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Modular theory and Eyvind Wichmann's contributions to modern particle physics theory *

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Dedicated to Prof. E. Wichmann his 70th birthday

Abstract

Some of the consequences of Eyvind Wichmann's contributions to modular theory and the QFT phase-space structure are presented. In order to show the power of those ideas in contemporary problems, I selected the issue of algebraic holography as well as a new nonperturbative constructive approach (based on the modular structure of wedge-localized algebras and modular inclusions) and show that these ideas are recent consequences of the pathbreaking work which Wichmann together with his collaborator Bisognano initiated in the mid seventies.

1 Wichmann's Influence on Ideas in Local Quantum Physics

Looking at the various contributions to post-perturbative quantum field theory, one could try to group the ideas that shaped the form of our present understanding into three classes. There are first those ideas on nonperturbative frameworks whose usefulness was immediately obvious. The prime example would be the time-dependent scattering theory of Lehmann, Symanzik and Zimmermann[5][1], which, together with the resulting stationary scattering formulas and combined with the dispersion-theoretical framework, marked the beginning of at least the (kinematical) setting of nonperturbative model-independent thinking. This theory (or rather this framework) used the formalism of (interpolating) local fields and their correlations. Fortunately there already existed at the time of LSZ a mathematically as well as conceptually very concise setting for doing quantum field theory, namely the characterization (of what was from the outset an

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[1] Since I did not intend this essay to become a review article, I will use textbook references wherever possible (from which the reader may find an exposition of the content of the referred articles as well the reference to the original paper(s)).

operator theory) in terms of Wightman functions, which gave an excellent understanding of the general singular nature of the correlation functions, as well as of their analytic continuation aspects [4]. The conceptually important aspect of that framework was of course the reconstructability of operators, states and Hilbert spaces from those functions; physicists traditionally, since the times of Feynman's great contributions, prefer to deal with functions and without that framework could not have been completely sure that their description was complete. The addition of the LSZ formalism also bridged the gap between Wightman's pure field theory setting and the 1939 theory of one-particle spaces as irreducible representation spaces of the Poincaré group. This connection of field theory with particle theory was later amplified in work by Weinberg [5], which linked Wigner's particle theory with Feynman's perturbation theory (in fact the exposition of the Wigner theory, the historical remarks and the presentation of the QED calculations constitute the high points of Weinberg's textbook). From there on it was clear that QFT, in contrast to classical field theory (viz. the particle models of the electron of Poincaré and Lorentz), did already contain in principle all particle (including multi-particle scattering) aspects, and even more, that there were methods and beautiful formulas which explicitly allowed extraction of those particle properties of fields.

In a way the Wigner theory of relativistic free particles was the first successful attempt to present relativistic particles without referring to quantization (in this case of relativistic classical mechanics), and in this way it was doing justice to the more fundamental nature of quantum over classical theory. Because of this, Wigner's theory became exemplary for all attempts to formulate and solve problems of local quantum physics (LQP) without relying on the quantization of classical expressions. One of its immediate successes was the classification of the plethora of physically equivalent field equations, found in the aftermath of Dirac's discovery, for the description of relativistic electrons/positrons.

This trend of finding more intrinsic descriptions for local quantum physics was one of the motives behind the algebraic approach initiated by Haag, which reached its first stage of mathematical maturity ("the framework of Algebraic QFT") in a often-cited paper of Haag and Kastler. The guiding principle there was a vast generalization of the idea of Wick, Wightman and Wigner on superselection sectors from their univalence rule in the direction of superselected generalized charges. The emerging paradigm of algebraic QFT was to view all of QFT, i.e. including the issue of spin and statistics and scattering theory, as the synthesis of the superselected representation theory of an underlying (model dependent) observable algebra with the localization structure imposed by Einstein causality [1, 2, 3].

This program required a formulation which was independent of "field coordinatizations" from the very beginning. Wightman's approach [4], on the other hand, was more conservative in that it used field operators which in contradistinction to the neutral observables were allowed to carry charge and to appear in multiplets acted upon by internal symmetry groups, with the effect that for most of the time it stayed closer to the very successful perturbative Lagrangian QFT. The conceptual relation between these two approaches was clarified by Doplicher, Haag and Roberts as well as Borchers [1]; their detailed mathematical connection offers some challenges to date. But even in the first comprehensive account of the Wightman theory [4], these authors already went some distance to counteract the tendency of overemphasizing the role of fields by pointing out that what is really relevant are equivalence classes (Borchers classes) of relatively local fields.

While touching the issue of scattering theory, one should not leave unmentioned another well-embraced idea of those early times: the S-matrix bootstrap approach. This was mainly Chew's idea (with some prior attempts in this direction by Heisenberg). In its extreme form of cleansing quantum fields and the locality principle altogether from the arena of quantum physics, it eventually failed. But, even apart from the useful pictures about effective interactions (as they were later obtained by qualitatively reading back certain on-shell properties into QFT by Weinberg), it left some interesting structural elements behind (see later). In fact one may call it the most successful among all failed theories of this century, a statement whose content will only become comprehensible in the light of its relation to modular theory sketched at the end of this essay.

The reason I mention these trends and achievements of the fifties through seventies is that, unlike most of his contemporaries in general QFT, Eyvind Wichmann, in whose honor I wrote the present essay, did not enter general QFT directly but rather started his carrier with very detailed QED radiative correction calculations and came, after passing actively through other particle physics problems, permanently to Berkeley as a result of his active interest in S-matrix theory and the dispersion relation approach.

But now it is time to get to the idea which is inexorably linked with the name of the Jubilar[2], namely the observation that QFT localization is related in a very deep way with the fundamental mathematical modular theory for von Neumann algebras.

The first achievement in this direction culminates in a 1975 seminal paper by Wichmann and (his at that time Ph.D. student) Bisognano [1]. It relates Tomita's 1967 modular theory, dealing with basic structural properties in von Neumann algebras, with fundamental structures of nonperturbative QFT[3]. Already at the time of Tomita's presentation of his theory at the 1967 Baton Rouge conference in the US, there was a physics discovery which preempted certain aspects of Tomita's theory. This was the analysis by Haag, Hugenholtz and Winnink of thermal quantum physics directly done in the so-called thermodynamic limit of an infinite extended QFT[4]. Their crucial observation consisted in

[2]In trying to translate the German "Jubilar" into short and precise English, my dictionary said: "the person whose anniversary is being celebrated". This is precisely the meaning, but not the type of short expression I was looking for.

[3]Nowadays mostly referred to as the Tomita-Takesaki modular theory because it was M. Takesaki, who by his penetrating analysis and improvements of Tomita's result, contributed to its widespread acceptance within the community of operator algebraists (and finally led A. Connes among other things to his classification of type III von Neumann factors).

