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HÖLDER SOLUTIONS FOR THE AMORPHOUS SILICON SYSTEM AND RELATED PROBLEMS

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Abstract. We present existence of solutions and other results for the partial differential equation system with memory which models amorphous silicon devices and related problems in $\mathbb{R}^3$. Our approach employs only classical estimates and Degree Theory; it shows the existence of $C^{\alpha,\alpha/2}$ solutions for some $\alpha > 0$. In view of the mixed boundary conditions, this is the maximum regularity that can be expected.

1. Introduction

In the past few years micro-electronic devices employing amorphous silicon as the semiconductor material have shown promise in a variety of applications such as liquid crystal displays, image sensors and solar cells. The mathematical model usually employed to simulate such devices involves drift-diffusion equations as well as equations describing the density of trapped charges, [3, 8]. The latter may be explicitly integrated in time, giving a drift-diffusion system with integral (i.e. "memory") terms. Specifically, if we assume only one trapped charge state and set all mathematically irrelevant coefficients to unity, we obtain:

$$-\Delta\varphi = \Big(p - n + C_1(x) + \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta} d\xi\Big) \tag{1'}$$

$$\begin{aligned}\frac{\partial n}{\partial t} &- \nabla[D_n(x,t,n,p,|\nabla\varphi|)\nabla n - n\mu_n(x,n,p,|\nabla\varphi|)\nabla\varphi] \qquad (2')\\ &= 1 - \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta} d\xi - n\Big[1 + \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta} d\xi\Big]\end{aligned}$$

$$\begin{aligned}\frac{\partial p}{\partial t} &- \nabla[D_p(x,n,p,|\nabla\varphi|)\nabla p + p\mu_p(x,n,p,|\nabla\varphi|)\nabla\varphi] \qquad (3')\\ &= 1 + \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta} d\xi - p\Big[1 - \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta} d\xi\Big]\end{aligned}$$

to be satisfied in a smooth domain $\Omega \subset R^3$. We observe that the factor "2" is present in the various integrals in equations (1')-(3') to ensure charge conservation, [8]. With equations (1')–(3') we associate initial/mixed boundary conditions as

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follows: Let $\partial\Omega = \Gamma_D \cup \Gamma_N$ with Γ_D a smooth nonempty closed sub-manifold in which Dirichlet conditions are to hold:

$$\varphi(x,t) = \overline{\varphi}(x), \quad n(x,t) = \overline{n}(x), \quad p(x,t) = \overline{p}(x)$$

for all t, while Neumann conditions are to hold on $\Gamma_N = \partial\Omega - \Gamma_D$:

$$\frac{\partial \varphi}{\partial \vec{\nu}} = \frac{\partial n}{\partial \vec{\nu}} = \frac{\partial p}{\partial \vec{\nu}} = 0\,.$$

With n, p we also associate initial conditions. Both to avoid technical difficulties and in keeping with the situation in the physical problems we also ask that $n(x,0) = \overline{n}(x)$, $p(x,0) = \overline{p}(x)$, with $\overline{n}(x)$, $\overline{p}(x) \in C^1(\overline{\Omega})$ and $\overline{n}, \overline{p} \geq 0$. These may be weakened in what follows without essential proof changes, but they do simplify the presentation.

For the same reason, we assume throughout the paper that all equation coefficients are smooth in their variables. Note that since n, p are densities we shall only seek solutions with $n, p \geq 0$. The behavior of the equation coefficients in (n,p) for $n, p < 0$ can thus be chosen for convenience. Some regularity is also needed for $\partial(\Gamma_N) \cap \Gamma_D$. Intuitively if $x \in \partial(\Gamma_N) \cap \Gamma_D$ and $\mathcal{N}$ is a small neighborhood of x, we require that regularity considerations for $\mathcal{N} \cap \partial\Omega$ be reduced via bi-Lipschitz (resp. smooth) coordinate maps to similar problems on quarter-spheres (resp. hemispheres). The reader interested in the explicit formulations of such conditions may find them for example in [7, 11, 13, 16].

