

Consistent histories: Description of a world with increasing entropy *

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Abstract

A distinction is made between two kinds of consistent histories: (1) robust histories consistent by virtue of decoherence, and (2) verifiable histories consistent through the existence of accessible records. It is events in verifiable histories which describe amplified quantum fluctuations. If the consistent-history formalism is to improve on the Copenhagen interpretation by providing a self-contained quantum representation of the quasi-classical world, the appropriate quantum state must track closely all macroscopic phenomena, and the von Neumann entropy of that quantum state ought to change in the same direction as the statistical entropy of the macro-world. Since the von Neumann entropy tends to decrease under successive branchings, the evolution of an entropy-increasing quasi-classical world is not described by a process of branchings only: mergings of previously separate histories must also occur. As a consequence, the number of possible quasi-classical worlds does not have to grow indefinitely as in the many-world picture.

1 Self-contained quantum description

The consistent-history program [3, 5, 8] represents a more concerted effort than previous attempts to overcome what some workers regard as the main defect of the Copenhagen interpretation: the need to invoke the existence of a classical world as a pre-condition to the meaningful interpretation of quantum mechanics. Hence, just as for any other attempt at a self-contained quantum description, the first order of business for this program is to represent the classical world correctly. It is precisely on this point, however, that the consistent-histories program has not yet been carried to completion to the satisfaction of all concerned. It is not the purpose of this note to discuss the nature of the difficulties encountered; rather, assuming that there is a quantum description of the quasi-classical world along the lines of consistent-histories, I ask what the description will be like. In particular, I investigate the rule of association between the histories in

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a suitable family and the quasi-classical worlds that the formalism is supposed to describe.

A family of consistent histories is specified by an initial state $\rho(0)$, the Hamiltonian of the world H , and sequences of events. Each sequence is represented by a chain of Heisenberg-picture projectors:

$$C(\alpha_1\alpha_2\dots\alpha_n) \equiv E_1^{\alpha_1}(t_1)E_2^{\alpha_2}(t_2)\dots E_n^{\alpha_n}(t_n), \quad (1)$$

where the subscript on the projector $E_i^{\alpha_i}$ refers to the nature of the resolution of the identity, and the superscript to the specific element within that set of projectors. The probability $P(C)$ for the occurrence of a history corresponding to the chain C is:

$$P(C) = \text{TR}(C^+\rho(0)C). \quad (2)$$

All histories within a family must fulfill certain consistency conditions so that the probabilities for their being realized satisfy normal additivity rules. The Eq.(1) has been given in the simplest form which does not take the possible branch-dependence of projections into account, but I will later return to the issue of branch-dependence where it might be relevant to the subject of this note.

Consistency alone does not guarantee that the events in a history correspond to what in the Copenhagen interpretation would be called “actualized” or “registered” events. In the absence of external observers, it appears that a selection criterion needs to be added to the consistent-histories formalism to characterize such special events, which correspond to the ordinary experience that these occurrences actually happened. It turns out that a mathematical formulation of such a criterion is no easy task¹ But the physical basis that qualifies certain types of events as actualized can be examined, and is discussed in some detail in the Appendix. Roughly speaking, the essence of actualization is verifiability from accessible records; and events which may be regarded as having been actualized or registered will be called “verifiable” from here on. A verifiable event is different from a robust event², the consistency of which is maintained only by decoherence. Decoherence ensures that the different alternatives will not later interfere, but does not guarantee that only one of the alternatives is “actually realized.” It is only when the accessible records in a branch can corroborate which alternative is realized that one has a verifiable event. A similar point has been made by Zurek and co-workers within the framework of what they call “the existential interpretation of quantum mechanics” [9], although it is not always clear whether they want to make a sharp distinction between a robust event, where the “record” may consist of little more than scattered photons which escape to infinity, and a verifiable event supported by accessible records. The main point of this note is sufficiently simple that the precise

¹Hartle and Gell-Mann have presented various sets of criteria for classicity. In a telephone conversation, Jim Hartle informed me in 1998 that none of these fully satisfied them. For other views on some of the difficulties, see, e.g., the reference [2].

²The distinction between robust histories and verifiable histories has been emphasized by the author in Found. of Phys. Lett., **6**, 275 (1993).

mathematical conditions qualifying an event as verifiable, which have resisted formulation so far, are not needed; but it is necessary to assume that a selection criterion does exist. The formulation of a set of clearly stated conditions for distinguishing certain events as verifiable is absolutely essential for the completion of the consistent-histories program, because this program tries to describe the classical world with events, and the quantum events which are directly relevant to the description of occurrences in the observable world are exclusively verifiable events.