[4]When the standard box-quantization produces a conceptual clash with other ideas, as is the case for e.g. (time-dependent) scattering theory or phase transition properties in statistical mechanics, it is clearly preferable to understand the infinitely extended translationally

noticing that a calculational trick, which appeared in previous works of Martin, Kubo and Schwinger (refereed to as the KMS condition) takes on a fundamental conceptual role in the physics of translationally-invariant local statistical quantum systems [1]. Whereas Winnink began to elaborate the deep connections in his thesis with Hugenholtz immediately after the Baton Rouge conference, where both results were presented independently, Haag and his collaborators succeeded in deriving the KMS condition from the stability properties of statistical mechanics equilibrium states [1]. This line of thinking culminated in the work of Pusz and Woronowicz [1] by which the abstract Tomita modular theory, if enriched by the physical idea of locality, got directly linked with the second fundamental law in thermodynamics, i.e., the impossibility of constructing perpetuum mobiles and all that. As a corollary, the TCP symmetry of local QFT developed another "modular" relation (in addition to the "detailed balance" relation) to equilibrium statistical mechanics and in particular to the second fundamental law.

So at the time when Wichmann with his collaborator Bisognano discovered the connection between wedge localization of quantum fields and the modular objects of Tomita for this situation[5], some of the thermal (heat-bath) aspects were already well understood. It is very natural that this situation called for an analysis of the Hawking-Unruh effect in those modular terms. But the first paper in this direction was a contribution by G. Sewell [1].

Another line of research of Wichmann was his collaboration with D. Buchholz, who, being familiar with prior work by Haag and Swieca, brought Wichmann's attention to this problem; the fruits of the joint discussions finally led to further significant contributions to the clarification of the degrees of freedom or phase space structure problems in QFT [1]. As many analogies of QM with LQP did not persist under closer conceptual scrutiny, the clarification of the nature of LQP phase space was an important issue. Already noted in the seminal work of Haag and Swieca [1], the counting with a "relativistic box", i.e., for a Minkowski space double cone region together with a sharp energy cutoff, could not give a finite number of degrees of freedom per phase space cell as in the case of the box in QM. Their rather rough methods and estimates were improved in the Buchholz-Wichmann work, and it became clear that the correct counting could not be better than "nuclear"[1], not even in the interaction-free case. It also gave a deeper insight into a prior conjecture that quantum field theory should exhibit the "split property," which is the statement that by allowing a "collar" region between the inside and the outside world of a double cone (which is the QFT counterpart of the quantum mechanical space box), the total algebra allows a tensor factorization of the inside and outside algebras which

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invariant systems directly and to view finite systems as open subsystems.

[5]In mathematical terms the problem was to compute the modular objects (modular group and modular involution) for the algebra generated by quantum fields smeared with functions which have their supports in a wedge, within the vacuum representation. The modular group turned out to be the wedge-associated Lorentz boost, and the modular involution was the TCP-like antiunitary reflection along the ridge of the wedge, which maps the wedge into its Einstein-causal complement. Certain very special free-field modular localization aspects were already noticed before[12].

is not possible without that collar. Later it turned out that this structure also had deep relations with modular theory. A third line of research of the Jubilar, which probably brought him to Berkeley, was on S-matrix theory. Here I am in the comfortable position of being able to refer to Weinberg's book where justice is done to Wichmann's S-matrix articles [5].

Wichmann's contributions definitely belong to those ideas which, unlike e.g. LSZ, did not have a visible connection to the immediate problems of particle physics; nor could one draw upon them as a framework in QFT as with the aforementioned contributions of Wightman, Haag, Kastler, Jost, and others. Like most of Borchers' contributions, they were less systematic and encyclopedic, and rather more of an enigmatic nature. Their power only unfolded slowly with time, and even to date we are still witnessing an accelerating unfolding of the physical consequences of modular structures; a fact which probably even Wichmann did not expect when he investigated this subject in the mid seventies.

The enigmatic power of Wichmann's modular ideas has been brought to light in many articles ranging from thermal QFT to superselection structure, i.e., the reconstruction of charge-carrying fields including their statistics, symmetry properties, and TCP structure, which especially in low dimensions (omitted in Wightman's framework) gives rise to very new and nontrivial problems related to modular theory [6, 7, 9][6].

Rather than continuing historical exposition, I would like to try in the next section to exemplify the power of modular ideas in the context of two very recent (and still unfolding) ideas: holography and a new nonperturbative method (which still lacks a catchy name) based on the structure of those Bisognano-Wichmann wedge algebras.

Since holography, in the sense of encoding the content of a QFT into a lower-dimensional one has, especially in the context of QFT in AdS (Anti-deSitter spacetime), attracted a lot of attention, this may be a good vehicle for demonstrating the power of modular ideas to a broader QFT-knowledgeable public. This is particularly the case in view of the perplexing aspects which holography and in particular quantum matter in AdS presents to the standard quantization (Lagrangian or functional interaction) approach. Most of these paradoxical aspects are naturally solved by algebraic QFT, with the modular enrichment originating from the Bisognano and Wichmann work being the essential link between local quantum physics and geometry of AdS [10, 11]. These are methods and concepts which are not yet known outside a small circle of specialists, although the latter paper makes great efforts to use rigorous physical arguments and avoids the explicit evocation of modular theory (but rather derives it, because all the arguments are of a general QFT nature). And even if, after all, the conceptual barriers for many particle physicists will remain high (please, dear reader, remember that you did not learn the sophisticated differential geometry and topology many of you are using these days in less than a year!), their enthusiasm for the AdS-conformal QFT connection may be of help here.

[6]Here the interested reader may also consult my 62-page review article (also containing additional more recent references), dedicated to Eyvind Wichmann and accepted for publication in Journal of Physics A.

The constructive approach via the modular properties of wedge algebras which is taken as our second illustration (which I initiated and pursued over the last years, partially with my collaborator H-W Wiesbrock) [12, 13, 14], is best thought of as the inverse of the Bisognano-Wichmann theorem, namely using the modular theory for the actual construction of quantum field theories. It shares with the holography issue the fact that a real understanding inside any framework based on "field-coordinatizations," in particular in any quantization approach, is impossible[7] (or only possible with artistic imagination). Here one is really hitting the limits of capabilities of what the quantization access to QFT can do, and although analogies are always dangerous, I feel tempted to make a historical comparison with Bohr-Sommerfeld rules (please don't look down on them, they where quite successful for many problems, but they demanded some artistry) versus full quantum mechanics.

Indeed everything you learned in text books (and which proved so useful in the pursuit of perturbation theory) about standard quantum field theory – interaction picture, time ordered correlations, canonical quantization or quantization through Euclidean functional integrals – all that is of no avail here. Of course none of these well-known standard tools is unique to LQP[8]; all of them can be (and have been) applied to QM as well, either in its Schrödinger or its so-called second quantized form. *But the modular structure in QFT on the other hand is totally characteristic* and not shared by any other kind of quantum theory.

