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FIXED POINT RESULTS FOR COMPLETE DISLOCATED G_d-METRIC SPACE VIA C-CLASS FUNCTIONS

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Abstract. In this paper, we discuss unique fixed point results for mappings satisfying contractive condition via C-class functions for a complete dislocated G_d-metric space. Example is also given which shows the novelty of our work. Our results improve/generalize several well known recent and classical results.

1. **Introduction and Basic Concepts**

In the field of analysis the notion of metric spaces plays an important role in pure and applied science such as biology, physics and computer science. The notion of a G-metric space was introduced by Mustafa et al. [29].

A point $x \in X$ is said to be a fixed point of mapping $T : X \to X$, if $x = Tx$. Many results appeared related to fixed point for mappings satisfying certain contractive conditions in complete G-metric spaces and dislocated metric spaces(see [1]-[43]). Recently, dislocated quasi G-metric space was introduced by Shoaib et al. [37, 39], which is a generalization of both G-metric spaces and dislocated metric spaces. A class of new C-class functions was recently introduced by Ansari et al. [6].

In this paper, we have obtained fixed point results for contractive self mappings in a complete dislocated G_d-metric space via C-class functions which extend and improve the recent fixed point results proved by Karapınar et al. [23]. An example is also given to support our results.

Definition 1.1 Let X be a nonempty set, and let $G_d : X \times X \times X \to [0, \infty)$, be a function satisfying the following properties:

(G_1) If $G_d(a, b, c) = 0$, then $a = b = c$;

(G_2) $G_d(a, a, b) \leq G_d(a, b, c)$, for all $a,\ b,\ c \in X$ with $b \neq c$;

(G_3) $G_d(a, b, c) = G_d(a, c, b) = G_d(b, a, c) = G_d(b, c, a) = G_d(c, a, b) = G_d(c, b, a)$ for all $a, b, c \in X$;

(G_4) $G_d(a, b, c) \leq G_d(a, d, d) + G_d(d, b, c)$, for all $a, b, c, d \in X$, (rectangle inequality).

Then the function G_d is called a dislocated G_d-metric on X and the pair (X, G_d) is called dislocated G_d-metric space.

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Example 1.2 Let $X = [0, \infty)$ be a nonempty set and $G_d : X \times X \times X \to [0, \infty)$ be a function defined by

$$G_d(a, b, c) = \max\{a, b, c\}, \text{ for all } a, b, c \in X.$$

Then clearly $G_d : X \times X \times X \to [0, \infty)$ is dislocated G_d-metric space.
Definition 1.3 Let (X, G_d) be a dislocated G_d-metric space, and let $\{x_n\}$ be a sequence of points in X, a point x in X is said to be the limit of the sequence $\{x_n\}$ if $\lim_{m,n\to\infty} G_d(x, x_n, x_m) = 0$, and one says that sequence $\{x_n\}$ is G_d-convergent to x. Thus, if $x_n \to x$ in a dislocated G_d-metric space (X, G_d), then for any $\epsilon > 0$, there exist $n, m \in N$ such that $G_d(x, x_n, x_m) < \epsilon$, for all $n, m \geq N$.
Definition 1.4 Let (X, G_d) be a dislocated G_d-metric space. A sequence $\{x_n\}$ is called G_d-Cauchy sequence if, for $\epsilon > 0$ there exists a positive integer $n^\star \in N$ such that $G_d(x_n, x_m, x_l) < \epsilon$ for all $n, l, m \geq n^\star$; or $G_d(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.
Definition 1.5 A dislocated G_d-metric space (X, G_d) is said to be G_d-complete if every G_d-Cauchy sequence in (X, G_d) is G_d-convergent in X.
Proposition 1.6 Let (X, G_d) be a dislocated G_d-metric space, then the following are equivalent:
(i) $\{x_n\}$ is G_d convergent to x.
(ii) $G_d(x_n, x_n, x) \to 0$ as $n \to \infty$.
(iii) $G_d(x_n, x, x) \to 0$ as $n \to \infty$.
(iv) $G_d(x_n, x_m, x) \to 0$ as $m\ n \to \infty$.
Lemma 1.7 Let (X, G_d) be a dislocated G_d-metric space and $\{x_n\}$ be a sequence in X such that $\{G_d(x_n, x_n, x_{n+1})\}$ is decreasing and