Already in classical physics it is imagined that there exists an underlying fine-grained structure, the complete details of which can never be checked within the limitations imposed by the availability of resources. Turning to a quantum description of the world adds the need to consider superpositions with phase correlations, and subtracts the possibility of certain types of simultaneously precise data, but it is not greatly different in spirit as far as entertaining the idea of the world as a closed system is concerned. In particular, among classical statistical physics expositions there are statements like “the entropy of the world, regarded as a closed system, is non-decreasing.” The question this article addresses is: What is the nature of the appropriate coarse-grained quantum state, so as to describe a quasi-classical world in which entropy increases?

It should be made clear at the outset that the goal here is only to clarify the nature of the appropriate quantum representation. There is no claim that this quantum description provides any *a priori* explanation for irreversibility – the common assumption that the world, under suitable coarse-graining, was in a particularly low entropy state once, and is still relatively low in entropy today, as the cause for the validity of the second law in the present era, will again be needed (in the form of an assumption on the near saturation of records, see section 3). But even with such a modest goal something can be learned. Because the aim of a self-contained quantum description of the world is so ambitious, whereas the language of consistent histories is so very economical – thus far the branching of histories is essentially the only type of events seriously considered – it turns out that even the mere task of describing entropy increase requires some modification of that perspective. The modification consists of the realization that the process of merging two or more histories together is as relevant to describing the observable world as the opposite process of splitting a history into separate branches. As a consequence, the number of possible quasi-classical branches does not increase indefinitely. In other words, there is no population explosion of the kind in Everett’s many-world picture.

2 Average entropy is non-increasing under branching

Although some workers prefer to speak of only histories and not the instantaneous density matrix of the world, most researchers, once taking the plunge of considering a self-contained quantum description of the world as a whole, do not

seem to object to this notion. Since the purpose here is to provide a quantum representation of the change in time of the macroscopic world of verifiable facts, it is appropriate to have an instantaneous density matrix to correspond to the situation at a specific instant. The consistent-histories formalism suggests a natural density matrix to consider. The matrix $\rho(C, t_n)$ defined by

$$\rho(C, t_n) = C^+(\alpha_1 \dots \alpha_n) \rho(0) C(\alpha_1 \dots \alpha_n) / \text{TR}[C^+(\alpha_1 \dots \alpha_n) \rho(0) C(\alpha_1 \dots \alpha_n)] \quad (3a)$$

is a suitable candidate, because it has the property that the conditional probability for the outcome of the next event to be that particular alternative represented by $E_{n+1}^{\alpha_{n+1}}(t_{n+1})$, given the fact that the past history is already specified by $C(\alpha_1 \dots \alpha_n)$, equals appropriately $\text{TR}[\rho(C, t_n) E_{n+1}^{\alpha_{n+1}}(t_{n+1})]$. The density matrix is alternatively defined by the recursive relation

$$\rho(C(\alpha_1 \dots \alpha_n), t_n) = E_n^{\alpha_n}(t_n) \rho(C(\alpha_1 \dots \alpha_{n-1}), t_{n-1}) E_n^{\alpha_n}(t_n) / N$$

where

$$N \equiv \text{TR}[E_n^{\alpha_n}(t_n) \rho(C(\alpha_1 \dots \alpha_{n-1}), t_{n-1})]. \quad (3b)$$

The way this density matrix enters the probability for the next event recommends it as a candidate for a quantum representation for the instantaneous state of the world, provided $\rho(0)$ is a suitable initial state of the world.

Only three things enter into $\rho(C, t_n)$: the initial density matrix, the Hamiltonian, and the chain of projections. The Hamiltonian is presumably fixed once and for all, and so is the initial density matrix. What changes as time progresses is, in the usual consistent-histories formalism, an indefinite elongation of the chain of projections. For a verifiable history, the projections are all supported by classical records, and hence the increase in the information content of the records in the classical world which $\rho(C, t_n)$ is supposed to describe, due to the occurrence of a new event, is no less than the increase in the information content of $\rho(C, t_n)$ itself. The increase of the information content of classical records can be greater, however, because for the quantum state some of the initial information may have been destroyed by the new projection (noncommutivity). One notes that the information content of classical records is really no different from that of the classical world. Mallarme famously said: "Everything there is in the world exists to be put in a book," but the "book" should be generalized to include all types of classical records.