Anybody who is familiar with the conceptual framework of LQP knows that this is deeply related to Einstein causality and the polarization structure of the vacuum resulting from that causality in the presence of interactions, the denseness of localized states generated by acting with operators associated with nonvanishing regions[9] whose causal complement is nontrivial, or the totally different nature of local algebras as compared to algebras of QM with consequences for quantum measurement theory [14]. Especially these last structural differences may be somewhat surprising (in the sense of low credibility) to somebody who has always thought about QFT just as a relativistic continuation of QM. Hyperfinite type III_1 von Neumann factors, as the wedge localized algebras of QFT, have indeed very different properties from the type I algebras of QM. They do not permit pure states at all[10], and they also, together with their commutants, do not admit the notion of tensor product factorization of the total algebra B(H), disentanglement and all the other old notions from von Neumann's exposition of QM, which recently have been propped up for "quantum

[7] At least if one does not solve the Lagrangian theory nonperturbatively and explicitly, and then reprocesses the resulting dynamical variables; an altogether impossible task.

[8] The reader may have noticed that whenever we want to emphasize the concepts of QFT but not necessarily their present textbook implementations, we prefer the terminology LQP (local quantum physics). This diminishes the unwarranted (almost subconscious) tendency of the reader to equate QFT with Euclidean functional representations and all that.

[9] This property has been colloquially termed the "particle behind the moon" paradox, better known as the Reeh-Schlieder property, or, in mathematical language of von Neumann algebras, as the cyclic and separating property of the vacuum with respect to local algebras.

[10] We follow here the by now standard terminology to omit the prefix "normal" for states, and add the prefix singular for the rare case of nonnormal states on von Neumann algebras.

computation" [15]. In order to recover the usual quantum mechanical structure of the inside/outside factorization of a Schrödinger box, one has to work quite hard and use the Buchholz-Wichmann nuclearity property for the control of local degrees of freedom [1]. The resulting "relativistic box" consists of a the inner region of a smaller double cone and the outer (infinite) spacetime of a larger double cone, but note that the inside and the outside need to be separated by a "collar region" in order to attenuate the uncontrolled vacuum fluctuation caused by sharp boundaries (the latter trouble already having been known to Heisenberg in his study of vacuum polarizations). As mentioned before, this is known under the name of split situation or split inclusions.

There are many recent results from modular theory which all point in the same direction and contain the same general message, namely one is dealing here with structures which, if at all, are visible from a quantization framework only with a superhuman hindsight and extraordinary stretch of imagination. Another convincing illustration, not presented here, are the recently-found "hidden symmetries" [16][17], where the word hidden is used in the sense of hidden to the Lagrangian-Noether framework (and of course not in the sense of the modular framework within which they were discovered). We also refrain from presenting some very surprising results about the possibility of creating a local net in LQP together with the full Poincaré symmetry from just a few (for chiral conformal theory, 2; for d=1+2 theory, 3; and for d=1+3 theory, 6) algebras in a certain modular position to each other [18, 19]. Although algebraic QFT, unlike string theory, is not designed to be quantum gravity, these findings about totally unexpected relations between raw (highly noncommutative) algebraic data and spacetime geometry, although not being directly related to quantum gravity, should be taken seriously in any attempt towards quantum gravity.

Another very important consequence of modular theory is the already mentioned thermal aspect which it attributes to localization. In the case of "natural localizations" related to classical bifurcated Killing horizons as they occur in black hole physics, this thermal aspect was discovered before modular theory. But for the general localization in QFT, which cannot be described in such classical metric terms, one really needs modular theory. The modular localized subspaces are dense in the Hilbert space of the full theory, but there is a natural "thermal" scalar product (the graph metric defined by the unbounded Tomita operator S of that region) in terms of which it is closed. This thermal inner product changes with the localization region of the local algebras, and it turns out to be related to the domain problems of Wightman's theory[11] and possibly also to the construction of pointlike covariant fields from the net of local algebras. There is a speculative remark of Fredenhagen (private communication) which fits in very nicely with these physical aspects of field domains and ranges of actions of algebras on the vacuum. It is the idea that a pointlike field, or rather the one-field subspace obtained by its application to the vacuum, can

[11] Actually the Wightman domain is related to the intersections of all (thermal) modular domains. This is quite interesting, since many particle physicist in my generation were told not to worry to much about these domain problems and accept them as a technical mathematical assumption void of any direct physical interpretation.

be characterized as the carrier of an irreducible representation of some (infinite dimensional) "universal modular group". The latter is generated by all the one-parameter modular groups for all spacetime regions[12]. This, if true (it is true for QFT sketched in the next section), would make a rather pivotal addition (if not revolution) in QFT as it has been hitherto understood, since it attributes to pointlike fields an intrinsic physical role analogous to the Wigner positive energy representation theory of the Poincaré group. In this way the "fields" would recover some of their lost ground (at least in the form of the mentioned field spaces), when from the viewpoint of AQFT they became relegated to mere "coordinates" of algebras. And much more: since the modular groups and their unitary implementers are expected to contain the crucial information on interaction, they would gain, in addition to the geometric properties they already had in the quantization approach, the status of an intrinsic modular-based concept of interaction. To put it into the context of the more concrete constructive nonperturbative modular setting of the next section, the incoming particle content of the interacting field (in terms of its form factor spaces which appear in their decomposition) would be governed by a new and subtle (hidden) infinite-dimensional group theory as an analogue of the (overt) diffeomorphism group in chiral conformal field theory[13]. The characterization of special operators in such an algebra would then require the study of the relation to modular subgroups belonging to finer localizations inside the chosen one.

2 Holography and the Constructive Approach to Wedge Algebras

Holography is the conjectured correspondence between higher- and (conformal) lower-dimensional QFT (or a family of lower dimensional ones). The attractive aspect of such a correspondence is that a lot more is known about low-dimensional QFT, in particular conformal QFT, which could be of use for the construction of higher dimensional QFT's. Historically the idea can be traced back to the thermal and geometric behavior of black hole (classical) entropies (Bekenstein, Hawking). Since the temperature aspects were understood in the setting of (free) QFT in CST, it was only natural to look for an explanation of the surface proportionality of entropy in terms of quantum degrees of freedom at or near the horizon. In contrast to the understanding of the Hawking-Unruh temperature as originating from the causal localization behind a (Killing) horizon, the entropy problem was less susceptible to explicit calculation involving (free) quantum matter in a black hole background. But it is clear that if one could understand the surface nature of the degrees of freedom, then the entropy should follow suit. In the Lagrangian formulation of QFT the elusive "light-

[12] The dynamics would then be carried by the representations of the local modular groups instead of the global time-frame dependent Hamiltonian.