In the next sections we introduce and analyze a system of equations which contains as special cases not only (1')–(3') but also the standard drift-diffusion equations. While we are not aware of an earlier study of such a system with "memory terms", we point out there have been numerous results on the non-augmented system in recent years. It is not possible to present a detailed analysis of previous results here, but we refer the interested reader in particular to the papers [2, 4, 5], to the books [9, 10, 12] and the references therein. It is the paper by Fang and Ito [2], and da Veiga, [15] which furnished in part the motivation for this work, and which are closest to the assumptions made. Indeed the regularity of n, p shown here is conjectured in [15]. In general terms, the approaches usually employed in the past are based on time discretization, on semigroup analysis, on fixed point theorems and weak solutions are found in suitable spaces. Often techniques involving maximum principles, the Einstein relations and the introduction of quasi Fermi variables, and coefficient truncation were employed. In this paper we use none of these tools, and it is not clear how useful many of these would be in our situation, given the memory term present in $(1')$. Instead we employ Degree Theory and work directly with $C^{\alpha,\alpha/2}$ spaces. Not only does this simplify considerably the presentation but the solutions we find are of the regularity one would expect from the physical point of view. More global regularity cannot be realized in general due to the mixed boundary conditions for n, p, φ.

Our procedures are based on simple arguments involving classical results ([6, 17]) which are well-known although in themselves far from simple. As presented, the results are given for $\Omega \subset R^3$ – the physically interesting case. We conjecture that similar results hold for $\Omega \subset R^N$, $N \neq 3$.

As a final observation, note that as mentioned above, we do not make use of the Einstein Relations connecting D_n, D_p and μ_n, μ_p as we shall have no need of them. It follows that the results also hold if the system does not admit " quasi-Fermi " variables.

II. Analysis

Based on the model system considered in the Introduction, we introduce the following equations:

$$-\Delta\varphi = f\big(x,t,p-n,h(p,n)\big) \tag{1}$$

$$\begin{aligned}\frac{\partial n}{\partial t} &- \nabla[D_n(x,t,n,p,|\nabla\varphi|)\nabla n - n\mu_n\big(x,t,n,p,|\nabla\varphi|\big)\nabla\varphi] \\ &= 1-h(p,n)-n[1+h(p,n)]+R_n(x,t,n,p)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial p}{\partial t} &- \nabla\big[D_p(x,t,n,p,|\nabla\varphi|)\nabla p + p\mu_p\big(x,t,n,p,|\nabla\varphi|\big)\nabla\varphi\big] \\ &= 1+h(p,n)-p[1-h(p,n)]+R_p(x,t,n,p)\end{aligned} \tag{3}$$

with

$$h(p,n) = \int_0^t (p-n)e^{-\int_\xi^t (n+p+2)d\eta}d\xi\,.$$

We keep the initial/boundary conditions given on (n,p,φ) in the introduction as well as the requirement that $D_n, D_p, \mu_n, \mu_p, R_n, R_p$ be smooth functions of their respective arguments (at least for $n,\,p \geq 0$). We now introduce the following growth conditions on $\Omega\times(0,T)$, which may depend on T.

(A) There exist positive constants α,β such that

$$\alpha \leq D_n(x,t,n,p,|\nabla\varphi|), \quad D_p(x,t,n,p,|\nabla\varphi|) \leq \beta$$

(B) $R_n(x,t,n,p) = R_{n,1}(x,t,n,p) - nR_{n,2}(x,t,n,p)$ with $R_{n,1},\ R_{n,2}$ nonnegative, bounded, smooth if $n,p\geq 0$. We assume that R_p admits a similar decomposition into $R_{p,1}-pR_{p,2}$ with and $R_{p,1},\ R_{p,2}$ nonnegative, bounded, smooth.