As to the quantum state, since the choice of an initial state must ultimately be justified by empirical support, from the position of trying to obtain a quantum description of the classical world that stays as close to empirical facts as possible, there is no good reason to treat the initial state and later events by different principles. If the later events are factual only by virtue of the records available today, then the initial state is to be determined by the same criterion: maximum ignorance consistent with present data. Indeed, one could let $\rho(0)$ be proportional to identity, and put in the earliest known information through the first projection at time t_1 , and in this way the initial data and subsequent events are treated on the same footing. Formally, the method of choosing a

least-biased state by a variation principle subject to constraints is well known, although in practice this will be incredibly complicated for the real world. If one wants, however, to have a quantum description that is as close to the classical world as possible, leaving out no relevant classical data but also putting in as little extraneous input as possible, then it is the maximally detailed verifiable histories with the least-biased initial state that are relevant. In other words, in principle one would like to have every piece of factual data in the classical world represented in the quantum description: if a classical datum arises out of deterministic evolution, the information resides in the initial state and earlier events; and if it arises out of an amplified quantum fluctuation, then the datum emerges at the point of a new verifiable event. The objective of the consistent-histories program to have a self-contained quantum description of the quasi-classical world is not realized unless there is a family of such “least-biased and maximally detailed verifiable histories.” It will be understood from here on that this is the kind of history with which we are concerned.

From the consideration of information gain, one might think that for such a least-biased and maximally-detailed verifiable history the change in the von Neumann entropy $s[\rho(C, t_n)] \equiv -\text{TR}[\rho(C, t_n)\log\rho(C, t_n)]$ due to the occurrence of a new verifiable event has at least the same sign as the change in the statistical entropy of the classical world that this history describes. Making this association, however, gives rise to an immediate difficulty. We state this difficulty as follows:

Proposition 1 *Suppose that (i) there is a faithful quantum description of quasi-classical worlds by means of a family of least-biased and maximally-detailed verifiable histories, which undergo only branchings, and (ii) the change in the von Neumann entropy of the density matrix of Eq.(3) due to a verifiable event is in the same direction as the change in the classical statistical entropy of the world being described. Then on the average the second law is violated in these worlds.*

This proposition follows directly from a theorem of Groenewold and Lindblad. Groenewold first conjectured [4] that under a branching the average entropy of a branch is equal to or less than that of the parent-state, but a geometrical approach to proving this conjecture turned out to be difficult. Lindblad [7] then gave an elegant proof drawing on some previous results.

Remark Readers interested in a quick grasp of the entropy non-increasing nature of branching may consider the quadratic entropy $s'[\rho] = -\text{TR}\rho^2$ instead of the von Neumann entropy, for in that case the analog of the Lanford-Robinson result used by Lindblad, viz., $s'[A + B] \leq s'[A] + s'[B]$ for positive trace-class operators A and B , is trivially true.

Theorem 1 (Groenewold-Lindblad) *If $\rho'_\alpha \equiv E^\alpha \rho E^\alpha / p_\alpha$ and $p_\alpha \equiv \text{TR}(E^\alpha \rho)$, then*

$$\overline{s[\rho']} \equiv \sum_\alpha p_\alpha s[\rho'_\alpha] \leq s[\rho]. \tag{4}$$

It is understood here that the set $\{E^\alpha\}$ consists of orthogonal projectors which resolve the identity. Since any chain of projections entering into the specification of a history is built up from projections satisfying the conditions of this theorem, it follows that, when averaged over the branches of each splitting, the entropy for the density matrix $\rho(C, t_n)$ is non-increasing with n , regardless of what kind of coarse-graining has been built into the projections. This theorem does not rule out the possibility that along a particular branch the entropy is non-decreasing, but that would be an exceptional situation requiring an explanation, for it goes against the average trend of Eq.(4), which is for the entropy associated with $\rho(C, t_n)$ not to increase with increasing n .

3 The merging of histories

We have used expressions like “the observable world” and “accessible records” without being specific on “observable” and “accessible” to whom. This is because a detailed examination would have involved a lengthy digression, and I decided to relegate all such matters to an appendix. At this point let it be tentatively supposed that these expressions have sensible meanings. Then the entropy-decreasing tendency of branching is at least intuitively comprehensible from the information-gain argument made earlier. This argument suggests that entropy-decrease becomes unavoidable for a history in which all the events are verifiable. The quoted theorem says that entropy tends to decrease even when earlier information can be partially destroyed, but for verifiable events the records keep the earlier information intact, thus making it even more likely that entropy would decrease with the prolongation of a history, i.e., with the accumulation of more and more records.