[13] There are strong arguments that those chiral diffeomorphism groups are of modular origin and belong to multi-interval algebras together with specially chosen (non-vacuum) states [19].

cone physics" preempted some aspects of this idea, and is not surprising that 't Hooft [21], who on various occasions used light-cone quantization, in more recent times suggested interpreting Bekenstein's classical observation on black hole entropy in terms of quantum "holography".

A problem like the present one, where field coordinates are not transformed into each other, but rather degrees of freedom become transmuted in a way which is hard to describe in terms of pointlike field concepts, is bound to cause trouble within the usual quantization formalism. To be sure, problems with the use of one set of field coordinates versus another one already appeared before in QFT, although in the early sixties they were sometimes the source of some prejudices about Lagrangian fields being in some sense "better" than any other composites (carrying the same charges). This was part of a bigger confusion about particles versus fields; the elementary versus bound state hierarchy of QM tacitly entered QFT where it should have been replaced by the hierarchy of superselected generalized charges and their fusion (including those nonabelian Casimir charges which underlies nonabelian internal symmetries). For example in connection with the PCAC, physicists in Lagrangian field theory had to take notice of the fact that one is not slavishly bound to those field coordinates in terms of which one has written a particular Lagrangian. From my time in Illinois as a collaborator of Rudolf Haag, I remember a conversation between Murray Gell-Mann and Rudolf Haag which ended with some astonishment on the side of Gell-Mann. Nowadays the extreme insensitivity of on-shell objects like the S-matrix to changes of field coordinatizations has become a commonplace even in Lagrangian QFT, especially after Weinberg taught physicists how to formally handle this problem in perturbation theory. However the morphisms and isomorphisms needed in order to understand holography are of a different caliber.

Progress in physics is to a large part the liberation from prejudices (including one's own). Algebraic QFT theory had the big advantage that there was no place for prejudices about fields, because there were no fields in its formulation. As a result, the problem with this "field-coordinate-free formulation" was shifted somewhere else. Namely, although it was desired to implement the same physical principles that underlie the quantization approach in a conceptually different and mathematically more controllable way, it was not clear that one had actually kept a connection to the same particle physics. But it became soon clear that the gains of working with nets of operator algebras instead of fields (e.g., the manifest independence of the S-matrix from the choice of particular interpolating operators taken from the local algebras) were not offset by an undesired vagueness or unintended invention of new physical content. This was established beyond reasonable doubt, and in the case of chiral conformal QFT there is even a rigorous proof for the equivalence of the Wightman description with the algebraic framework [20]. As mentioned before, algebraic QFT based on modular methods wants to stay laboratory physics, and not aim, like string theory, at quantum gravity.

Already in the early stages of the theory there were concepts, questions, and techniques which transcended the Lagrangian framework and even that of

Wightman. One could, e.g., ask about the possibilities of particle statistics compatible with the Einstein causality of observables. This goes certainly beyond the Wightman theory, which includes charge-carrying fields that ab initio are assumed to have $\pm$ commutation relations for spacelike separation. The Spin-Statistics theorem just selects the correct one of these two possibilities. In algebraic quantum field theory, one computes the field statistics without such restrictive assumptions on nonobservable quantities (over which in $d < 1+3$ one has no a priori control). In the intermediate steps of the conceptually and mathematically rich DHR and DR constructions [1], there appear parastatistics fields which belong to nonabelian Young tableaux of the permutation group. They do not permit quasiclassical limits and Lagrangians, but are reasonable objects in algebraic QFT (in the sense that the charge-carrying parastatistics fields have enough locality in order to admit a reasonable physical interpretation, albeit one which is much more noncommutative. Only after enlarging the Hilbert space by the introduction of multiplicities (i.e. indices on which symmetry-groups can act), does one make contact with quasiclassics. On the other hand, writing down a Lagrangian has already preempted the answer before having been able to ask the question. An ardent philosophical empiricist may point to the fact that there was never any practical need for asking such a question since the usual formulation with built in multiplicities and Bose/Fermi statistics works nicely. But he would have a hard time in, say, d=1+2 theories with braid group statistics, where it can be mathematically demonstrated that those plektonic objects will never fit into a Lagrangian quantization approach with field multiplicities.

Of course such an empirical fundamentalist may then retort that d=1+2 models do not constitute particle physics. In that case one could, assuming that he does not also declare the present AdS discussion in connection with holography irrelevant in particle physics, point out to him that, although there is no satisfactory solution of the paradoxical situation of holography in any quantization approach[14], the solution which was given in algebraic QFT by Rehren [10] is conceptually clear and mathematically rigorous. The main point in Rehren's presentation is that the adapted Bisognano-Wichmann theorem allows understanding of an isomorphism between AdS_{n+1} and conformal Minkowski spacetime M_n which is not a pointwise geometric mapping (diffeomorphism) but rather a set mapping between modular localization regions of algebras. In fact, the notion of "weak locality" [4] used in his paper is completely equivalent to the spatial part (the thermal subspaces of the total Hilbert space which are closed in the modular graph norm) in my constructive approach built on modular wedge localization [3]. If the reader finds the small amount of modular concepts inaccessible because he lacks mathematical understanding of LQP concepts, he

[14]Quantization difficulties have been mentioned in Witten's papers [22]. Actually AdS only exposes the *general limitation of quantization* which always exists in any interacting QFT, once one leaves the realm of perturbation and quasiclassical approximations. The entrance into QFT from the noncommutative side of modular theory, i.e., without the classical parallelism (called quantization) may be more difficult and unusual, but does certainly not suffer from those limitations.

may have another chance by looking at the closely related paper of Buchholz, Florig and Summers [11]. These authors explain the LQP in an AdS space-time together with a rigorous physical account of what is necessary to know about modular theory without assuming (in principle) prior knowledge. We will not try to reproduce these results here, since the clarity of the papers makes this a sacrilege.

As far as I could see, the only open problem in the BFS work is the question of whether there can be any genuine interaction at all in such an AdS world with that causality paradox mentioned in their paper. This question is reminiscent of a problem I encountered in my collaboration with Swieca at the beginning of the seventies. At that time there existed the challenge of understanding the (global) "causality paradox of conformal QFT" [25], i.e. the apparent contradiction between being able to conformally transform oneself globally from space-like separations via the light-like infinity into the time-like region and the fact that certain interacting "would-be" conformal models, as, e.g., the massless Thirring model, did not comply with the Huygens principle calling for vanishing time-like (anti)commutators which was required in order to avoid contradictions with that global transformation property. In fact the only known d=1+1 models which did not generate this paradox were free fields with Fourier transforms on the light cone, as conformal currents or energy-momentum tensors. The resolution [26, 27] of this paradox turned out to consist in realizing the important role of the conformal covering space in that those paradoxical looking fields as the Thirring field were not (as everybody believed up to that time) globally irreducible, but rather had a rich decomposition with respect to the center of the conformal covering group. The irreducible components in this decomposition (there simply called "nonlocal components") became known 10 years later as the conformal blocks in the famous Belavin-Polyakov-Zamolodchikov paper. One reason why we only looked at the Thirring-like exponential boson fields was (besides the fact that they already were available) that these new irreducible component objects were outside the range of euclideanization and even outside the Wightman framework[15] since their algebra admits local annihilators. My impression after having read [11] is that the AdS situation has analogous causality problems. In fact, using the Rehren isomorphism for AdS(1,1), one would expect to be able to lift the solution of the old conformal paradox directly into the new AdS(1,1) realm.