(C) $\mu_n = \mu_{n,1}+\mu_{n,2}$, with $\mu_{n,1}$ a positive constant and $\mu_{n,2} = \mu_{n,2}\big(x,t,n,p,|\nabla\varphi|\big)$ such that $|\mu_{n,2}\nabla\varphi| \leq a_n$ for some positive constants a_n if $\big(x,t\big)$ are bounded. Similarly, $\mu_p = \mu_{p,1}+\mu_{p,2}$ with $0<\mu_{p,1}$ constant and $|\mu_{p,2}\nabla\varphi| \leq a_p$.

(D) There exist positive smooth functions M_1, M_2 of (x,t) such that

$$\big|f(x,t,\xi_1,\xi_2) - M_1|\xi_1|^{\alpha_2}\,\mathrm{sign}\,\xi_1\big| \leq M_2$$

for some $\alpha_2\geq 1$ and all $(x,t)\in\Omega\times[0,T], \quad 0\leq\xi_2\leq 1$.

Observe that system (1)–(3) with conditions (A – D) includes both the standard Drift-Diffusion model and the amorphous silicon model.

We choose and fix a parameter τ with $3<\tau<4$, set $Q_T = \Omega\times(0,T)$ and recall $\Omega\subset R^3$. We observe the following results

Lemma 0.

(a) *Let $-\Delta u(x) = f_1(x)$ in Ω, with $f_1\in L^\tau(\Omega)$. If $u=\overline{u}(x)\in C^1$ in Γ_D, $\quad \frac{\partial u}{\partial n}=0$ on Γ_N then $u\in H^{1,\tau}(\Omega)$ and*

$$\|\nabla u\|_{L^\tau(\Omega)} \leq C\Big[\|f_1\|_{L^\tau(\Omega)} + \|\overline{u}\|_{C^1(\Omega)}\Big]$$

(b) *Let v be a generalized solution of*

$$v_t - \nabla[w\nabla v + \vec{\delta}v] + mv = f_2 \tag{4}$$

with $0 < \alpha < w(x,t) < \beta$ *(*α*,* β *constants) and* $|\vec{\delta}|^2$*,* m*,* f_2 *in* $L^{q,r}(Q_T)$ *for some* $q \in (\frac{n}{2}, \infty]$*,* $r \in (1,\infty]$*,* $\frac{1}{r} + \frac{n}{2q} < 1$*. Suppose* v *satisfies the initial/boundary conditions:* $v = \overline{v}(x) \in C^1$ *on* $\{\Gamma_D \times (0,T)\} \cup \{\Omega \times \{0\}\}$*,* $\frac{\partial v}{\partial \vec{\nu}} = 0$ *on* $\Gamma_N \times (0,T)$ *and* v *is bounded in* $L^2(Q_T)$*. Then there exists an* $\alpha_0 > 0$ *(independent of* v*) such that* $v \in C^{\alpha_0,\alpha_0/2}(\overline{Q}_T)$*.*

(c) *If* v *solves* (4) *with the given initial/boundary conditions and* $\|v\|_{L^2(\Omega)}(t)$ *is bounded, then* v *is globally bounded in* L^∞*.*

Proof. Part (a) is immediate from the results of Shamir, [13].