The most natural way out of the quandary created by a tendency for the von Neumann entropy to decrease under branching is to take into account the fact that in a world of increasing entropy records do decay. If a branching in the past is still relevant later because of the persistence of its associated accessible records, then the decay of records, by their becoming inaccessible as evidence, would partially undo the effect of that branching. Such partial undoing is what I call “merging.” It is desirable to incorporate merging without spoiling the simplicity of the consistent-histories approach. The following proposal for bringing the deterioration of records into the quantum description of a closed system is guided by two considerations: (a) the verifiability of a given event is a time-dependent property, since records change with time; and (b) the decay of the records concerning a past event is not the same as a quantum erasure. The implication of the first premise is that, unlike a branching which involves just one time, a merging commonly involves two times: an earlier moment when the event and its registration occurred, and the later moment when the records concerning the outcome of that event are obliterated. The implication of the second premise is that the decay of records does not completely undo a projection. If one were to represent the decay of the records concerning an event at time t_i by removing the projection $E_i^{\alpha_i}(t_i)$ altogether from the chain C which

helps to define $\rho(C, t_n)$, it would be as if no event happened at t_i at all; whereas the decay of records presupposes that an event did occur, and it was even verifiable at one stage, and the different outcomes were decoherent. When the records decay at time t_n and it is no longer possible to verify the outcome of what actually happened at t_i , the relative likelihood of the various alternatives being the best that can be deduced from the surviving evidence, then these alternatives are to be incoherently summed. In contrast, removal of the projection at t_i would correspond to a quantum erasure, with the alternative components added back together with exactly the correct phases, which is imaginable in a laboratory setting but is not realistic for the overwhelming majority of actual events. The expression “merging of histories” for describing the incoherent summation seems appropriate because of the following consideration. The most common pictorial representation for the structure of a family of histories is that of a branching tree. Although the summation over those branches which the records can no longer distinguish is akin to bundling several branches together rather than fusing them, the subsequent offshoots undergo a real reduction in number. For example, a branching into n branches followed by a second splitting each into m branches would result in a total of nm alternatives; but if the records for the first branching are later destroyed, the final outcome leaves only m alternatives, and in that sense the nm branches have merged into the m branches. The situation described corresponds to a family structure where the projections are not branch-dependent. If the later projections depend on which branch the event is taking place in, the situation becomes more complicated. But the existence of branch-dependence by itself provides some sort of a record, and therefore in considering the complete destruction of records, we limit ourselves to situations where branch-dependence is absent.

The mathematical formulation of merging is relatively straightforward. Thus the erosion at time t_n of the records concerning an event at t_i , where $t_n > t_i$, is to be represented by the transformation

$$\begin{aligned} T : \rho(C(\alpha_1 \dots \alpha_i \dots \alpha_{n-1}), t_{n-1}) &\rightarrow \rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_{n-1}), t_n) \\ &\equiv \sum_{\alpha_i} b_{\alpha_1 \dots \alpha_i \dots \alpha_{n-1}} e^{-iH(t_n - t_{n-1})} \rho(C(\alpha_1 \dots \alpha_i \dots \alpha_{n-1}), t_{n-1}) e^{iH(t_n - t_{n-1})} \end{aligned}$$

where

$$\begin{aligned} p_{\alpha_1 \dots \alpha_i \dots \alpha_{n-1}} &\equiv \text{TR}[C^+(\alpha_1 \dots \alpha_i \dots \alpha_{n-1}) \rho(0) C(\alpha_1 \dots \alpha_i \dots \alpha_{n-1})] \\ b_{\alpha_1 \dots \alpha_i \dots \alpha_{n-1}} &\equiv p_{\alpha_1 \dots \alpha_i \dots \alpha_{n-1}} / \sum_{\alpha'_i} p_{\alpha_1 \dots \alpha'_i \dots \alpha_{n-1}}. \end{aligned} \quad (5)$$