$$\lim_{n\to\infty} G_d(x_n, x_n, x_{n+1}) = 0.$$

If $\{x_{2n}\}$ is not a G_d-Cauchy sequence, then there exist an $\epsilon > 0$ and $\{m_k\}$ and $\{n_k\}$ of positive integers such that the following sequences $\{G_d(x_{m_k}, x_{n_k}, x_{n_k})\}$, $\{G_d(x_{m_k}, x_{n_k+1}, x_{n_k+1})\}$, $\{G_d(x_{m_k-1}, x_{n_k}, x_{n_k})\}$, $\{G_d(x_{m_k-1}, x_{n_k+1}, x_{n_k+1})\}$ and $\{G_d(x_{m_k}, x_{n_k+1}, x_{n_k+1})\}$ tend to $\epsilon > 0$, when $k \to \infty$.
Definition 1.8 [6] A mapping $F : [0, \infty)^2 \to \mathbb{R}$ is called a C-class function if it is continuous and satisfies the following axioms:
(i) $F(s, t) \leq s$ for all $s, t \in [0, \infty)$;
(ii) $F(s, t) = s$ implies that either $s = 0$ or $t = 0$.
Mention that some C-class function F verifies $F(0, 0) = 0$. We denote by $\mathcal{C}$ the set of C-class functions.
Example 1.9 [6] Following examples show that the class $\mathcal{C}$ is nonempty:
(i) $F(s, t) = s - t$.
(ii) $F(s, t) = ms$,for some $m \in (0, 1)$.
(iii) $F(s, t) = \frac{s}{1+t}$.
[6] Let Φ_u denote the class of all functions $\varphi : [0, \infty) \to [0, \infty)$ which satisfy the following conditions:
(i) φ is continuous ;
(ii) $\varphi(t) > 0, t > 0$ and $\varphi(0) \geq 0$.

2. Main Result

Theorem 2.1: Let (X, G_d) be a complete dislocated G_d-metric space, let $T : X \longrightarrow X$ be a mapping satisfying

$$G_d(Ta, Tb, Tc) \leq F(W(a, b, c), \varphi(W(a, b, c))) \tag{2.1}$$

for all $a, b, c \in X$, where $\varphi \in \Phi_u$, and F is a C class function.
Here,

$$\begin{aligned} W(a,b,c) \quad = \quad & \frac{1}{2}\max\{G_d(b, T^2a, Tb), G_d(Ta, T^2a, Tb), G_d(a, Ta, b), G_d(a, Ta, c), \\ & G_d(c, T^2a, Tc), G_d(b, Ta, Tb), G_d(Ta, T^2a, Tc), G_d(c, Ta, Tb), \\ & G_d(a, b, c), G_d(a, Ta, Ta), G_d(b, Tb, Tb), G_d(c, Tc, Tc), \\ & G_d(a, Tb, Tb), G_d(b, Tc, Tc), G_d(c, Ta, Ta)\}. \end{aligned} \tag{2.2}$$

Then, there exists a unique fixed point $a \in X$ such that $Ta = a$.
Proof: Consider a Picard sequence $\{a_n\}$ with initial guess $a_0 \in X$ such that

$$a_{n+1} = Ta_n, \text{ for all } n \in N.$$

Suppose $a_{n+1} \neq a_n$, for all $n \in N \cup \{0\}$. Now, consider the relation

$$\begin{aligned} G_d(a_n, a_{n+1}, a_{n+1}) \quad & = \quad G_d(Ta_{n-1}, Ta_n, Ta_n) \\ & \leq \quad F(W(a_{n-1}, a_n, a_n), \varphi(W(a_{n-1}, a_n, a_n))). \end{aligned} \tag{2.3}$$

From (2.2),

$$\begin{aligned} W(a_{n-1}, a_n, a_n) \quad = \quad & \frac{1}{2}\max\{G_d(a_{n-1}, a_n, a_n), G_d(a_n, a_{n+1}, a_{n+1}), G_d(a_n, a_n, a_{n+1}), \\ & G_d(a_{n-1}, a_{n+1}, a_{n+1}), G_d(a_n, a_n, a_n)\}. \end{aligned}$$