Part (b) follows from e.g. [6, Theorem 10.1, p. 205] (see also [1]) and a reflection process to establish the needed regularity on $\overline{\Gamma}_N \cap \Gamma_D$, [7], [16], and Part (c) follows from [6, p. 192]. More explicitly, let v satisfy (4) and suppose first that $\Omega_0 \subset\subset \Omega$. Then for Ω_0 Parts (b), (c) are found explicitly in [6]. Next, if $P \in \Gamma_N$ then we map a neighborhood $\mathcal{N}$ of P by a bi-Lipschitz map L to a sphere S with $L(\Gamma_N \cap \mathcal{N}) \subset \{x \mid x_3 = 0\}$, $L(P) = 0$ and $L(\Omega \cap \mathcal{N}) \subset \{x \mid x_3 > 0\}$. We extend v as an even function to the whole of S and the coefficients as in [14], [16] so that the extended function $\widehat{v}$ satisfies (the extended) (4) in S. We can now use the interior/initial results to conclude first that $\widehat{v}$ is bounded in L^∞ and then that $\widehat{v} \in C^{\alpha_0,\alpha_0/2}$ in a neighborhood of $0 \times [0,T]$. Applying L^{-1} then shows the result for $\mathcal{N}$. If $P \in \overline{\Gamma}_N \cap \Gamma_D$ then the process is the same except now $L(\Omega \cap \mathcal{N}) \subset \{x \mid x_2 > 0,\ x_3 > 0\}$, $L(\Gamma_N \cap \mathcal{N}\} \subset \{x \mid x_2 = 0\}$, $L(\Gamma_D \cap \mathcal{N}) \subset \{x \mid x_3 = 0\}$. We first extend v as an even function to the upper hemisphere and then apply the Dirichlet problem results. The results for P on the Dirichlet Boundary are in [6].

In summary, for each $P \in \overline{\Omega}$, there exists a neighborhood $\mathcal{M}$ such that $u \in C^{\alpha_0,\alpha_0/2}(\mathcal{M} \times [0,T])$ and thus $u \in C^{\alpha_0,\alpha_0/2}(\overline{Q}_T)$ by boundary regularity. The same arguments also show Part (c).

Theorem 1. *There exist* $\alpha_1 > 0$ *and* $K > 0$ *such that all solutions of* $(1-3)$ *in* $C^{\alpha,\alpha/2}(\overline{Q}_T)$ *with* $0 < \alpha < \alpha_1$ *and* $n, p \geq 0$ *actually satisfy*

$$\|n\|_{C^{\alpha_1,\alpha_1/2}} + \|p\|_{C^{\alpha_1,\alpha_1/2}} + \|\varphi\|_{C^{\alpha_1,\alpha_1/2}} \leq K.$$

Proof. Let (n,p,φ) represent a solution in $C^{\alpha,\alpha/2}$ for some $\alpha > 0$. First note that

$$\frac{\partial h}{\partial t} + (n+p+2)h = p - n.$$

Since p, n are assumed nonnegative and $h(x,0) = 0$, we immediately conclude that $|h| \leq 1$. We next show that p, n are bounded in $L^\xi(L^\xi)$ for some large ξ. Assume without loss of generality that $\mu_{n,1} = \mu_{p,1} = \mu_1$. Otherwise we multiply the "n equation" in procedures that follow by $\frac{\mu_{p,1}}{\mu_{n,1}}$ and repeat.

Put $E = \max[\|\overline{n} + \overline{p}\|_{L^\infty}, 1]$ and let $n = Ew$, $p = Ez$ in equations $(1-3)$. We then have $0 < w,\ z < 1$ on Γ_D and equations (1 - 3) yield

$$-\Delta\varphi = f\big(x,t,E(z-w),h(p,n)\big) \tag{5}$$

$$\frac{\partial w}{\partial t} - \nabla[D_n\nabla w - w\mu_n\nabla\varphi] \leq \frac{R_{n,1}+2}{E} \tag{6}$$

$$\frac{\partial z}{\partial t} - \nabla[D_p\nabla z + z\mu_p\nabla\varphi] \leq \frac{R_{p,1}+2}{E}. \tag{7}$$

