

Numerical studies of vertically propagating acoustic and magneto-acoustic waves in an isothermal atmosphere *

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Abstract

In this paper we investigate numerically the effect of viscosity and Newtonian cooling on upward and downward propagating magneto-acoustic waves, resulting from a uniform horizontal magnetic field in an isothermal atmosphere. The results of the numerical computations are compared with those of asymptotic evaluations. It is shown that the presence of a small viscosity creates a layer which acts like an absorbing and reflecting barrier for waves generated below it and that the presence of the magnetic field produces a reflecting layer only. The addition of Newtonian cooling affects mainly the lower region in which it produces waves attenuation and alters the wavelength. If the Newtonian cooling coefficient is large compared with the frequency of the waves, the temperature in the lower region evens out and the wave motion approaches an isothermal one. This eliminates the attenuation in the wave amplitude since the isothermal region is dissipationless. This problem is solved analytically and numerically. The results of the numerical computation are in a complete agreement with the analytical results.

1 Introduction

The propagation of atmospheric waves, both in isothermal and in non-isothermal atmospheres, has been investigated extensively in recent years. The discovery of hydro-magnetic waves was followed by an extensive study of magneto-acoustic waves in an isothermal atmosphere. Much of the motivation of these studies comes from their relevance to phenomena in compressible ionized fluids, such as solar, stellar, earth's atmospheres and to certain phenomena in ocean dynamics (see Alkahby [2]-[6], Yanowitch [9]-[11] for references).

It is well known that the solar corona is extremely hot, typical temperatures are 10^6 K compared with 5×10^3 K in the photosphere. Consequently, thermal energy must be continually supplied to maintain this temperature against

* 1991 Mathematics Subject Classifications: 76N10, 76Q35.

Key words and phrases: magneto-acoustic waves, Newtonian cooling coefficient, wave propagation, reflection coefficient.

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Published December 9, 1999.

radiative cooling. Early theories of coronal heating were essentially based on the dissipation of acoustic waves or shock waves. Recent theories involve magnetic energy dissipation as the source of thermal energy. These two questions must be answered: how is magnetic energy supplied to the corona, and how is it dissipated? To answer these questions, many mathematical models and dissipative mechanisms are suggested (see Alkahlby [1]-[8], Yanowitch [9]-[11] for references).

The aim of this study is to obtain numerical data for the effects of the viscosity, Newtonian cooling and magnetic field on the reflection and dissipation of an upward and a downward propagating sound wave in an isothermal atmosphere for practical purposes. To obtain a full understanding of the nature of the waves propagation, reflection, the effect of Newtonian cooling on the waves below the reflecting barrier and the nature of the reflecting layer, the values of viscosity and magnetic field were taken to be small, arbitrary and positive. The variation of the values of the Newtonian cooling coefficient allows us to determine the maximum and minimum values of the attenuating factor in the amplitude of the upward and downward propagating waves, maximum and minimum values of the reflection coefficient, the cutoff frequencies and the change of wavelength from the adiabatic values to the isothermal ones. Upon using central differences, the differential equation (3.14) is replaced by a difference equation, which is solved by backward and forward integration. In the computation process, the magnitude of the reflection coefficient is determined from the ratios of the maximum with respect to the minimum values of the kinetic and magnetic energies below the reflecting layer.

The results of the numerical computation are described in Section [5] by six figures. The computation shows that: (a) when the viscosity dominates the oscillation process, the maximum value of the reflection is $\exp(-\pi\beta_a)$ and is attained when the oscillatory process is adiabatic and the minimum is $\exp(-\pi\beta_i)$ and is attained when the Newtonian cooling coefficient is large compared with the adiabatic cutoff frequency (Note that β_a and β_i are the adiabatic and isothermal wave numbers, with $\beta_i > \beta_a$ and defined in Section [3]). If values of the Newtonian cooling coefficient are small compared with the adiabatic cutoff frequency of the wave, the magnitude of the reflection coefficient is less than $\exp(-\pi\beta_a)$ and greater than $\exp(-\pi\beta_i)$; (b) when the magnetic field effect dominates oscillatory process, below the reflecting layer, the magnitude of the reflection coefficient is always one for all positive values of the Newtonian cooling coefficient. Since the wave number changes from β_a to β_i , it follows from (a) and (b) that the presence of small viscosity creates a reflecting and an absorbing reflecting barrier. On the contrary, the presence of the magnetic produces a reflecting layer only. This result is expected because of the dissipationless nature of the magnetic field and that the change of the oscillatory process from the adiabatic form to the isothermal one and vice versa do not influence the nature of the reflection but affect only the reflecting layer produced by the effect of viscosity. In addition, the oscillatory process changes from the adiabatic form to the isothermal one below the reflecting layer. This change can easily be deduced from the change in the wavelength of the wave. Moreover, the