By Definition 1.1, we have

$$G_d(a_n, a_n, a_n) \leq G_d(a_n, a_{n+1}, a_{n+1}).$$

So,

$$\begin{aligned} W(a_{n-1}, a_n, a_n) \quad \leq \quad & \frac{1}{2}\max\{G_d(a_{n-1}, a_n, a_n), G_d(a_n, a_{n+1}, a_{n+1}), \\ & G_d(a_n, a_n, a_{n+1}), G_d(a_{n-1}, a_{n+1}, a_{n+1})\}. \end{aligned}$$

In first case, if

$$W(a_{n-1}, a_n, a_n) = \frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}),$$

then, by (2.3)

$$\begin{aligned} \frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}) \quad & \leq \quad G_d(a_n, a_{n+1}, a_{n+1}) \\ & \leq \quad F(\frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}), \varphi(\frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}))) \\ & \leq \quad \frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}). \end{aligned}$$

Then

$$F(\frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}), \varphi(\frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}))) = \frac{1}{2}G_d(a_n, a_{n+1}, a_{n+1}).$$

By the property of F, we get

$$\frac{1}{2}G_d(a_n,a_{n+1},a_{n+1})=0 \text{ or } \varphi(\frac{1}{2}G_d(a_n,a_{n+1},a_{n+1}))=0.$$

Then,

$$G_d(a_n,a_{n+1},a_{n+1})=0.$$

It is a contradiction because $a_{n+1}\neq a_n$. Now, in second case, if

$$W(a_{n-1},a_n,a_n)=\frac{1}{2}G_d(a_n,a_n,a_{n+1}),$$

then, we have

$$\begin{aligned}\frac{1}{2}G_d(a_n,a_n,a_{n+1}) &\leq G_d(a_n,a_n,a_{n+1})\leq G_d(a_n,a_{n+1},a_{n+1})\\ &\leq F(\frac{1}{2}G_d(a_n,a_n,a_{n+1}),\varphi(\frac{1}{2}G_d(a_n,a_n,a_{n+1})))\\ &\leq \frac{1}{2}G_d(a_n,a_n,a_{n+1}),\end{aligned}$$

which implies

$$F(\frac{1}{2}G_d(a_n,a_n,a_{n+1}),\varphi(\frac{1}{2}G_d(a_n,a_n,a_{n+1})))=\frac{1}{2}G_d(a_n,a_n,a_{n+1}).$$

By the property of F, we get

$$\frac{1}{2}G_d(a_n,a_n,a_{n+1})=0 \text{ or } \varphi(\frac{1}{2}G_d(a_n,a_n,a_{n+1}))=0.$$

Then,

$$\frac{1}{2}G_d(a_n,a_n,a_{n+1})=0.$$

It is a contradiction because $a_{n+1}\neq a_n$. In third case, if

$$W(a_{n-1},a_n,a_n)=\frac{1}{2}G_d(a_{n-1},a_n,a_n),$$

then, we have

$$\begin{aligned}G_d(a_n,a_{n+1},a_{n+1}) &\leq F(\frac{1}{2}G_d(a_{n-1},a_n,a_n),\varphi\frac{1}{2}G_d(a_{n-1},a_n,a_n)))\\ &\leq \frac{1}{2}G_d(a_{n-1},a_n,a_n)\\ &\leq G_d(a_{n-1},a_n,a_n). \qquad (2.4)\end{aligned}$$

In fourth case, if

$$W(a_{n-1},a_n,a_n)=G_d(a_{n-1},a_{n+1},a_{n+1}),$$

then,

$$\begin{aligned}G_d(a_n,a_{n+1},a_{n+1}) &\leq F(\frac{1}{2}G_d(a_{n-1},a_{n+1},a_{n+1}),\varphi(\frac{1}{2}G_d(a_{n-1},a_{n+1},a_{n+1})))\\ &\leq \frac{1}{2}G_d(a_{n-1},a_{n+1},a_{n+1})\\ &\leq \frac{G_d(a_{n-1},a_n,a_n)+G_d(a_n,a_{n+1},a_{n+1})}{2}\\ G_d(a_n,a_{n+1},a_{n+1}) &\leq G_d(a_{n-1},a_n,a_n). \qquad (2.5)\end{aligned}$$