Let $g^{(1)} = (g-1)^+$ for any function g, C denote an arbitrary constant and use the Steklov average of $[(w^{(1)})]^\theta$, $[(z^{(1)}]^\theta$ as test functions in equations (6), (7) respectively for some $\theta > 1$. Since n, p are assumed of class $C^{\alpha,\alpha/2}$, these are suitable test functions. We find from assumptions (A), (B), (C) that:

$$\begin{aligned}
&\frac{1}{(\theta+1)}\int_\Omega [w^{(1)}]^{\theta+1}\Big|_{t_1}^{t_2} + \frac{4\theta}{(\theta+1)^2}\int_{t_1}^{t_2}\int_\Omega \alpha\Big|\nabla\left([w^{(1)}]^{\frac{\theta+1}{2}}\right)\Big|^2 \\
&-\int_{t_1}^{t_2}\int_\Omega a_n\theta\{[w^{(1)}]^\theta + [w^{(1)}]^{\theta-1}\}|\nabla w^{(1)}| \\
&-\int_{t_1}^{t_2}\int_\Omega \mu_1\theta[(w^{(1)})^\theta + (w^{(1)})^{\theta-1}]\nabla\varphi\nabla w^{(1)} \\
&\le \int_{t_1}^{t_2}\int_\Omega \frac{C}{E}(w^{(1)})^\theta.
\end{aligned}$$

We repeat with equation (7) and add to obtain

$$\begin{aligned}
&\frac{1}{(\theta+1)}\int_\Omega \left\{(w^{(1)})^{\theta+1} + (z^{(1)})^{\theta+1}\right\}\Big|_{t_1}^{t_2} \\
&+\frac{4\theta}{(\theta+1)^2}\int_{t_1}^{t_2}\int_\Omega \alpha\left\{\big|\nabla([w^{(1)}]^{\frac{\theta+1}{2}})\big|^2 + \big|\nabla([z^{(1)}]^{\frac{\theta+1}{2}})\big|^2\right\} \\
&-\int_{t_1}^{t_2}\int_\Omega \theta(a_n+b_n)\big[\{[w^{(1)}]^\theta + [w^{(1)}]^{\theta-1}\}|\nabla w^{(1)}| + \{[z^{(1)}]^\theta + [z^{(1)}]^{\theta-1}\}|\nabla z^{(1)}|\big] \\
&-\int_{t_1}^{t_2}\int_\Omega \mu_1\theta\nabla\varphi\nabla\left[\frac{(w^{(1)})^{\theta+1}}{\theta+1} + \frac{(w^{(1)})^\theta}{\theta} - \frac{(z^{(1)})^{\theta+1}}{\theta+1} - \frac{(z^{(1)})^\theta}{\theta}\right] \\
&\le \int_{t_1}^{t_2}\int_\Omega \frac{C}{E}[(w^{(1)})^\theta + (z^{(1)})^\theta]. \qquad (8)
\end{aligned}$$

Let $I_1,\ I_2,\ I_3,\ I_4$ denote the four integrals on the left hand side of (8). $I_3,\ I_4$ can be estimated by elementary means as follows. The first part of I_3 can be estimated by

$$\begin{aligned}
&\theta a_n\int_{t_1}^{t_2}\int_\Omega \{[w^{(1)}]^\theta + [w^{(1)}]^{\theta-1}\}|\nabla w^{(1)}| \\
&\quad\le \frac{2\theta a_n}{\theta+1}\int_{t_1}^{t_2}\int_\Omega \{[w^{(1)}]^{\frac{\theta+1}{2}} + [w^{(1)}]^{\frac{\theta-1}{2}}\}\big|\nabla[(w^{(1)})^{\frac{\theta+1}{2}}]\big| \\
&\quad\le \frac{2\theta a_n}{\theta+1}\left[\frac{1}{2\varepsilon}\int_{t_1}^{t_2}\int_\Omega \{[w^{(1)}])^{\theta+1} + [w^{(1)}]^{\theta-1}\} + \frac{\varepsilon}{2}\int_{t_1}^{t_2}\int_\Omega \big|\nabla[(w^{(1)})^{\frac{\theta+1}{2}}]\big|^2\right].
\end{aligned}$$