computation shows that the resonance may occur for infinitely many values of the magnetic field and the frequency of the wave. Finally, the asymptotic and numerical results are almost in a complete agreement for 5 places.

2 Mathematical formulation of the problems

The hydro-magnetic equations of motion for pulsating stars consist of the momentum equation, the continuity equation, the induction equation, and the pressure and energy equations, which can be written as follows:

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \right] + \nabla P = \rho \mathbf{g} + \frac{4}{3} \mu \nabla^2 \mathbf{V} + \frac{1}{4\pi} [\nabla \mathbf{x} \mathbf{B} \mathbf{x} \mathbf{B}] , \quad (1)$$

$$\rho_t + \nabla(\rho \cdot \mathbf{V}) = 0 , \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \mathbf{x} (\mathbf{V} \mathbf{x} \mathbf{B}) = 0 , \quad (3)$$

$$P = R\rho T , \quad (4)$$

$$\rho T \frac{Ds}{Dt} = -\nabla \cdot \mathbf{q} - L_r + \frac{j^2}{\sigma} + H_{ts} . \quad (5)$$

In the above equations (1-5), ρ means density, \mathbf{V} is the fluid vertical velocity, P is the pressure, \mathbf{g} is the gravitational acceleration, μ is the dynamic viscosity coefficient, \mathbf{B} is the magnetic field strength, R is the gas constant, T is the temperature, S is the entropy per unit mass of the plasma, \mathbf{q} is the heat flux due to partial conduction, L_r is the net radiation, $\frac{j^2}{\sigma}$ is the ohms dissipation, and H_{ts} represents the sum of all the other heating sources.

The equations of motion form a system of nonlinear partial differential equations which, in most cases, cannot be solved. For small-amplitude oscillations the dependent variables can be written as the sum of a mean value and a small perturbation. The equations are then simplified to a linear system by neglecting all products of perturbation terms. Let P , ρ , \mathbf{V} , T , and \mathbf{B} be the perturbations in the pressure, density, vertical velocity, temperature, and the magnetic field strength and P_0 , ρ_0 , T_0 , B_0 be the equilibrium quantities. Also we restrict our investigation to an isothermal atmosphere permeated by an uniform horizontal magnetic field and it has an infinite electrical conductivity. In addition, we investigate small oscillations $z \geq 0$. As a result of the above restriction, equilibrium pressure, temperature and density satisfy the gas law $P_0 = R\rho_0 T_0$ and the hydrostatic equation $P'_0 + g\rho_0 = 0$. Consequently, the pressure and density are given by

$$\begin{aligned} P_0(z) &= P_0(0) \exp(-z/H), \\ \rho_0(z) &= \rho_0 \exp(-z/H), \end{aligned}$$

where H is the density scale height and defined by $H = RT_0/g$. Consequently, the density scale height is not constant in the solar atmosphere, i.e., each region has its own density scale height because of the change in the temperature and the acceleration from one region to another. This observation also necessitates the

study of the effect of the Newtonian cooling on the acoustic waves propagation in the solar atmosphere and its influence on the heating mechanism. Moreover, the linearized equations of motion can be written like

$$\rho_0 V_t + P_z + \rho g + \left(\frac{B_0}{4\pi} B_z\right) = \frac{4}{3}\mu V_{zz}, \quad (6)$$