I now would like to explain some of the modular ideas which I used recently in a constructive program for interacting LQP models which is based on the use of algebras and is completely free of field coordinates (although I will think of the reader, and use field notation wherever possible). Of course one must first test these ideas in the interaction-free case. This I did by showing that the Wigner representation theory can be directly used for the construction of the local nets without, e.g., using Weinberg's formalism of first constructing free fields which then would generate these local algebras. In this way one obtains an intrinsic

[15]Wightman fields do not come with source and range projectors as those nonlocal components, for an explicit illustration see [28].

description of noninteracting theories which restores (or rather maintains) the uniqueness[16] of the (m,s) Wigner representations and avoids the plethora of covariant associated field coordinatizations [12, 13, 14].

This first step may be viewed as analogous to the intrinsic coordinate-free description of geometry. It uses a kind of inverse of the Bisognano-Wichmann theorem, i.e., the known modular theory for the wedge, in order to obtain the operator algebra localized in the wedge. It may be viewed as a refinement of Weinberg's exposition of the Wigner theory mentioned in the first section, by implementing the idea of modular localized subspaces directly in the Wigner momentum space description without the use of covariant x-space wave functions or the noncovariant Newton-Wigner localization. This baby-version of modular theory can be understood without knowing anything about the Tomita modular theory and as such furnishes an excellent pedagogical example of the power of modular localization and the Bisognano-Wichmann theorem.

The next step, namely to construct interacting nets in this intrinsic manner, is more difficult. Of course one could follow Weinberg for the construction of free fields from Wigner particles, select some free fields corresponding to (m,s), couple them to a scalar Wick-ordered interaction density W(x) (which one may call $\mathcal{L}_{int}$, but the existence of an $\mathcal{L}_0$ is not necessary, see previous footnote) which is then plugged into the causal perturbative machine whose heartpiece is the perturbative transition operator $S(g)$. From there one obtains the retarded representations of interacting fields in terms of free field in Fock space, which in turn (or by direct use of the $S(g)$) generate the localized algebras after suitable test-function smearing. But this way of constructing local algebras would amount to just an exercise in semantics and go against the spirit of LQP.

Let me explain the gist of the correct idea with the help of a two-dimensional representation and using standard field theoretical language wherever it is possible.

Let $A(x)$ be a d=1+1 massive free scalar field with the following notation:

$$\begin{aligned} A(x) &= \frac{1}{\sqrt{2\pi}}\int (e^{-ipx}a(p) + h.a.)\frac{dp}{2\omega} \\ &= \frac{1}{\sqrt{2\pi}}\int (e^{-im\rho sh(\chi-\theta)}a(\theta) + h.a.)d\theta,\ x^2<0 \\ &= \frac{1}{\sqrt{2\pi}}\int_{\mathsf{C}} e^{-im\rho sh(\chi-\theta)}a(\theta)d\theta,\ \ \mathsf{C} = \mathbb{R}\cup\{-i\pi+\mathbb{R}\} \end{aligned} \tag{1}$$

where in the second line we have introduced the position- and momentum-space rapidities and specialized to the case of spacelike x, and in the third line we used the analytic properties of the exponential factors in order to arrive at

[16]The uniqueness on the field level was lost because there are infinitely many u- and v- intertwiners from the unique Wigner representation to the plethora of covariant L-representations. Weinberg prefers the one (those) for which there is a free Lagrangians, since he wants to use fields for a Euclidean functional integral representation. Real time causal perturbation theory on the other hand can be done in any field coordinatization. The best way would be do use none at all.

a compact and (as it will turn out) useful contour representation. Note that the analytic continuation refers to the c-number function, whereas the formula $a(\theta - i\pi) \equiv a^*(\theta)$ is a definition and has nothing to do with analytic continuations of operators[17].

With this notational matter out of the way, we now write down our ansatz for nonlocal but (as it turns out) still wedge-localized fields using the same notation:

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathsf{C}} e^{-im\rho sh(\chi-\theta)} Z(\theta) d\theta, \quad Z(\theta)\Omega = 0 \tag{2}$$

$$\begin{aligned} Z(\theta_1)Z(\theta_2) &= S_{Z,Z}(\theta_1 - \theta_2)Z(\theta_2)Z(\theta_1) \\ Z(\theta_1)Z^*(\theta_2) &= \delta(\theta_1 - \theta_2)\mathbf{1} + S_{Z,Z^*}(\theta_1 - \theta_2)Z^*(\theta_2)Z(\theta_1) \end{aligned} \tag{3}$$

For the moment the $S^{'}s$ are simply Lorentz-covariant (only rapidity differences appear) functions which for algebraic consistency fulfill unitarity $\overline{S(\theta)} = S(-\theta)$. We assume (for simplicity) that the state space contains only one type of particle.

Before continuing with the special situation we introduce a useful general definition.

Definition A field operator F(x) is called "one-particle **p**olarization **f**ree" if F(x)Ω and F*(x)Ω have only one-particle components (for any one of the irreducible particle spaces in the theory).