Hence, by combining (2.4) and (2.5), we have

$$G_d(a_n, a_{n+1}, a_{n+1}) \leq G_d(a_{n-1}, a_n, a_n) \to d.$$

Now, by inequality (2.3) with $n \to \infty$, we have

$$d \leq F(d, \varphi(d)),$$

then,

$$d = 0 \quad \text{or} \quad \varphi(d) = 0.$$

So, we have

$$\lim_{n\to\infty} G_d(a_n, a_{n+1}, a_{n+1}) = 0.$$

We shall show that $\{a_n\}$ is a G_d-Cauchy sequence. Suppose that $\{a_{2n}\}$ is not a G_d-Cauchy sequence and from Lemma 1.7, there exists $\epsilon > 0$ such that

$$G_d(a_{m_k+1}, a_{n_k+1}, a_{n_k+1})) \leq F(W(a_{m_k}, a_{n_k}, a_{n_k}), \varphi(W(a_{m_k}, a_{n_k}, a_{n_k}))). \quad (2.6)$$

Now, by using (2.6) as $k \to \infty$, then

$$\epsilon \leq F(\epsilon, \varphi(\epsilon)) \leq \epsilon.$$

By the property of F, we get

$$\varepsilon = 0 \ \text{ or } \ \varphi(\varepsilon) = 0.$$

Then, $\varepsilon = 0$, which is a contradiction. This proves that $\{a_{2n}\}$ is a G_d-Cauchy sequence and hence $\{a_n\}$ is a G_d-Cauchy sequence. So, we have

$$G_d(a_n, a_m, a_m) \to 0, \text{ as } n \to \infty.$$

Therefore, Picard sequence $\{a_n\}$ is Cauchy sequence in X. Hence, $a_n \to a$ as $n \to \infty$. In general it is clear that,

$$\lim_{n\to\infty} G_d(a_n, a, a) = \lim_{n\to\infty} G_d(a, a_n, a_n) = 0. \quad (2.7)$$

To check either $a \in X$ is a fixed point of T or not, we consider

$$\begin{aligned} G_d(a, Ta, Ta) &\leq G_d(a, a_{n+1}, a_{n+1}) + G_d(a_{n+1}, Ta, Ta) \\ &\leq G_d(a, a_{n+1}, a_{n+1}) + F(W(a_n, a, a), \varphi(W(a_n, a, a))). \quad (2.6) \end{aligned}$$

From (2.2),

$$\begin{aligned} W(a_n, a, a) &= \frac{1}{2}\max\{G_d(a, T^2a_n, Ta), G_d(Ta_n, T^2a_n, Ta), G_d(a_n, Ta_n, a), \\ &\quad G_d(a_n, Ta_n, a), G_d(a, T^2a_n, Ta), G_d(a, Ta_n, Ta), \\ &\quad G_d(Ta_n, T^2a_n, Ta), G_d(a, Ta_n, Ta), G_d(a_n, a, a), \\ &\quad G_d(a_n, Ta_n, Ta_n), G_d(a, Ta, Ta), G_d(a, Ta, Ta), \\ &\quad G_d(a_n, Ta, Ta), G_d(a, Ta, Ta), G_d(a, Ta_n, Ta_n)\} \end{aligned}$$

$$\begin{aligned} W(a_n, a, a) &= \frac{1}{2}\max\{G_d(a, a_{n+2}, Ta), G_d(a_{n+1}, a_{n+2}, Ta), G_d(a_n, a_{n+1}, a), \\ &\quad G_d(a_n, a_{n+1}, a), G_d(a, a_{n+2}, Ta), G_d(a, a_{n+1}, Ta), \\ &\quad G_d(a_{n+1}, a_{n+2}, Ta), G_d(a, a_{n+1}, Ta), G_d(a_n, a, a), \\ &\quad G_d(a_n, a_{n+1}, a_{n+1}), G_d(a, Ta, Ta), G_d(a, Ta, Ta), \\ &\quad G_d(a_n, Ta, Ta), G_d(a, Ta, Ta), G_d(a, a_{n+1}, a_{n+1})\} \end{aligned}$$

$$\begin{aligned} W(a_n,a,a) &= \frac{1}{2}\max\{G_d(a,a_{n+2},Ta),G_d(a_{n+1},a_{n+2},Ta),G_d(a_n,a_{n+1},a), \\ & G_d(a,a_{n+1},Ta),G_d(a_n,a,a),G_d(a_n,a_{n+1},a_{n+1}), \\ & G_d(a,Ta,Ta),G_d(a_n,Ta,Ta),G_d(a,a_{n+1},a_{n+1})\}. \end{aligned} \tag{2.7}$$

