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ON WEAKLY RICCI φ-SYMMETRIC δ-LORENTZIAN TRANS-SASAKIAN MANIFOLDS

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Abstract. The purpose of the paper is to introduce the notion of weakly Ricci φ- symmetric δ- Lorentzian trans-Sasakian manifolds and study characteristic properties of locally φ- Ricci symmetric and φ-recurrent spaces. Finally, local symmetry of a generalized recurrent weakly symmetric δ- Lorentzian trans-Sasakian manifolds is discussed.

1. Introduction

Many authors recently have studied Lorentzian α - Sasakian manifolds [19] and Lorentzian β- Kenmotsu manifolds [14],[13]. In 2011, S.S.Pujar and V.J. Khairnar [7] have initiated the study of Lorentzian Trans-Sasakian manifolds and studied the basic results with some of its properties. Earlier to this , S. S. Pujar [9] has initiated the study of δ -Lorentzian α- Sasakian manifolds [5] and δ Lorentzian β-Kenmotsu manifolds [7]. Also , S. S. Pujar and V.J.Khairnar [8] have continued the work on Lorentzian manifolds and in fact studied the properties of weak symmetries of Lorentzian manifolds. More about the weakly symmetric spaces can be seen in a paper by Ralph R,Gomez [10].

In 2010, S.S. Shukla and D.D.Singh [12] have introduced the notion of (ϵ)- trans-Sasakian manifolds and studied its basic results and using these results studied some of its properties. Earlier to this in 1969 Takahashi [16] had introduced the notion of almost contact metric manifold equipped with pseudo Riemannian metric. In particular , he studied the Sasakian manifolds equipped with Riemannian metric g.These indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known as (ϵ)-almost contact metric manifolds and (ϵ)- Sasakian manifolds respectively.

Recently [15] and [17], we have observed that there does not exists a light like surface in the (ϵ)- Sasakian manifolds . On the other hand in almost para contact

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manifold defined by Motsumoto [5], the semi Riemannian manifold has the index 1 and the structure vector field ξ is always a time like.This motivated the Thripathi and others [17] to introduce (ϵ)-almost para contact structure where the vector field ξ is space like or time like according as $\epsilon = 1$ or $\epsilon = -1$.

In this paper, in section 2, we have listed the various preliminary results (for instance please see [7]) of δ- Lorentzian trans- Sasakian manifolds which are needed in the rest of sections .

In section 3, characterization of locally φ- Ricci -symmetric and φ- recurrent spaces are discussed.It is established that if the weakly Ricci φ-symmetric δ-Lorentzian trans-Sasakian manifold of non zero ξ-sectional curvature is locally φ- Ricci -symmetric , then the sum of the associated 1-forms A,B and C is zero everywhere. Also it is proved in section4 that if the weakly Ricci φ-symmetric δ-Lorentzian trans-Sasakian manifold of non zero ξ-sectional curvature is φ- Ricci -recurrent , then the associated 1-forms B and C are in the opposite directions".

Finally,in section 5, local symmetry of a generalized recurrent weakly symmetric δ- Lorentzian trans-Sasakian manifolds is discussed.

2. Preliminaries

In this section , we list the basic definitions and known results of δ- Lorentzian trans-Sasakian manifolds.

Definition 2.1. [7]. A (2n+1) dimensional manifold M, is said to be the δ- almost contact metric manifold if it admits a 1-1 tensor field φ, a structure tensor field ξ , a 1-form η and an indefinite metric g such that

$$\varphi^2 X = X + \eta(X)\xi, \eta(\xi) = -1$$
$$g(\xi, \xi) = -\delta, \eta(X) = \delta g(X, \xi)$$
$$g(\varphi(X), \varphi(Y)) = g(X, Y) + \delta\eta(X)\eta(Y),$$

for all vector fields X and Y on M , where δ is such that $\delta^2 = 1$ so that $\delta = \pm 1$. The above structure $(\varphi, \xi, \eta, g, \delta))$ on M is called the the δ Lorentzian structure on M. If $\delta = 1$ and this is the usual Lorentzian structure [7] on M , the vector field ξ is the time like [19], that is M contains a time like vector field.