For polarization free $F(x)'s$ the vector $F^{\#}(x)\Omega$ is on mass-shell, i.e., has a Fourier transform in terms of $Z^*(\theta)\Omega$, with $Z(\theta)\Omega = 0$. Note that the definition does not yet require that $F(x)$ itself to be on-shell. We are, however, interested in $F(x)'s$ which upon smearing with test functions restricted to a subspace $\mathcal{L}$ generate algebras

$$\mathcal{A} = alg\left\{ F(\hat{f}) = \int F(x)\hat{f}(x)d^dx \mid \hat{f} \in \mathcal{L} \right\} \tag{4}$$

which on the one hand are big enough in order to create a dense set of states if applied to Ω, but on the other hand allow for an equally big commutant algebra $\mathcal{A}'$. In short, the PF's should generate an $\mathcal{A}$ which is cyclic and separating with respect to the vacuum. As a result of $F(\hat{f})A'\Omega = A'F(\hat{f})\Omega$ for $A' \in \mathcal{A}'$, the on-shell aspect of the vectors is transferred to the operators, i.e. formula (2) for $F(x)$ is valid. The $\mathcal{L}'s$ we have in mind are subspaces of localized test functions $\mathcal{L} = \left\{ \hat{f} \mid supp\hat{f} \subset \mathcal{O} \right\}$. But as a consequence of an old theorem by Jost and the

[17]Operators in QFT never possess analytic properties in x- or p-space. The notation and terminology in conformal field theory is a bit confusing, because although it is used for operators it really should refer to vector states and expectation values in certain representations of the abstract operators. The use of modular methods require more conceptual conciseness than standard methods.

present author [4], this immediately limits the admissible localization properties. If the field is pointlike local, this theorem forces the F to be a free field, and by a slight massaging of the proof this would continue to hold for $F's$ which have a compact Minkowski space localization. Even for noncompact localizations which are properly contained in a wedge (i.e., a Lorentz transformation of the standard wedge $x_1 > |x_0|$) this clash with interactions continues[18], and the only consistent value of the S-functions in the above ansatz are $S = \pm 1$, i.e., free Bosons/Fermions. The smallest region for which these arguments break down are full wedges. The following theorem shows that indeed wedge localization in d=1+1 is consistent with nontrivial interactions, and the result emerging from the above ansatz in formula (3) is quite surprising.

One finds that the coefficients are related to each other and fulfill the complete Zamolodchikov-Faddeev algebra if and only if the $F(\hat{f})'s$ with $supp\hat{f} \in W$ generate a wedge localized algebra, thus unraveling the physical significance of this formally-introduced algebraic structure in terms of wedge localization [12, 13].

This is not the first time in physics that wedges play a prominent role. In Unruh's Minkowski space illustration of the origin of thermal aspects of quantum matter encapsulated behind a horizon, in the first application of Tomita's modular theory by Bisognano and Wichmann, and now in the inverse use of the Bisognano-Wichmann theorem for the direct construction of local algebras, in all cases one encounters the fundamental role of wedge localization and wedge algebras. In the present case we find [3, 13, 14]

Proposition 1 *The requirement of wedge localization of a polarization free operator $F(f) = \int F(x)\hat{f}(x)d^2x$, $suppf \in W$ with F fulfilling formula 3 is equivalent to the Zamolodchikov-Faddeev structure of the Z-algebra. In particular, the thermal (Hawking-Unruh) KMS condition on their Wightman correlation functions corresponds to the crossing symmetry of the S-coefficient functions. The corresponding F's cannot be localized in smaller regions, i.e., the localization of $F(\hat{f})$ with supp $\hat{f} \in \mathcal{O} \subset W$ is not in $\mathcal{O}$, but still uses all of W.*

The reader can find the proof, which amounts to a simple computation, in [13, 14]. Of course the F's are not ordinary (Lagrangian or Wightman) fields, since there localization does not follow the decreasing support properties of f's inside the wedge, and therefore F(f) is a better notation than F(x). Since **p**olarization **f**ree **g**enerators F will only play a role as wedge generators, we will simply use the abbreviation **PFG** standing for "polarization-free-wedge-generators".

A moment's thought about the special situation reveals that the modular structure, i.e., the existence of the antilinear unbounded Tomita involution S_T (the subscript serves to distinguish this time-honored modular notation from the equally time-honored notation for the scattering operator), is the general cause underlying the above observation. In fact the modular "basic law" for the

[18] I owe this general model-independent insight to D. Buchholz, private remark, unpublished.

physical wedge algebra is:

$$S_T A\Omega = A^*\Omega, \;\; A \in \mathcal{A}(W) \tag{5}$$

which defines the antilinear, unbounded, closable, involutive (on its domain) Tomita operator S_T. Its polar decomposition

$$S_T = J\Delta^{\frac{1}{2}} \tag{6}$$

defines a positive unbounded $\Delta^{\frac{1}{2}}$ and an antiunitary involutive J and the nontrivial part of Tomita's theorem (with improvements by Takesaki) is that the unitary Δ^{it} implements an automorphism of the algebra, i.e., $\sigma_t(\mathcal{A}) \equiv \Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}$ and the J maps antiunitarily into its commutant $j(\mathcal{A}) \equiv J\mathcal{A}J = \mathcal{A}'$. For the case at hand (the Bisognano-Wichmann situation) these operators have their following physical aliases:

$$\begin{aligned} \Delta^{it} &= U(\Lambda(-2\pi t)) = U_{in}(\Lambda(-2\pi t)) \\ J &= SJ_{in} \end{aligned} \tag{7}$$

where $U_{in}(\cdot)$ is the unitary representation of the Poincaré-group in the incoming Fock space, and $J(J_{in})$ is the TCP operator (its free field incoming version). The last relation shows clearly that the S-matrix is a relative modular invariant of the wedge algebra.

The wedge situation is a special illustration for the Tomita theory covered by the Bisognano-Wichmann theorem [1]. In that case both operators have well-known physical aliases; the modular group is the one-parametric wedge affiliated Lorentz boost group $\Delta^{it} = U(\Lambda(-2\pi t)$, and the J in $d = 1 + 1$ LQP's is the fundamental TCP-operator as derived from first principles by R. Jost [1]; in higher dimensions it is only different from TCP by a π-rotation around the spatial wedge axis. The formula for the modular operator in terms of the scattering matrix (which contains the information about the interaction) is not part of that theorem and as such is new. However it turns out to be just a modular adapted transcription of the TCP transformation law of the textbooks [4]. The prerequisite for the general Tomita situation is that the vector in the pair {algebra, reference vector} is cyclic and separating, i.e., there is no annihilation operator in the von Neumann algebra, or equivalently: its commutant is cyclic relative to the reference vector. In LQP these properties are guaranteed for localization regions $\mathcal{O}$ with nontrivial causal complement $\mathcal{O}'$, thanks to the Reeh-Schlieder theorem. In terms of the correlation functions of the generators, the wedge localization affiliation of the generated algebra is nothing but the KMS condition (which is checked in the above-mentioned proof [19]).

The construction of the local QFT behind the S-matrix of the above model is of course not finished with that of its wedge algebra. The essential next step is the construction of its double-cone algebras[19] via the demonstration of the

[19]The sharpening of localization via algebraic intersections is the essential difference from usual QFT even including Wightman's approach. I was quite surprised when in this way I obtained the same recursion formula [12] which occurs in Smirnov's "axiomatic" approach [24] (not the usual QFT axioms but rather some calculational recipes designed for factorizing models).

nontriviality ($\neq C \cdot \mathbf{1}$) of the intersection of the right wedge algebra with its translated opposite left wedge. It is precisely here where the idea of holography enters the game. It is much easier to show the nontriviality of the holographic image of this situation.