If we choose ε small enough (depending on $a_n,\ \alpha,\theta$) then the second integral on the right hand side has coefficient smaller than the corresponding term in I_2. Observe that if $\frac{\alpha}{a_n(\theta+1)}$ is big enough, then we can also employ to advantage the estimate

$$\int_\Omega (w^{(1)})^{\theta+1} \le \frac{1}{\rho_1}\int_\Omega \big|\nabla[(w^{(1)})^{\frac{\theta+1}{2}}]\big|^2$$

where ρ_1 denotes the least eigenvalue of $-\Delta$ with mixed boundary conditions. While this comment is irrelevant here, it is useful both for the existence of steady state solutions and of an absorbing set. The second part of I_3 is treated identically, with z replacing w. Next:

$$-I_4 = \int_{t_1}^{t_2} \int_\Omega \mu_1 \theta \Big\{ -\frac{(w^{(1)})^{\theta+1}}{\theta+1} - \frac{(w^{(1)})^{\theta}}{\theta} + \frac{(z^{(1)})^{\theta+1}}{\theta+1} + \frac{(z^{(1)})^{\theta}}{\theta} \Big\} f\big(x, t, E(z-w), h\big)$$

Without loss of generality, at any given point (x, t) we may first assume $z^{(1)} > w^{(1)}$ with $z^{(1)} > 0$ and also note that $(z-w)^{\alpha_2} \geq (z^{(1)} - w^{(1)})^{\alpha_2}$ and recall $\alpha_2 \geq 1$. We then have from (5)

$$\begin{aligned}
&\frac{1}{\theta+1}[(z^{(1)})^{\theta+1} - (w^{(1)})^{\theta+1}][M_1 E^{\alpha_2}(z^{(1)} - w^{(1)})^{\alpha_2} - M_2] \\
&\quad \geq \frac{1}{\theta+1}[(z^{(1)})^{\theta+1} - (w^{(1)})^{\theta+1}][M_1 E^{\alpha_2}(z^{(1)} - w^{(1)}) - (M_2 + M_1 E^{\alpha_2})] \\
&\quad \geq -\frac{1}{\theta+1} \frac{[(z^{(1)})^{\theta+1} - (w^{(1)})^{\theta+1}]}{(z^{(1)} - w^{(1)})} \frac{(M_1 E^{\alpha_2} + M_2)^2}{4 M_1 E^{\alpha_2}} \\
&\quad \geq -\frac{(M_1 E^{\alpha_2} + M_2)^2}{4 M_1 E^{\alpha_2}} [(z^{(1)})^{\theta} + (w^{(1)})^{\theta}].
\end{aligned}$$

An identical estimate, with θ replaced by $\theta - 1$, holds for the other two terms in the integrand of I_4 and for the points where $w^{(1)} > z^{(1)}$. Thus:

$$-I_4 \geq -C \int_{t_1}^{t_2} \int_\Omega \{(z^{(1)})^{\theta} + (w^{(1)})^{\theta} + (z^{(1)})^{\theta-1} + (w^{(1)})^{\theta-1}\}$$

with a calculable constant C. In summary, setting $s = (w^{(1)})^{\frac{\theta+1}{2}}$ and $r = (z^{(1)})^{\frac{\theta+1}{2}}$, we obtain from equation (8):

$$\int_\Omega (s^2 + r^2)\Big|_{t_1}^{t_2} + c_0 \int_{t_1}^{t_2} \int_\Omega [|\nabla s|^2 + |\nabla r|^2] \leq c_1 \int_{t_1}^{t_2} \int_\Omega (s^2 + r^2) + c_2$$

with calculable positive constants $c_0,\ c_1,\ c_2$. We thus have that $(z^{(1)})^{(\theta+1)/2}$ and $(w^{(1)})^{(\theta+1)/2}$ are bounded in $C(L^2) \cap L^2(H^{1,2})$ and thus, see e.g. [6], $w^{\theta+1},\ z^{\theta+1}$ are bounded in $L^{10/3}(L^{10/3})$, i.e., n, p are bounded in $L^\xi(L^\xi)$ for any large chosen ξ. In particular, f is bounded in $L^\xi(L^\xi)$ and thus $|\nabla\varphi|$ is bounded in $L^\xi(L^\tau)$ for ξ large, where we recall $3 < \tau < 4$. We now employ [6] and Lemma 0 to conclude that n, p (and thus φ) are bounded in $C^{\alpha_1, \alpha_1/2}$ with α_1 and bound independent of n, p.