This corresponds to a situation where the last step in the destruction of records is through the deterministic processes which result from the evolution generated by $\exp[-iH(t_n - t_{n-1})]$. If, on the contrary, the destruction of records is itself accompanied by the actualization of a new quantum event, then one has instead:

$$T' : \rho(C(\alpha_1 \dots \alpha_i \dots \alpha_{n-1}), t_{n-1}) \rightarrow \rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_n), t_n)$$

$$\equiv \sum_{\alpha_i} b_{\alpha_1 \dots \alpha_i \dots \alpha_n} \rho(C(\alpha_1 \dots \alpha_i \dots \alpha_n), t_n)$$

where

$$\begin{aligned} p_{\alpha_1 \dots \alpha_i \dots \alpha_n} &\equiv \text{TR}[C^+(\alpha_1 \dots \alpha_i \dots \alpha_n) \rho(0) C(\alpha_1 \dots \alpha_i \dots \alpha_n)] \\ b_{\alpha_1 \dots \alpha_i \dots \alpha_n} &\equiv p_{\alpha_1 \dots \alpha_i \dots \alpha_n} / \sum_{\alpha'_i} p_{\alpha_1 \dots \alpha'_i \dots \alpha_n}. \end{aligned} \quad (6)$$

The proposal is that the instantaneous quantum state suitable for describing what is happening in the macro-world corresponds to a density matrix of the form $\rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_n), t_n)$ rather than of the form $\rho(C(\alpha_1 \dots \alpha_i \dots \alpha_n), t_n)$. In other words, the coarse-graining necessary for describing the observable world cannot be all effected through the use of suitably coarse projections alone: certain coarse-graining requires convex summations, as in Eq.(5) and Eq.(6). The resulting change in the formalism is very minor, and in particular the transformation corresponding to a branching is given by:

$$\rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_{n-1}), t_{n-1}) \rightarrow E_n^{\alpha_n}(t_n) \rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_{n-1}), t_{n-1}) E_n^{\alpha_n}(t_n) / N$$

where

$$N \equiv \text{TR}[E_n^{\alpha_n}(t_n) \rho(\bar{C}(\alpha_1 \dots \bar{\alpha}_i \dots \alpha_{n-1}), t_{n-1})], \quad (7)$$

identical in structure to Eq.(3b). It should be noted, however, that the original chains $C(\alpha_1 \dots \alpha_n)$ are still relevant because the additional consistency conditions to be fulfilled with the introduction of any new projection $E_n^{\alpha_n}(t_n)$ are in terms of the individual $C(\alpha_1 \dots \alpha_i \dots \alpha_n)$ and not in terms of a sum over α_i .

The processes represented by Eq.(5) are clearly entropy non-decreasing. For the process of Eq.(6) where $t_n > t_i$, or processes where Eqs.(5),(6) and (7) all contribute, there is competition between the entropy non-decreasing convex summations and the average entropy non-increasing additional branchings. Without further input it is not possible to say which tendency wins. If, however, the world's capability for carrying records is already near saturation, in the sense that the creation of new records requires in most cases the destruction of some existing records, then records are almost continuously being created and destroyed. Over the course of many such creations and destructions there is a sense in which the entropy-increasing trend wins. This is because of the fact that the average decrease in entropy due to a branching event is necessarily less than or equal in magnitude to the average increase in entropy when subsequently the records corroborating this event are destroyed.

This can be proved as follows. With the notation of Eq.(4), the absolute magnitude of the average decrease in entropy as a result of a branching is $s[\rho] - \sum p_\alpha s[\rho'_\alpha]$, and the average entropy increase accompanying the destruction of those records which verify the outcome of the branching is $s[\sum p_\alpha \rho'_\alpha] - \sum p_\alpha s[\rho'_\alpha]$. The fact that the entropy is non-decreasing as a result of these two steps follows trivially since $s[\sum p_\alpha \rho'_\alpha] \geq s[\rho]$.

Thus, analogous to the usual assumption that the world is currently in a low-entropy state, the operative assumption here that can lead to a tendency

for entropy increase is that the present world is already near saturation for record-keeping, so that records are continuously being generated and destroyed. The degree of plausibility of this assumption is also discussed in the Appendix.