From the above equations , one can deduce that

$$\varphi\xi = 0, \eta(\varphi(X)) = 0$$

Example: Let us consider the 3-dimensional manifold $M = \{(x, y, z)\epsilon R^3\}$, where x,y,z are the co-ordinates of a point in R^3.Let $\{e_1, e_2, e_3\}$ be the global frames on M given by

$$e_1 = e^z(\frac{\partial}{\partial x} + y\frac{\partial}{\partial z}), e_2 = e^z\frac{\partial}{\partial y}, e_3 = e^z\frac{\partial}{\partial z}$$

Let g be the δ- Lorentzian metric on M defined by

$$g(e_1, e_2) = g(e_2, e_3) = g(e_1, e_3) = 0$$

and

$$g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -\delta$$

where $\delta = \pm 1$. Then δ-Lorentzian indefinite metric g on M is in the following form:

$$g = \{e^{-2z} - \delta y^2\}(dx)^2 + e^{-2z}(dy)^2 - \delta e^{-2z}(dz)^2 + 2\delta y e^{-z} dxdy$$

Let $e_3 = \xi$. Let η be the 1-form defined by

$$\eta(U) = \delta g(U, e_3),$$

for any vector field U on M. Let φ be the 1-1 tensor field defined by

$$\varphi(e_1) = e_2, \varphi(e_2) = e_1, \varphi(e_3) = 0$$

Then using linearity of φ and g and taking $e_3 = \xi$, one obtains

$$\eta(e_3) = -1, \varphi^2 U = U + \eta(U)e_3$$

and

$$g(\varphi(U), \varphi(W)) = g(U, W) + \delta\eta(U)\eta(W), \tag{2.1}$$

for any vector fields X and Y on M. Also putting $W = \xi$ in (2.1),one can see that

$$\eta(U) = \delta g(U, \xi). \tag{2.2}$$

Putting $W = U = \xi$ in(2.1) and (2.2) respectively, we have

$$g(\xi, \xi) = -\delta$$

and

$$\eta(\xi) = -1$$

Clearly from(2.1) φ is symmetric. Thus $(\varphi, \xi, \eta, g, \delta)$ defines a δ- Lorentzian contact metric structure on M.

In Tanno.S.[18], classified the connected almost contact metric manifolds whose automorphism groups possesses the maximum dimension. For such a manifold the sectional curvature of the plane section containing ξ is constant, say c. He showed that they can be divided into three classes. (1) homogeneous normal contact Riemannian manifolds with $c > 0$. It is known that the manifolds of class (1) are characterized by admitting a Sasakian structure. Other two classes can be seen in Tanno [21].

In Grey and Harvella [3], the classification of almost Hremitian manifolds , there appears a class W_4 of Hermitian manifolds which are closely related to the conformal Kaehler manifolds .The class $C_6 \oplus C_5$ [4] coincides with the class of the trans-Sasakian structure of type (α, β). In fact , the local nature of the two sub classes, namely C_6 and $C_5$of trans-Sasakian structures are characterized completely . An almost contact metric structure on M is called a trans-Sasakian (please see details in [6] and [3]) if (MxR, J, G) belongs to the class W_4, where J is the almost complex structure on MxR defined by

$$J(X, f\frac{d}{dt}) = (\varphi(X) - f\xi, \eta(X)\frac{d}{dt})$$

for all vector fields X on M and smooth function f on MxR and G is the product metric on MxR. This may be expressed by the condition

$$(\nabla_X \varphi)(Y) = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\varphi(X), Y)\xi - \eta(Y)\varphi(X)\}, \tag{2.3}$$

for any vector fields X and Y on M , ∇ denotes the Levi-Civita connection with respect to g , α and β are smooth functions on M. The existence of condition (2.3) is ensured by the above discussion .

With the above literature now we define the δ -Lorentzian trans-Sasakian manifold as follows.

Definition 2.2. . A δ- Lorentzian manifold with structure $(\varphi, \xi, \eta, g, \delta)$ is said to be δ- Lorentzian trans Sasakian manifold M of type (α, β) if it satisfies the condition .

$$(\nabla_X \varphi)(Y) = \alpha\{g(X,Y)\xi - \delta\eta(Y)X\} + \beta\{g(\varphi(X),Y)\xi - \delta\eta(Y)\varphi(X)\}, \tag{2.4}$$

for any vector fields X and Y on M

If $\delta = 1$, then the δ- Lorentzian trans Sasakian manifold is the usual Lorentzian trans -Sasakian manifold of type (α, β) [7]. δ-Lorentzian trans-Sasakian manifold of type $(0,0), (0,\beta), (\alpha, 0)$ are the Lorentzian cosympletic,Lorentzian β- Kenmotsu and Lorentzian α- Sasakian manifolds respectively. In particular if $\alpha = 1$, $\beta = 0$,and $\alpha = 0$, $\beta = 1$, then δ- Lorentzian trans -Sasakian manifold reduces to δ- Lorentzian Sasakian and δ- Lorentzian Kenmotsu manifolds respectively.