The crucial idea is to look at the relative commutant for light-like translations for, say, $a_+ = (1,1)$

$$\mathcal{A}(W_+)' \cap \mathcal{A}(W) \tag{8}$$

where $\mathcal{A}(W_+)$ is the a_+-shifted wedge algebra. $\mathcal{A}(W_+) \subset \mathcal{A}(W)$ is almost a modular inclusion, i.e., the modular group of $\mathcal{A}(W)$, i.e., the Lorentz-boost in one direction, acts on $\mathcal{A}(W_+)$ as a compression into itself. The only missing property is the standardness of the relative commutant in H with respect to Ω. But this is easily achieved by projection onto the cyclicity space $\mathcal{M}_+\Omega$

$$H_+ = P_+ H \subset H = \overline{\mathcal{A}(W)\Omega} \tag{9}$$

Using a theorem of Takesaki, the reduced inclusion defines again a modular inclusion in its own right, from which one may reconstruct a positive energy translation $\hat{U}(a)$, which then can be used to define a reduced net indexed by intervals

$$\begin{gathered}\mathcal{A}(I_{a,e^{2\pi t}+a}) = \hat{U}(a)\Delta^{-it} E_+ \left(\mathcal{A}(W_{a_+})' \cap \mathcal{A}(W)\right) \Delta^{it} \hat{U}^{-1}(a,a) \\ \mathcal{M}_+ \equiv \cup_t A(I_{0,e^{2\pi t}}),\ E_+(\mathcal{A}(W)) = \mathcal{M}_+ = P_+ \mathcal{A}(W) P_+\end{gathered} \tag{10}$$

The reduced net can be shown to be "standard," and the set of standard modular inclusions is known to be isomorphic to the set of all chiral conformal field theories (S-W). Therefore, each d=1+1 net comes associated with a "satellite" chiral conformal net. This is the rigorous modular version of holography, and again, as in the AdS case treated by Rehren, this association is outside the range of Lagrangian quantization, since there is no Lagrangian or Euclidean functional integral process which can describe properly this transmutation of degrees of freedom. At the time of writing of this essay, the computations for the existence proof of the factorizing models (double-cone algebra nontriviality) have not been finished.

The use of the holography idea for higher dimensional QFT's is more involved. If one carries out the previous modular inclusion construction, one realizes that, because of the transversal indeterminacy of the chiral conformal theory attached by modular inclusion (which is localization-wise really attached to a whole light front rather than a light ray), the chiral theory is, contrary to Rehren's AdS treatment, not yet sufficient for a reconstruction of the original theory from the holographic image. It turns out that by tilting the Lorentz-boost of the original wedge around one of its light rays one generates a "stalk" of conformal theories which, more analogous to a scanning process than holography, allows the reconstruction [16] (called "blow-up" in the paper) of the full net theory. It uses an enrichment of modular inclusion, called modular intersections, which in its geometrical interpretation corresponds to the interaction of two different wedges which have one light ray in common. Modular inclusions and modular intersections constitute presently the most powerful conceptual/mathematical instruments which LQP has to offer.

It should be clear to those who know a bit about the two-dimensional bootstrap-form factor constructive program that the two-dimensional modular method for factorizing models (i.e., those which are defined by a matrix-generalization of the Ansatz at the beginning of this section) is what lies behind the form factor program initiated by Karowski-Weisz and Smirnov [23, 24]. It does, however, more than only justify those very successful collection of non-Lagrangian cooking recipes, in that it promises to solve the difficult and not fully understood problem of the correlation functions of pointlike quantum fields (using again standard QFT concepts) in those models. In fact this difficulty, known to every expert of the bootstrap-form factor program in the conventional setting, results from the fact that there is no natural basis in interacting field space as the Wick composites for free fields. Therefore it is better to avoid fields altogether and characterize the physical content of a theory in terms of its basis-independent double-cone algebras. In their nontriviality demonstration as well as in their actual construction, the holography, as we have argued, plays an essential role.

Since Chew's S-matrix bootstrap program (i.e., the formulation of the nonlinear S-matrix axioms as well as the actual construction of interesting examples) only works for the d=1+1 factorizing models, there is no hope to do higher dimensional QFT with modular methods in a two-step process of calculating first S and then the associated wedge algebra. Rather, one has to understand the structure of correlation functions of the wedge generators $F(f)$ (which turn out to be uniquely fixed in terms of S) and of S itself simultaneously. This is presently only imaginable in a perturbative spirit. But note that this would be a perturbative approach for wedge algebras and not for individual fields, i.e., technically speaking for the whole space of form factors generated from the PFG's sandwiched between incoming particle ket vectors and outgoing bra vectors, without any natural way of distinguishing individual elements [14]. Refinements and distinctions have to go via improvements of localizations, which in modular theory can only happen through algebraic intersections.

In order to return at the end of my essay to the Wichmann's S-matrix research[20] carried out at the beginning of his research carrier at Berkeley, I would like to use some recent personal experience of my own as a vehicle to recapture some flavor of those times.

Shortly after string theorists picked the big Latin letter "M" for one of their recent inventions, but before the much clearer AdS proposal (note the small d there!) attracted attention, I was struck by the wealth of coincidences of some of the string theoretic statements, especially in connection with transmutations and counting of degrees of freedom, light cone physics (in particular their Galilean group affiliated with the light cone) and ideas on holography, with

[20] I do not know whether Wichmann, while working on wedge localization and modular theory, was aware of these strong connections with his previous S-matrix research. It may have been another instance of the role of the subconscious on scientific research.

recent results about consequences of modular theory. Although I admittedly do not understand string theory from a physical point of view, I do think (most of my colleagues from algebraic QFT do not share such optimistic ideas) that the differential-geometric quantization formalism, together with the relics of physical locality and spectral properties which such a generalized standard formalism inexorably contains and which are (even in arguing along differential geometric instead of local quantum physical lines) hard to lose, constitutes a powerful mathematical machinery for the discovery of new structures; even though physical interpretations which could reconcile string theory with the physically (but not mathematically!) more conservative local quantum physics are hard to see, and even when faintly visible, probably not always correct. Apparently the string formalism uses ideas (hidden to me) which carry string physicists beyond the confines of possibilities allowed by Lagrangian quantization, and in this way achieves similar "miracles" as modular theory (which originated by faithfulness to all the principles underlying standard QFT, but not its formalism).

But my proposal to include "modular" (in the sense of d=1+3 LQP without "curling up" unwanted dimensions by semiclassical Klein-Kaluza ideas) in the list of possible interpretations of the letter "M" did not find the approval of the referee, who claimed that all this is accidental and spurious (whereupon I withdrew my admittedly rather speculative notes from the hep-th server).

Fortunately the arrival of the clearer AdS structures has made it possible to have at least some realistic comparisons[11, 10] if not directly of string theory, then at least of some of what are believed to be consequences of its underlying philosophy.