$$\rho_t + (\rho_0 V)_z = 0, \quad (7)$$

$$B_t + B_0 V_z = 0, \quad (8)$$

$$P = R(\rho_0 T + T + T_0 \rho), \quad (9)$$

$$c_V(T_t + qT) + gHV_z = 0. \quad (10)$$

The subscripts z and t denote the differentiation of the independent variables with respect to z and t respectively, c_V denotes the specific heat at constant volume and q is the Newtonian cooling coefficient which refers to the heat exchange between hot and cold regions. We consider solutions which are harmonic in time, i.e., $V(z, t) = V(z) \exp(-\sigma t)$ and $T(z, t) = T^*(z) \exp(-\sigma t)$ where σ denotes the frequency of the wave. It is more convenient to rewrite the equation of motion in dimensionless form: $z^* = z/H$, $\sigma_a = c/2H$ is the adiabatic cutoff frequency, where $c^2 = \gamma RT_0 = \gamma gH$ is the adiabatic sound speed $V^* = V/c$, $\mu^* = 2\mu/3\rho_0 cH$, $\sigma^* = \sigma/\sigma_a$, $t^* = t\sigma_a$, $a_1 = a_A^2/c^2$, $T^* = T/2\gamma T_0$, $q^* = q/\sigma_a$, $a = a_1 - i\sigma^* \mu^*$. The star can be omitted, since all variables are written in dimensionless form from now on. Moreover, ρ , p , and B can be eliminated from equation (6) by differentiating it with respect to t and substituting equations (7–10) to obtain a system of differential equations from $V(z)$ and $T(z)$.

$$(D^2 - D + \gamma\sigma^2/4)V(z) + \gamma a e^z D^2 V(z) + i\gamma(D-1)T = 0, \quad (11)$$

$$DV(z) = \gamma(i\sigma - q)T(z)/(\gamma - 1) \quad (12)$$

where $D = d/dz$. Moreover, $V(z)$ can be eliminated from equation (11) to obtain a second order equation for $T(z)$.

$$\begin{aligned} & [\gamma\sigma(D^2 - D + \sigma^2/4) + iq(D^2 - D) + \gamma\sigma^2/4] \\ & + \gamma(\sigma + iq)ae^z(D^2 + D)]T(z) = 0. \end{aligned} \quad (13)$$

In addition, the first two terms can be combined to give the following differential equation

$$[(D^2 - D + Q\sigma^2/4) + Qae^z(D^2 + D)]T(z) = 0, \quad (14)$$

where the parameter Q is defined by

$$Q = \gamma(\sigma + iq)/(\gamma\sigma + iq).$$

It is clear that the parameter $Q = 1$ when $q = 0$ and $Q = \gamma = 1.4$ as $q \rightarrow \infty$. This must indicate some changes in the physical nature of the problem and these changes will influence mainly the wavelength, the magnitude of the reflection coefficient and the nature of the reflecting layer. These changes will be clear, asymptotically and numerically, in the following sections.

Boundary Condition: To obtain a unique solution for the differential equation (13), physically relevant conditions must be imposed. When $B = \mu = 0$, there is no need for physical mechanism to be used to determine a unique solution. In this case, the only boundary condition is the radiation condition which will ensure a unique solution. When $q = m = 0$, the acoustic waves are only influenced by the effect of the magnetic field. As a result, there is no dissipative mechanization and the only condition which will be used to ensure a unique solution is the magnetic energy condition. This condition can be expressed mathematically in the following form:

$$\int_0^{\infty} |V_z|^2 dz < \infty . \quad (15)$$

Moreover, when $q \geq 0$ and $B \neq 0$, the magnetic energy condition is still the only upper boundary condition. When μ , the dissipative mechanism (????? couldn't read this word) because of the effect of the viscosity. As a result, a unique solution is obtained from the requirement that the average (per period) rate of energy dissipation in a column of fluid should be finite. Since the dissipation function depends on the square of the velocity gradients, this implies

$$\mu \int_0^{\infty} |V_z|^2 dz < \infty . \quad (16)$$

Consequently, the energy condition and the dissipation conditions are mathematically equivalent. It follows from equation (12) that

$$\int_0^{\infty} |T|^2 dz < \infty . \quad (17)$$

It will be seen that the upper boundary conditions in connection with boundary conditions at $z = 0$, determine a unique solution The boundary condition at the ground can always be made $T(0) = 1$ by suitably normalizing $T(0)$. Finally, it has to be noted that the dissipation condition is necessary and sufficient condition to determine a unique solution.