After applying limit $n \to \infty$, by (2.8), for every selection of $W(a_n,a,a)$ from (2.9) and by using the fact that G_d is symmetry, we get

$$G_d(a,Ta,Ta) \le F(G_d(a,Ta,Ta),\varphi(G_d(a,Ta,Ta))).$$

By the property of F, we get

$$G_d(a,Ta,Ta)=0 \text{ or } \varphi(G_d(a,Ta,Ta))=0.$$

That is

$$G_d(a,Ta,Ta)=0.$$

Hence, $Ta=a$ where $a \in X$ is a fixed point for T. For uniqueness of fixed point, consider $a,b \in X$ be two distinct fixed points. So consider the relation,

$$\begin{aligned} G_d(a,b,b) &= G_d(Ta,Tb,Tb) \\ G_d(a,b,b) &\le F(W(a,b,b),\varphi(W(a,b,b))). \end{aligned} \tag{2.8}$$

From (2.2),

$$\begin{aligned} W(a,b,b) &= \frac{1}{2}\max\{G_d(a,b,b),G_d(b,a,b,),G_d(a,a,b), \\ & G_d(b,a,a),G_d(a,a,a),G_d(b,b,b)\}. \end{aligned} \tag{2.9}$$

Also,

$$\begin{aligned} G_d(a,a,a) &\le G_d(a,b,b), \\ G_d(b,b,b) &\le G_d(a,b,b), \\ G_d(a,a,b) &\le G_d(a,b,b), \end{aligned}$$

and

$$G_d(b,a,a) \le G_d(a,b,b).$$

Hence, (2.11) gives

$$W(a,b,b)=\frac{1}{2}G_d(a,b,b).$$

$$\begin{aligned} G_d(a,b,b) &\le F(\frac{1}{2}(G_d(a,b,b),\varphi(\frac{1}{2}(G_d(a,b,b))), \\ &\le \frac{1}{2}(G_d(a,b,b), \end{aligned}$$

which implies

$$G_d(a,b,b)=0. \;\; or \;\; \varphi(G_d(a,b,b))=0.$$

That is

$$G_d(a,b,b)=0.$$

It is a contradiction to our assumption, that is $a \neq b$. So our supposition is wrong. Hence, $a \in X$ is a unique fixed point for T.

Example 2.2: Let $X=\{0,1,2,3,4\}$, and $G_d : X \times X \times X \longrightarrow X$, be a mapping defined by,

$$G_d(a,b,c)=\max\{a,b,c\} \quad \text{for all } a,b,c \in X$$

then, (X, G_d) is a complete dislocated G_d-metric space. Let, $T : X \to X$ be defined by,

$$Tx = \begin{cases} 0 & \text{if } x \in \{0,1,2\} \\ 1 & \text{if } x \in \{3,4\} \end{cases},$$

and

$$F(s,t) = s - t \quad \text{for all} \quad s,t \geq 0.$$

Take $\varphi(t) = \frac{t}{5}$ for all $t \geq 0$.
Case I: If $a = 0$, $b = 1$, and $c = 2$, then

$$\begin{aligned} G_d(Ta,Tb,Tc) &= \max\{0,0,0\} \\ &= 0. \end{aligned}$$

Moreover

$$\begin{aligned} W(a,b,c) &= \frac{1}{2}\max\{1,0,1,2,2,1,0,2,2,0,1,2,0,1,2\} \\ &= \frac{2}{2} = 1. \end{aligned}$$

Therefore

$$\begin{aligned} F(W(a,b,c),\varphi(W(a,b,c))) &= F(1,\varphi(1)) \\ &= F(1,\frac{1}{5}) \\ &= 1 - \frac{1}{5} \\ &= \frac{4}{5}. \end{aligned}$$