Bennett [1] and Landauer [6] have already pointed out that entropy increase is associated with the erasure of records. Their analysis was in a classical setting, and therefore they did not address the entropy-decreasing tendency associated with the branching of quantum histories, nor need they be concerned with the difference between classical erasure and quantum erasure. Their starting point involving Maxwell's demons and computers seemingly has very little in common with our starting point of formulating consistent histories in order to describe an entropy-non-decreasing world. Nevertheless their analysis is relevant to our consideration in two respects. One is Bennett's demonstration that it makes sense to speak of the entropy of a single system instead of that of an ensemble – the quantum history description of the quasi-classical world is most simply viewed as dealing with a single system. The second relevant point is their analysis showing that measurement and copy-making need not be accompanied by an entropy increase. Such reversible record-keeping is highly idealized, and most practical record-making is irreversible. Nonetheless as a matter of principle it is important to recognize that record-making is not unavoidably entropy-increasing, whereas record-erasing is. Our application of the average entropy-decrease of Eq.(4) to an amplified quantum fluctuation, i.e., to a measurement-like event, should be viewed as referring to a measurement with ideal record-making. Once the nature of entropy increase is clarified in such ideal events, then the additional entropy-increase associated with the difference between ideal record-making and realistic record-making can be accounted for as an accumulation of elementary events, each accompanied by the destruction of some previously existing records.

There is an alternative possibility for accommodating entropy increase as long as one is willing to add convex sums to the consistent-histories formalism. Noting the existence of many physical processes having outcomes that are decoherent but not verifiable, one may be tempted to adopt the rule that every time such an event occurs the state representing the quasi-classical world becomes an incoherent sum over these alternative outcomes. This modification will also bring about an entropy increase. The defect of this approach is the arbitrariness in the choice of “relevant variables” which decohere as a result of summing over the “irrelevant” variables, that is, an arbitrariness in the division between the environment and the subsystem of interest. If the alternatives involving the subsystem are not accessible to verification, why are they “relevant” and “of interest”? Remember that one is attempting a closed-system description here, and there is no pre-determined environment. Our proposal is close to what are verifiable, and requires incoherent sums only when there is a destruction of records, i.e., when formerly verifiable alternatives later become unverifiable.

Lastly, although I have argued that the dilemma posed by the Proposition is to be resolved by adding merging to branching, others may wish to avoid the conundrum altogether by arguing that the change of von Neumann entropy is

unrelated to that of the classical entropy³. If one were to look at the entropy changes during a measurement-like event, whether it occurred in nature or in the laboratory, by the usual way of reckoning, it would indeed appear that the latter position is obviously valid. For example, a quantum event having two possible outcomes in a particular measurement situation can bring about a decrease in the von Neumann entropy of the density matrix for the quantum system by at most $k \log 2$, but the recording of the outcome is usually associated with irreversible processes, causing a rise in the classical entropy of the world by an amount far exceeding $k \log 2$. Thus the signs are opposite and the magnitudes do not match. One must remember, however, from the analysis by Bennett and Landauer that irreversibility is ultimately attributable to erasure, that is to say, to processes described by Eq.(5) or Eq.(6). Hence the overall change in the statistical entropy of the world can be accounted for by the change in the von Neumann entropy of $\rho(\bar{C})$. In other words, the merging of histories has to be taken into account. The two changes may not be exactly equal because of the possibility of the destruction of some quantum information by a new projection, discussed earlier, but the inequality is such that an increase in classical statistical entropy requires a corresponding increase in the von Neumann entropy of $\rho(\bar{C})$ if the quantum representation is as close to a detailed description of the quasi-classical world as possible. On the other hand, if one insists on having only branching and no merging, then the change in the von Neumann entropy of $\rho(C)$ is indeed unrelated to the change in the classical entropy of the world. But to adopt this as the solution is to unnecessarily impoverish the consistent-histories formalism, rendering only a very partial quantum description of some aspects of the quasi-classical world. Such a skeletal description is likely to be insufficient as replacement for the classical underpinning in the Copenhagen interpretation.

In summary, the tendency for the von Neumann entropy to decrease under branching, on the average, is a fact to be reckoned with. This is regardless of how a coarse-grained quantum state is defined: as long as it undergoes only branchings, the average entropy tends to decrease with time. If the second law in the macro-world is taken as an input, and if a coarse-grained quantum state is to faithfully describe that macro-world so that changes of the von Neumann entropy of the quantum state track those of the classical entropy of the world, then its time evolution has to incorporate the merging of previously verifiably distinct histories. If the consistent-histories program is to improve on the Copenhagen interpretation, it has to provide a faithful description of the quasi-classical world strictly within a self-contained quantum theory. But that task is not finished even when a specific family of histories is singled out by some criterion as being suitable for describing quasi-classical physics, for there must be in addition a rule of association between the histories in that family and a quasi-classical world with all its macroscopic details. And our conclusion is that even if at one time a single history in this fixed family describes that world, it

³I thank Lawrence Landau for raising such objection during my oral presentation at the Symposium; the following elaboration was added subsequently.

will be the incoherent sum of several histories that has to be associated with the same world at a later time. Another way of saying this is that it is inadequate to use projections alone to represent all coarse-graining: there are events which require incoherent sums, besides coarse projectors, to represent what is happening to the quantum state. The suggested modifications are easy to incorporate and does not require any major overhaul of the consistent-histories formalism; but conceptually the picture of an ever multiplying number of potential quasi-classical worlds is changed to one where the population of quasi-classical worlds need not inexorably grow.