It is useful to embed $(1-3)$ and the associated boundary/initial conditions in the following system:

$$-\Delta\varphi = \lambda f(x, t, p - n, h(p^+, n^+)) \tag{9}$$

$$\frac{\partial n}{\partial t} - \nabla[D_n \nabla n - n\mu_n \nabla\varphi] = \lambda\{1 - h(p^+, n^+) - n^+[1 + h(p^+, n^+)] + \widetilde{R}_n\} \tag{10}$$

$$\frac{\partial p}{\partial t} - \nabla[D_p \nabla p + p\mu_p \nabla\varphi] = \lambda\{1 + h(p^+, n^+) - p^+[1 - h(p^+, n^+)] + \widetilde{R}_p\} \tag{11}$$

with boundary/initial Dirichlet conditions

$$\varphi = \lambda \overline{\varphi}, \quad n = \lambda \overline{n}, \quad p = \lambda \overline{p} \quad \text{on} \quad \Gamma_D; \quad n = \lambda \overline{n}, \quad p = \lambda \overline{p} \quad \text{at} \quad t = 0 \tag{12}$$

and $\widehat{R}_n = R_n(x,t,n^+,p^+)$, $\widehat{R}_p = R_p(x,t,n^+,p^+)$. Observe that for $\lambda = 1$ and $n,p \geq 0$, this reduces to the original problem, and the solutions n, p must be nonnegative by the weak maximum principle and equation (9).

Theorem 2. *There exists a $C^{\alpha,\alpha/2}$ solution (n,p,φ) of system (1,3) with the associated boundary/initial conditions for some $\alpha > 0$ independent of (n,p,φ), with (n,p) nonnegative. If D_n, D_p, μ_n, μ_p are only functions of (x,t), the solution is unique.*

Proof. We transform (9)–(12) into an operator equation in the usual way. Let (λ_0, n_0, p_0) be given with $(n_0,p_0) \in C^{\alpha,\alpha/2}$ with $\alpha > 0$ chosen, evaluate f at this point and calculate φ_0 from (9). Evaluate the coefficients D_n, D_p, μ_n, μ_p, h, boundary/initial conditions and the right hand sides of (10), (11) at $(\lambda_0, n_0, p_0, \varphi_0)$ and solve the now linear equations to obtain the new (n,p). We may express this process in the form:

$$\begin{aligned} \lambda &= \lambda_0 \\ \varphi &= \lambda T_0(n_0,p_0) \\ (n,p) &= T_1(n_0,p_0,T_0(n_0,p_0),\lambda). \end{aligned}$$

Observe that $T_1 : C^{\alpha,\alpha/2} \times [0,1] \to C^{\alpha_1,\alpha_1/2}$ whence if we choose some $\alpha < \alpha_1$ we have compactness, since $C^{\alpha_1,\alpha_1/2} \subset\subset C^{\alpha,\alpha/2}$ and the earlier estimates of Theorem 1 still hold (indeed the presence of λ helps as $0 \leq \lambda \leq 1$). Finally, that T_1 is continuous can be seen from lengthy but routine arguments. Note in particular that the compactness of T_1 implies that continuity need only be shown $C^{\alpha,\alpha/2} \times [0,1] \to L^2$ and that the coefficients D_n, μ_n, D_p, μ_p are assumed bounded and thus the Lebesgue Convergence Theorem can be applied, much as for example, was done in [2]. Once again the framework of $C^{\alpha,\alpha/2}$ spaces makes this process easier. The existence of a solution is then immediate by the Leray-Schauder Degree using λ as a homotopy parameter, [17]. The uniqueness of (n,p,φ) under the extra assumption on D_n, D_p, μ_n, μ_p, is immediate from Gronwall's Lemma (see once again, e.g., [2]), since if we let (n,p,φ) and $(\widehat{n},\widehat{p},\widehat{\varphi})$ denote two solutions we then observe the estimate:

$$\begin{aligned} |h(n,p) - h(\widehat{n},\widehat{p})| &= \bigg| \int_0^t ((p-\widehat{p}) - (n-\widehat{n})) e^{-\int_\xi^t (p+n+2) d\xi} \\ &\quad + \int_0^t (\widehat{p} - \widehat{n}) \Big[e^{-\int_\xi^t (p+n+2)} - e^{-\int_\xi^t (\widehat{p}+\widehat{n}+2)} \Big] \bigg| \\ &\leq [C_0 + C_1(n,p,\widehat{n},\widehat{p})t] \int_0^t |p-\widehat{p}| + |n-\widehat{n}| \end{aligned}$$

for some constant C_0. Choose T and let $0 \leq t \leq t_1 < T$. In view of the assumed regularity of the coefficient functions and employing the equations solved by φ, $\widehat{\varphi}$, we have

$$\frac{1}{2}\int_\Omega \{(n-\widehat{n})^2+(p-\widehat{p})^2\}\Big|_{t_1} - C_2\int_0^{t_1}\int_\Omega \{(n-\widehat{n})^2+(p-\widehat{p})^2\}$$
$$\leq \int_\Omega C_3\left[\int_0^{t_1}(|n-\widehat{n}|+|p-\widehat{p}|)\left(\int_0^t |n-\widehat{n}|+|p-\widehat{p}|\right)dt\right]$$
$$\leq C_4\int_0^{t_1}\int_\Omega \{|n-\widehat{n}|^2+|p-\widehat{p}|^2\}$$

with the constants C_i depending on $n,\ p,\ \widehat{n},\ \widehat{p},\ T$. We then have $n\equiv\widehat{n}$ and $p\equiv\widehat{p}$ for $t<T$ and thus for all t.

III. Global Results

In the earlier section, we cannot exclude the possibility that $n,p\to\infty$ as $t\to\infty$. However, it is easy to give conditions which ensure global boundedness and the existence of steady state solutions. Indeed, we need only show the local boundedness of (n,p) in L^ξ (L^ξ) for large ξ. After that, classical results (see again [6]) will ensure the conclusion. Observe in this regard that if a_n+b_n are sufficiently small then from simple modifications of the proof of Theorem 1, as mentioned above, it follows that we can estimate all of the "negative" integrals in (8) in terms of the positive ones and by repeating obtain the estimate

$$\int_\Omega [n^\theta+p^\theta]\Big|_t \leq C_0$$

for some $C_0>0$. Obviously a similar estimate holds for $\|n\|_{L^\infty}+\|p\|_{L^\infty}$. Furthermore a similar proof shows that in this case there exists at least one steady state solution $\widehat{n}$, $\widehat{p}$, $\widehat{\varphi}$ in $C^{\alpha_1,\alpha_1/2}(\Omega)$, with $h=\frac{p-n}{n+p+2}$,

Absorbing set considerations can also be based in this case directly on the proof of Theorem 1. Indeed, choose $E=\sup\left\{\|\overline{n}+\overline{p}\|_{L^\infty(\partial\Omega_D)},\ 1\right\}$. We then repeat and find that there exist a $K,\ t_0$ such that for $t\geq t_0$ we have $\|n+p\|_{L^\infty}\leq K$, where K depends only on $\|\overline{n}+\overline{p}\|_{L^\infty(\partial\Omega_D)}$ and t_0 on $\|\overline{n}+\overline{p}\|_{L^\infty(\Omega)}$. Some idea of the precise nature of the bounds K and t_0 can be obtained by following the various proofs in [6], [13] and Theorem 1. In general, however, precise estimates seem extremely difficult to obtain due to the difficulty in estimating the various constants.

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