Thus

$$G_d(Ta,Tb,Tc) = 0 < \frac{4}{5} = F(W(a,b,c),\varphi(W(a,b,c))),$$

that is, (2.1) holds.
Case II: If $a = 0$, $b = 1$, and $c = 3$, then

$$\begin{aligned} G_d(Ta,Tb,Tc) &= \max\{0,0,1\} \\ &= 1. \end{aligned}$$

Moreover

$$\begin{aligned} W(a,b,c) &= \frac{1}{2}\max\{1,0,1,3,3,1,1,3,3,0,1,3,0,1,3\} \\ &= \frac{3}{2}. \end{aligned}$$

Therefore

$$\begin{aligned} F(W(a,b,c),\varphi(W(a,b,c))) &= F(\frac{3}{2},\varphi(\frac{3}{2})) \\ &= F(\frac{3}{2},\frac{3}{10}) \\ &= \frac{3}{2} - \frac{3}{10} \\ &= \frac{6}{5}. \end{aligned}$$

Thus

$$G_d(Ta,Tb,Tc) = 1 < \frac{6}{5} = F(W(a,b,c),\varphi(W(a,b,c))),$$

that is, (2.1) holds.
Case III: If $a = 1$, $b = 1$, and $c = 1$, then

$$\begin{aligned} G_d(Ta,Tb,Tc) &= \max\{0,0,0\} \\ &= 0. \end{aligned}$$

Moreover

$$\begin{aligned} W(a,b,c) &= \frac{1}{2}\max\{1,0,1,1,1,1,0,1,1,1,1,1,1,1,1\} \\ &= \frac{1}{2}. \end{aligned}$$

Therefore

$$\begin{aligned} F(W(a,b,c),\varphi(W(a,b,c))) &= F(\frac{1}{2},\varphi(\frac{1}{2})) \\ &= F(\frac{1}{2},\frac{1}{10}) \\ &= \frac{1}{2}-\frac{1}{10} \\ &= \frac{2}{5}. \end{aligned}$$

Thus

$$G_d(Ta,Tb,Tc) = 0 < \frac{2}{5} = F(W(a,b,c),\varphi(W(a,b,c))),$$

that is, (2.1) holds.

It is clear from above cases, the contractive condition of Theorem 2.1 holds and similarly for other cases. Therefore, $0 \in X$, is a fixed point for T, such that $T0 = 0$.

In Theorem 2.1, $W(a,b,c)$ contains 15 elements. Hence many corollaries can be constructed by taking different subsets of $W(a,b,c)$. Some of them are given below.

Corollary 2.3: Let (X,G_d) be a complete dislocated G_d-metric space, let $T : X \longrightarrow X$ be a mapping satisfying

$$G_d(Ta,Tb,Tc) \leq F(\frac{1}{2}G_d(b,T^2a,Tb),\varphi(\frac{1}{2}G_d(b,T^2a,Tb)))$$

for all $a,b,c \in X$, where $\varphi \in \Phi_u$, and F is a C class function. Then, there exists a unique fixed point $a \in X$ such that $Ta = a$.

Corollary 2.4: Let (X,G_d) be a complete dislocated G_d-metric space, let $T : X \longrightarrow X$ be a mapping satisfying

$$G_d(Ta,Tb,Tc) \leq F(\frac{1}{2}G_d(Ta,T^2a,Tb),\varphi(\frac{1}{2}G_d(Ta,T^2a,Tb)))$$

for all $a,b,c \in X$, where $\varphi \in \Phi_u$, and F is a C class function. Then, there exists a unique fixed point $a \in X$ such that $Ta = a$.

Corollary 2.5: Let (X,G_d) be a complete dislocated G_d-metric space, let $T : X \longrightarrow X$ be a mapping satisfying

$$G_d(Ta,Tb,Tc) \leq F(\frac{1}{2}G_d(a,Ta,b),\varphi(\frac{1}{2}G_d(a,Ta,b)))$$

for all $a, b, c \in X$, where $\varphi \in \Phi_u$, and F is a C class function. Then, there exists a unique fixed point $a \in X$ such that $Ta = a$.

Competing Interest:

The authors declare that they have no competing interest.

Authors Contribution:

All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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