Professor Wichmann introduced me to the rich structures of various kinds of density matrices many years ago, and I hope it is not inappropriate to present this paper to him on the occasion of his seventieth birthday. Nonetheless I do so with some trepidation. Under his teaching I began to appreciate the merit of precise mathematical language: even though such language may seem dry and even pedantic at times, he taught that precision and clarity serve to promote intellectual honesty. The subject of this article is such, however, that although I hope to have adhered to the spirit of striving for as much clarity as possible, I was unable to avoid dealing with a variety of notions not yet formulated with mathematical precision.

4 Appendix

This article contains expressions like “observable world,” “information,” “verifiability,” and “accessible records,” which inevitably invite questions: Whose information? Observable and verifiable by whom and accessible to whom? How stable does a physical correlation have to be in order to count as a record? Discussion of these issues leads to what many physicists would dismiss as “philosophy,” and yet avoidance of these issues does not save time. As continuing controversies surrounding the subject of quantum measurement show, some knotty issues refuse to go away⁴. Even when the final answers are not available, it is better to state one’s tentative understanding as clearly and explicitly as possible.

One way to specify the meaning of such expressions is by linking them to operations. The basic issues as far as this paper is concerned are first of all what kind of an event can be said to have verifiably occurred, and subsequently when the event can be said to cease to be verifiable. In an operational sense, an event is verifiable if the community of scientists, on the basis of records, is capable of reaching a consensus on the outcome, provided efforts are devoted to the task of checking this event, limited only by the availability of natural resources. Similarly the information content of the observable world is interpreted to be the total database needed for a complete description of the macroscopic world, including every bit of datum that is verifiable. This way of describing the relation between records and the verifiability of an event does not require

⁴See for example the exchange between J.S. Bell and N.G. van Kampen, both long immersed in the subject, in *Physics World*, August 1990, p.33, and October 1990, p.20.

the records to be unchanging: they can be evolving in time, because all that is required is for the scientists to be able to use them to unequivocally interpret an event in the past. By the same token, the decay of records can take many forms: some due to the corruption of macroscopic information through classical processes, and some through random events in which quantum fluctuations play a role. That is why in the sentence following Eq.(5) a distinction is made between the decay of records through deterministic processes and through quantum fluctuations. That records can decay even when the process results from $\exp[-iH(t_n - t_{n-1})]$ is not a contradiction, because realistically the scientific community cannot completely evaluate this operator within the limits of finite resources, to the precision needed to overcome widespread deterministic chaos.

Note that it is potential verifiability that matters rather than actual verification: because rigorous verification is costly in labor and resources, a coarse-grain description in which every datum is actually verified, rather than just being verifiable, would turn out to be very crude indeed. In contrast, a description in terms of verifiable events can be much finer. The distinction between verified and verifiable events also helps to answer the following question: With the notion of information being so closely tied to what the scientific community can verify, how is the increase in scientific knowledge over time to be reconciled with entropy increase? The answer is that whereas the amount of scientifically verified data is increasing, the (much greater) amount of scientifically verifiable data is non-increasing when entropy is non-decreasing. But the idea of calling an event “verifiable” provided only that it can be actually verified when attention is focused on the task of its validation may seem to lead to a contradiction: the attention of the scientific community can after all be turned to checking the position of a particle with great accuracy or the momentum of a particle with great accuracy; hence would not the position and momentum be simultaneously verifiable? It must be remembered, however, that the branching structure of a consistent family is regarded as being first given, and the verifiability of the events entering a history is being considered in that context. The consistency conditions exclude histories with events which refer to the position and the momentum of a particle at the same time. As long as the focus of validation is directed to one or another of the events already in consistent histories, no contradiction would arise.