3 Solutions and some remarks about eqn. (14)

CASE ONE: In the lower region (i.e., region below the reflecting layer) where $|Qa| e^z \ll 1$, and small values of the Newtonian cooling coefficient q , the solution of equation (14) can be approximated by the solution of the following differential equation

$$4D^2T(z) - 4DT(z) + Q\sigma^2T(z) = 0 . \quad (18)$$

Consequently, when $q = 0$, the solution which satisfies the radiation condition (in this case the dissipation condition is not applicable because the atmosphere

is considered to be inviscid) and the lower boundary condition can be written as

$$T(z) = A_1 \exp\left[\left(\frac{1}{2} + o\beta_a\right)z\right], \quad (19)$$

where A_1 is a constant and $2\beta_q = \sqrt{\sigma^2 - 1}$ is the adiabatic wave number. This is exactly the solution of the first term in the differential equation (13). When $q \rightarrow \infty$ and $\sigma > 1/\sqrt{\gamma}$, the solution which satisfies the radiation condition can be written in the following form:

$$T(z) = A_2 \exp\left[\left(\frac{1}{2} + i\beta_i\right)z\right], \quad (20)$$

where A_2 is a constant and $2\beta_i = \sqrt{\gamma\sigma^2 - 1}$ is the isothermal wave number. As a result, one of the important effects of the heat radiation is to change the oscillatory process from the adiabatic form, below the reflecting layer, to an isothermal one.

When q is small compared with σ , the solution of the differential equation (18) which satisfies the radiation condition can be written as:

$$T(z) = A_3 \left[\left(\frac{1}{2} - d(q) + i\beta\right)z \right] \quad (21)$$

where A_3 is a constant, $d(q)$ is the damping factor in the amplitude of the wave, and β is the wave number for small values of q compared with σ . When $q = 0$, we have $d(q) = 0$ and $\beta = \beta_a$ for $\sigma > 1$ (because $Q = 1$, when $q = 0$). On the other hand, when $q = \infty$, i.e., q is large compared with σ , we have $d(q) = 0$, $\beta \rightarrow \beta_i$, (because $Q \rightarrow \gamma$ when $q \rightarrow \infty$). As a result, when $q = 0$ the cutoff frequency of the wave σ_a equals to 1. When $q \rightarrow \infty$, we have $Q \rightarrow \gamma$ and the isothermal cutoff frequency σ_i equals $1/\sqrt{\gamma}$. This indicates that we have three ranges for the frequency of the wave,

$$\sigma : \sigma_a = 1, \quad \sigma < \sigma_i = 1/\sqrt{\gamma} \quad \text{and} \quad \sigma_1 < \sigma < \sigma_a. \quad (22)$$

CASE TWO: When $|Qa|e^z \gg 1$, the structure of the problem is completely different mathematically and physically. To obtain a general solution of the differential equation (14) which satisfies the prescribed boundary conditions, let

$$\xi = -e^{-z}/qQ = -\exp(-z - \delta_1 - \text{Arg}(aQ)), \quad (23)$$

where $\delta_1 = \ln |aQ|$. The differential equation (14) will be transformed to

$$[\xi(1 - \xi)D^2 - 2\xi D - Q\sigma^2/4]T(\xi) = 0, \quad (24)$$

where $D = d/d\xi$ and $\arg(-\xi) = \arg(1/aQ)$. Equation (24) is a special case of the hyper-geometric equation

$$[\xi(1 - \xi)D^2 + (c - (a + b + 1)\xi)D - ab]T(\xi) = 0, \quad (25)$$

with $c = 0$, $a + b = Q\sigma^2/4$. Solving for parameters a and b , we have

$$a = (1 + \sqrt{1 - Q\sigma^2})/2 = \frac{1}{2} - d(q) + i\beta, \quad (26)$$

$$b = (1 + \sqrt{1 - Q\sigma^2})/2 = \frac{1}{2} - d(q) - i\beta. \quad (27)$$

Thus the differential equation (3.7) has three regular singular points as $\xi = 0$, $\xi = 1$ and at infinity. The point $\xi_0 = -1/aQ = -\exp[-(\sigma_1 + \text{Arg}(aQ))]$ corresponds to $z = 0$, and $\xi = 0$ corresponds to $z = \infty$. The point $\xi = 1$ corresponds to $\delta_1 = -\text{Arg}(aQ)$. This argument is valid because the problem is in dimensionless form. Moreover, the differential equation (24) has two linearly independent solutions which can be written in the following form

$$T_1(\xi) = \xi F(a + 1, b + 1, 2, \xi), \quad (28)$$

and

$$T_2(\xi) = T_1(\xi) \log \xi + 1/ab + \sum_{k=1}^{\infty} c_k \xi^k, \quad (29)$$

where $F(a + 1, b + 1, \xi)$ is the hyper-geometric function. It is evident that $T_1(z) = O(e^{-z})$ and $T_2(z) \rightarrow (1/ab)$. It follows from equation (1) that V_z is proportional to T , and this implies that $T_2(z)$ does not satisfy the dissipation condition. The solution of the differential equation (24) is, therefore, a multiple of $T_1(\xi)$.

