $]N, X[$ (for example $\hat{Q}_{ij}(k_1, x_2, x'_2, k_3)$) are precisely indicated with the help of equation (2.3). Figure 4 represents the manner of informing $\hat{Q}_{ij}(k_1, x_2, x'_2, k_3)$.

The IFMDP technique is thus more precise and more adequate for the partial non homogenous case. It has the advantage of making it possible to gradually take

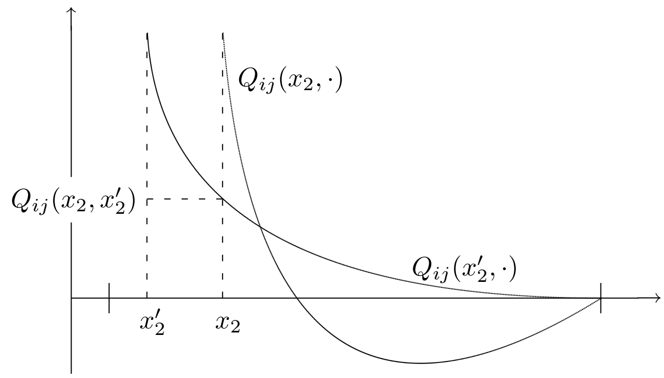

FIGURE 3. Construction of $\hat{Q}_{ij}(k_1, x_2, ., k_3)$ by IFMDP

into account the presence of the walls, it verifies both the two conditions (i) and (ii) and fundamental equation (2.3). We will further use IFMDP during numerical tests of validation of the new mathematical closing of the pressure problem.

3. Conclusion

The spectral nature of turbulence has been taking into account by using the translation of the equations in the quasi spectral space. This space, intermediary between physical and spectral spaces, is an additional field of study of turbulence and its evolution. The associated formulation especially makes it possible to divide, in stages, the consideration of the degrees of non homogeneity. It also changes the nature of the equations to be solved by decreasing considerably the problems of numerical "congestion" which generally characterize this type of treatments.Among the methods of production of data in a quasi-spectral space for the double correlations IFMDP technique is more precise and more adequate for the partial non homogenous case. It has the advantage of making it possible to gradually take into account the presence of the walls, it verifies both the two conditions (i) and (ii) and the fundamental equation (2.3).

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