By using potential verifiability rather than actual verification as a selection criterion, one can envision a relatively refined coarse-grain description of the observable world. In cosmological terms such a stage was reached probably only after the recombination era: the entropy non-increasing tendency of branching discussed in this article becomes relevant only after it is possible for branches to be sharply distinguished through the existence of records. Initially this tends to bring most branches towards low entropy states. It is only after a vast number of verifiable branchings already left a wealth of records, allowing a fairly detailed description of the macro-world, that new records are mostly made at the expense of erasing old ones. Even then, if an event has a huge number of redundant records, the erasing of a few of these will not destroy the credibility of the event; and therefore the hypothesis of “near saturation of record-keeping

capacity” presupposes that overall there are far more nonredundant records than redundant ones. In a refined description this is not implausible. The most obvious kinds of redundant records occur right here on earth, but biologically related individual organisms carry a great deal more data than their shared genetic information, and redundant records made by people also do not approach anything close to the capacity of refined classical information that the earth can carry.

One may ask why “scientists” are not replaced by the IGUSes (information gathering and utilizing systems) of Gell-Mann and Hartle. The answer is that the greater generality of IGUSes is not particularly helpful in this case: an ant is an IGUS, and presumably it has some notion of whether some kinds of events happened or not, but one would be most reluctant to add to the list of verifiable events something that only ants are aware of with no possibility of independent checks by human scientists, now or in the future, even when attention is directed to that event. In other words, even with the introduction of IGUSes, reference to the “community of scientists” would still be necessary.

With the above explanation of what is meant by “verifiable through records,” it is finally possible to specify, in principle, when a formerly verifiable event ceases to be verifiable: a verifiable event at t_i may be said to have become unverifiable at time t_n if the records in none of the future extensions from t_n onward would permit the scientists in those branches to corroborate the outcome of this event.

Although a scientific-community-based approach results in a verifiability criterion that is close to the common-sensical notion of “objective reality,” there are nonetheless some counter-intuitive consequences. By the expression “scientific community” one usually includes not only today’s scientists but also scientists of the future, because technology improves with time and defining verifiability as what current technology can ascertain is too restrictive. Furthermore, there were no scientists in the early stages of cosmological evolution, and yet some of those early occurrences are regarded as verifiable today, because scientists living considerably later than these events can still check them out from the records. Once it is accepted that future scientists must be included in the notion of a scientific community, one has to face the awkward fact that there are different future branches. Suppose, for example, that a continuation of our history into two different future branches, branch A and branch B, results in different consequences for life. Whereas in branch A life continues to thrive and instruments continue to improve, in branch B life is extinguished forever after a cataclysmic event. Then according to the scientific-community-based criterion, there may be current events which count as verifiable if our future is in branch A but not verifiable if our future is in branch B. Although from a pragmatic standpoint this difference is immaterial because present-day scientists cannot check these subtle events in either case, this example shows the value of an alternative, strictly objective standard that is independent of the existence of people. For example, one may try to abstract the essence of “verifiability by a scientific community” into a mathematical criterion for factuality that can be applied even when life does not exist. This objective has not yet been accomplished, and it

is not obvious that it can be reached at all; although a formulation in terms of suitable complexity measures, in effect having finite-resource computers filling in for scientists, appears promising, that approach suffers from the defect that some of the notions, such as minimal program length, though well defined cannot be operationally checked exactly. There can be heuristic checks, but heuristic standards are likely to be again evolving with time. As to conceptual rigor, separate from the question of mathematical rigor, one pitfall that is easy to fall into is to unconsciously slip in some criteria which are reasonable only because of our experience up to now, and then to regard the result as being more general than it really is. For example, the environment-induced superselection approach implicitly regards cuts between environment and system in ways that are based on our usual notion of what variables are essentially disentangled from the rest of the world, and the de Broglie-Bohm theory contains arbitrariness in giving spatial positions a privileged role. These arbitrary inputs all seem very reasonable on the basis of our experience up to now, but then we cannot yet claim that these theories are totally free from biases associated with people and their state of advancement.

In a way it is preferable to explicitly acknowledge this possible lack of finality. The dependence on the existence of a scientific community in the proposed verifiability criterion is similar to the need to refer to normal people for Locke's secondary qualities. Locke's definition is useful only if our standard for normal people is not going to change significantly in the future. Similarly if the standard of objectivity achieved by contemporary science is not going to undergo substantial improvements in whichever future branch our world evolves into, then the proposed criterion will be useful. Otherwise the utility of this criterion will be limited because of its still evolving nature.

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