To find the asymptotic behavior of the solution, $\text{Arg}(aQ)$ must be determined. The maximum value of $\text{Arg}(Q)$ is $\theta_0 = (\gamma - 1/2)\sqrt{\gamma}$, attained when $\sigma/q = \sqrt{\gamma}$. Also $\text{Arg}(a)$ (which is denoted by ϕ_1) is $-\pi/2 < \phi_1 \leq 0$. Consequently, $\text{Arg}(-\xi) = \text{Arg}(1/Qa)$ satisfies $-\phi_0 < \arg(-\xi) < \pi/2$, and thus will allow us to write the solution $T_1(z)$ (using the relation (23)) in the following form

$$\begin{aligned} T_1(z) &= F(a + 1, b + 1, 2, \xi) \\ &= \Gamma(b - a)/b\Gamma^2(b)\xi(-\xi)^{-(1+a)}F(a + 1, a - 1, 2a, \xi^{-1}) \\ &\quad + (\Gamma(a - b)/a\Gamma^2(a)\xi(-\xi)^{-(1+b)}F(b + 1, b - 1, 2b, \xi^{-1}). \end{aligned} \quad (30)$$

This argument is valid because $-\pi < \arg(-\xi) < \pi$ and has as $|Qa| \rightarrow 0$, the equation (24) can be written like

$$\begin{aligned} T(z) \sim & [\Gamma(b - a)/b\Gamma^2(b)] \exp\left[\left(\frac{1}{2} - d(q) + i\beta\right)(z + \delta_1 + \text{Arg}(aQ))\right] \\ & + [\Gamma(a - b)/a\Gamma^2(a)] \exp\left[\left(\frac{1}{2} + d(q) - i\beta\right)(z + \delta_1 + \text{Arg}(aQ))\right]. \end{aligned} \quad (31)$$

Equation (31) represents the solution of the differential equation (24) which satisfies the prescribed boundary conditions. The first term on the right represents an upward propagating wave, its amplitude decaying exponentially with the altitude as $\exp[-d(q)z]$. The second term is a downward traveling wave

decaying at the same rate. Moreover, the reflection takes place at the region $z = O(-\delta_1 - \phi)$ where $\phi = \phi_0\phi_1$ and (31) can be written as:

$$T(z) \sim [\Gamma(b-a)/b\Gamma^2(b)]\{\exp[(\frac{1}{2} - d(q) + i\beta)z + RC \exp[(\frac{1}{2} + d(q) - i\beta)z]]\}, \quad (32)$$

where RC denotes the reflection coefficient and it is given by

$$RC = \Gamma(a-b)/a\Gamma^2(a) \cdot b\Gamma^2(b)/\Gamma(b-a) \exp[(2d(q) - 2i\beta)(\delta_1 + \phi)], \quad (33)$$

which can be rewritten in the following form:

$$RC = \exp[(2d(q) - 2i\beta)(\delta_1 + i\text{Arg}(RC))]. \quad (34)$$

4 Magnitude of the reflection coefficient

It is well known that the pressure of the viscosity creates an absorbing and reflecting barrier. As a result, the atmosphere can be divided into two distinct regions. The lower region is adiabatic (when $q = 0$) and in it, the solution, which satisfies the upper boundary condition (15) can be written as a linear combination of an upward and a downward propagating wave. In the upper region, the solution decays as $\exp(-z)$. As a result, the magnitude of the reflection coefficient depends on the nature of the force which controls the oscillatory process in the lower regions and on the values of the Newtonian cooling coefficient compared with those of the frequency of the wave. Consequently, several cases must be considered to obtain the magnitude of the reflection coefficient. First, when the viscosity dominates the oscillatory process, (i.e., in the regions of the solar atmosphere where the effect of the magnetic field is negligible), below the reflecting barrier, we have the following cases:

- (A) When $q = 0$ and $\sigma > 1$, we have $a = \frac{1}{2} + i\beta_a$, $b = \frac{1}{2} - i\beta_a$, and $\text{Arg}(RC) = -\pi/2$. As a result, the magnitude of the reflection coefficient

$$|RC| = |RC_a| = \exp(-\pi\beta_a). \quad (35)$$

- (B) When $q \rightarrow \infty$, (i.e., when q is very large compared with the frequency of the wave, σ), and $\sigma > 1/\sqrt{\gamma}$, one obtains $a = \frac{1}{2} + i\beta_i$, $b = \frac{1}{2} - i\beta_i$ and $\text{Arg}(RC) = -\pi/2$. Consequently, the magnitude of the reflection coefficient

$$|RC| = |RC_i| = \exp(-\pi\beta_i). \quad (36)$$

- (C) When the values of the Newtonian cooling coefficient are not very large compared with the frequency of the wave, we have

$$\exp(-\pi\beta_i) \leq |RC| \leq \exp(-\pi\beta_a). \quad (37)$$

It is clear that the magnitude of the reflection coefficient is less than one. Consequently, part of the energy is absorbed in the reflecting layer and this may contribute to the heating of the solar atmosphere. The other part is reflected downward.

When the magnetic field dominates the oscillatory process in the lower region, in the regions like the sun spots, we have the following case:

(D) When $q = 0$ or $q \rightarrow \infty$, we have $Arg(RC) = 0$. Consequently, the magnitude of the reflecting coefficient:

$$|RC_a| = 1 . \quad (38)$$

As a result, the magnetic field creates a non-absorbing reflecting layer because of the dissipative nature of the magnetic field. In addition, the change of the oscillatory process from the adiabatic form to the isothermal one does not influence the nature of the reflecting layer produced by the magnetic field, which results from the dissipationless nature of the magnetic field. On the contrary, the change in the oscillatory process, from the adiabatic form to the isothermal one influences only the nature of the reflecting layer created by the effect of the viscosity. These observations are based on the reduction in magnitude of the reflection coefficient when the viscosity dominates the oscillatory process.

5 Computing scheme and results of computations

The results of the previous section are asymptotically valid as $|a| \rightarrow 0$ or $|Qa| \rightarrow 0$. In order to examine the nature of the reflecting layer and its influences on the reflection process, the problem is solved numerically and the results are compared with those of the previous sections. To obtain a reasonable result, the value of $|a|$ is taken to be a very small one in order to give a sufficient range for the waves to propagate below the reflecting layer because the existence of the lower region depends mainly on the range of $|a|$. Also to determine the values of q for which the oscillatory process, in the lower region, changes from an adiabatic form to an isothermal one. The boundary value problem is solved numerically for several values of a , μ , and q . Using central differences, we can replace the differential equation (24) by the difference equation

$$A_n T_{j+1} + B_n T_j + C_n T_{j-1} = 0 . \quad (39)$$

Using the substitution

$$T_{j-1} = a_{n-1} T_j + \beta_{n-1} , \quad (40)$$

then equation (5.1) can be written as:

$$T_j = a_n T_{j+1} + \beta_n , \quad (41)$$

where

$$a_n = -[B_n + C_n a_{n-1}]^{-1} A_n, \quad (42)$$

$$\beta_n = -[B_n + C_n a_{n-1}] C_n \beta_{n-1}. \quad (43)$$

The system was solved on an interval $0 \leq z \leq L$ sufficiently large enough to allow T to reach its limit values. The dissipation condition is replaced by $T_N = T_{N-1}$, where N is the index of the point $z = L$. The problem was solved by the standard method in which the T_j is computed from equation (5.3) by backward integration, while a_n and β_n in equations (5.4) and (5.5) are computed by forward integration. The problem was solved for $|a| = 10^{12}$ which is sufficiently small values to test the asymptotic formula, and for different values of q , a_1 , μ , and of the wavelength $2\pi/\beta$. A value of 30 or 40 was more than enough for L . The results of the computations are shown in figures 1, 2, 3, 4, 5, and 6. Moreover, let M and m denote the maximum and minimum values of the oscillation amplitude and $d = \sqrt{M/m}$, $|RC|$ can be computed from

$$|RC| = (d - 1)/(d + 1). \quad (44)$$

The numerical and asymptotic results are in agreement to five places.

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