My attitude towards the string/modular theory issue[21] did not change, in part due to the fact that I have much deeper, almost archaic reasons, which probably also the Jubilar can appreciate. I am referring to the mysterious role of crossing symmetry, which at the same time was (apart from unitarity) by far the physically most important input into the Berkeley (primarily Chew's) S-matrix bootstrap program; the analyticity, to the extent that it was not needed in the formulation of crossing, was more of a technical nature. Now, with hindsight and knowledge of the consequences of the Jubilar's modular contributions, it is becoming gradually clear that those two topics belong together, and that there could not have been conceptual progress on crossing symmetry without a comprehensive modular understanding of the wedge situation.

Let us follow the flow of history on crossing symmetry a bit more.

As everybody of that generation remembers, in order to lift some of the

[21] I still believe that both string theory and LQP leave the rather narrow confines of standard Lagrangian QFT, but for different reasons and with different aims. Whereas, for the followers of string theory, QFT was identified with the standard text book Lagrangian or functional quantization, and therefore the (revolutionary) departure happened on the physical side by keeping as much as possible of the standard formalism, LQP is totally conservative with respect to the underlying physical principles, but revolutionary on their mathematical and conceptual implementations. String theory, after its second revolution, wanted to be (or at least to incorporate) quantum gravity, whereas LQP definitely wants to stay with laboratory physics. A very instructive illustration of this difference is supplied by comparing Witten's approach [22] to AdS versus that in [10, 11].

mystery of crossing symmetry, Veneziano invented the dual model, and Virasoro observed subsequently that the (on-shell) S-matrix (still without its unitarity corrections) permitted the mathematical trick of a representation in terms of a lower-dimensional (off-shell) QFT. This was the birth of string theory, never mind its several revolutions and the semantic changes which happened in the course of its conversion from a nonperturbative proposal for a strongly interacting S-matrix[22] into a TOE including quantum gravity.

Unfortunately, full understanding of the notoriously difficult crossing symmetry (which most people thought of as an on-shell imprint of the off-shell Einstein causality), whose unraveling was worth any effort, was not obtained. Regrettably, the birth of string theory was for many, especially young theoreticians, also the point of departure into the physical blue yonder with little chance to return.

Now with the patient and precise early work of the Jubilar on modular theory bearing many fruits, and with the importance of crossing symmetry at the cradle of string theory on one's mind, the proposal to occupy part of the physically underpopulated M-universe with m as in modular theory may after all not turn out to be as outrageous as it appears on first sight.

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References

[1] R. Haag, *Local Quantum Physics*, Springer 1992

[2] D. Buchholz, *Current trends in axiomatic quantum field theory*, hep-th/9811233

[3] B. Schroer, AOP Vol. 255, No.2, (1997) 270

[4] R.F. Streater and A.S. Wightman, *PCT, Spin and Statistics and all That*, Benjamin 1964

[5] S. Weinberg, *The Quantum Theory of Fields, I,* Cambridge University Press 1995

[22]An S-matrix with an infinite tower of particles in a finite range of mass is of course not compatible with reasonable phase space behavior of quantum physics (Hagedorn temperature and worse), but as in Feynman's perturbation theory one would expect that from genus g=2 on, the tower (except a finite number of particles) would transmute into second Riemann sheet resonances. According to the best of my knowledge there is no known property of QFT which prohibits this. But in this case, what means "stringiness" versus QFT behavior?

[6] K. Fredenhagen, K.-H. Rehren and B. Schroer, Rev. Math. Phys., **Special Issue** (1992) 113 and references therein

[7] D. Guido and R. Longo, Commun. Math. Phys. **181** (1996) 11, and papers quoted therein

[8] K.-H. Rehren, Lett. Math. Phys. **46**, (1998) 95

[9] H.-J. Borchers, On the TCP-Theorem in the Theory of Local Observables, University of Göttingen preprint 1999 and references therein.

[10] K.-H. Rehren, *Algebraic Holography*, hep-th/9905179

[11] D. Buchholz, M. Florig and S.J. Summers, *Hawking-Unruh temperature and Einstein causality in anti-de Sitter space-time*, hep-th/9905178

[12] B. Schroer, *Modular Wedge Localization and the d=1+1 Form Factor Program*, in print, scheduled to appear in Annals of Physics **275**, (1999)

[13] B. Schroer and H.-W. Wiesbrock, *Modular Constructions of Quantum Field Theories with Interactions,* hep-th/9812251

[14] B. Schroer, *Test of Nonperturbative Concepts in d=1+1 Factorizing Models of QFT,* Notas de Fisica, CBPF-NF-019/99

[15] B. Schroer, *Basic Quantum Theory and Measurement from the Viewpoint of Local Quantum Physics*, quant-ph/9904072

[16] B. Schroer and H.-W. Wiesbrock, *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models*, hep-th/9901031, to be published in Rev. in Math. Phys.

[17] D. Buchholz, *O. Dreyer, M. Florig and S. Summers, Geometric Modular Action and Spacetime Symmetry groups*, hep-th/

[18] H.-W. Wiesbrock, Commun. Math. Phys. **157**, (1993) 83, Commun. Math. Phys. **193**, (1998) 269, to be published in Rev. in Math. Phys.

[19] B. Schroer and H.-W. Wiesbrock, *Modular Theory and Geometry*, math-th/9809003, to be published in Rev. in Math. Phys.

[20] K. Fredenhagen and M. Joerss, *conformal Haag-Kastler nets, pointlike localized fields and the existence of operator product expansions,* Commun. Math. Phys. **176** (1996) 541

[21] G. 't Hooft, *Dimensional reduction in Quantum Gravity*, University of Utrecht preprint TAU 93, gr-qc/9310000

[22] E. Witten, Adv. Theor. Math. Phys. **2**, (1998) 253

[23] M. Karowski and P. Weisz, Nucl. Phys. **B 139**, (1978) 445

[24] F. A. Smirnov, Int.J. Mod. Phys. **A4**, (1989) 156

[25] M. Hortacsu, R. Seiler and B. Schroer, Phys. Rev. D **5**, (1972) 2519

[26] B. Schroer and J. A. Swieca, Phys. Rev. D **10**, (1974) 480
B. Schroer, J. A. Swieca and A. H. Voelkel, Phys. Rev. D **11**, (1975) 1509

[27] B. Schroer, *Principles of General Quantum Field Theory versus New Intuition from Model Studies*, an essay dedicated to the memory of Jorge Andre Swieca (1936-1980), in: *The Algebraic Theory of Superselection Sectors, Introduction and Recent Results*, page 573-604, ed. Daniel Kastler, World Scientific 1990

[28] K.-H. Rehren and B. Schroer, Phys. Lett. **198B**, (1987) 84

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