For a δ- Lorentzian trans Sasakian manifold , we have (please see with $\delta = 1$ [7]) and calculations give the following with δ.

$$\nabla_X \xi = \delta\{-\alpha\varphi(X) - \beta(X + \eta(X)\xi)\},$$
$$(\nabla_X \eta)(Y) = \alpha g(\varphi(X),Y) + \beta\{g(X,Y) + \delta\eta(X)\eta(Y)\}$$

$$\begin{aligned}
R(X,Y)\xi &= (\alpha^2+\beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\varphi(X) - \eta(X)\varphi(Y)\} \\
&\quad + \delta\{-X\alpha)\varphi(Y) + (Y\alpha)\varphi(X) - (X\beta)\varphi^2 Y + (Y\beta)\varphi^2 X\},
\end{aligned}$$

$$\begin{aligned}
R(\xi,Y)X &= (\alpha^2+\beta^2)\{\delta g(X,Y)\xi - \eta(X)Y\} \\
&\quad + \delta(X\alpha)\varphi(Y) + \delta g(\varphi(X),Y)(grad\alpha) \\
&\quad + \delta(X\beta)(Y + \eta(Y)\xi) - \delta g(\varphi(Y),\varphi(X))(grad(\beta) \\
&\quad + 2\alpha\beta\{\delta g(\varphi(X),Y)\xi + \eta(X)\varphi(Y)\},
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\eta(R(X,Y)Z) &= \delta(\alpha^2+\beta^2)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)] \\
&\quad + 2\delta\alpha\beta[-\eta(X)g(\varphi(Y),Z) + \eta(Y)g(\varphi(X),Z)] \\
&\quad - [(Y\alpha)(g(\varphi(X),Z) + (X\alpha)g(Y,\varphi(Z)) \\
&\quad - (Y\beta)g(\varphi^2 X, Z) + (X\beta)g(\varphi^2 Y, Z)],
\end{aligned}$$

$$\begin{aligned}
S(X,\xi) &= \{2n(\alpha^2+\beta^2) - \delta(\xi\beta)\}\eta(X) + (2n-1)\delta(X\beta) \\
&\quad + \{2\alpha\beta\eta(X) + \delta(X\alpha)\}f + \delta(\varphi(X))\alpha.
\end{aligned} \tag{2.6}$$

$$S(\xi,\xi) = -2n(\alpha^2 + \beta^2 - \delta\xi\beta). \tag{2.7}$$

$$2\alpha\beta - \delta\xi\alpha = 0 \tag{2.8}$$

for any vector fields X,Y,Z on M

In this section , we define the ξ- sectional curvature of δ- Lorentzian trans-Sasakian manifold .

Definition 2.3. [11] The ξ- sectional curvature of δ- Lorentzian trans- Sasakian manifold for a unit vector field X orthogonal to ξ is defined by

$$K(\xi, X) = R(\xi, X, \xi, X) \tag{2.9}$$

From (2.5), we have

$$R(\xi, X, \xi, X) = \{(\alpha^2 + \beta^2 - \delta(\xi\beta)\}g(\varphi^2 X, X) + (2\alpha\beta - \delta(\xi\alpha))g(\varphi(X), X).$$

Finally by virtue of (2.8),the ξ-sectional curvature is given by

$$K(\xi, X) = \alpha^2 + \beta^2 - \delta(\xi\beta). \tag{2.10}$$

If $\alpha^2 + \beta^2 - \delta(\xi\beta) \neq 0$, then M is of nonvanishing .ξ-sectional curvature.

3. Weakly Ricci φ-symmetric and locally Ricci φ-symmetric spaces

In this section, we introduce the notion of weakly Ricci φ - symmetric δ - Lorentzian trans -Sasakian manifolds and in such a space characterization of locally Ricci φ-symmetric property is discussed.

Definition 3.1. A δ-Lorentzian trans -Sasakian manifold M ($n > 1$) is said to be weakly Ricci φ-symmetric if the non zero Ricci curvature tensor Q of type(1,1)satisfies the condition

$$\varphi^2(\nabla_X Q)(Y) = A(X)Q(Y) + B(Y)Q(X) + g(QX, Y)\rho \tag{3.1}$$

where the vector fields X and Y on M, ρ is a vector field such that $g(\rho, V) = C(V)$, A and B are associated vector fields (not simultaneously zero) and φ is tensor field of type(1,1) on M.

Definition 3.2. A weakly Ricci φ - symmetric δ -Lorentzian trans -Sasakian manifolds (M, g)($n > 1$) is said to be locally Ricci φ-symmetric if

$$\varphi^2(\nabla Q) = 0$$

.

We proceed with these definitions. Equation (3.1) can be written as

$$g(\varphi^2(\nabla_X Q)(Y), V) = A(X)S(Y, V)) + B(Y)S(X, V) + S(X, Y)C(V) \tag{3.2}$$

Suppose weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M ($n > 1$) is locally Ricci φ symmetric. Then from(3.2)and definition, we have

$$A(X)S(Y, V) + B(Y)S(X, V) + C(V)S(X, Y) = 0 \tag{3.3}$$

Setting $X = Y = V = \xi$ in (3.3), we find

$$A(\xi) + B(\xi) + C(\xi) = 0, \tag{3.4}$$

provided $\alpha^2 + \beta^2 - \delta(\xi\beta) \neq 0$.Hence one can state the following theorem.

Theorem 3.3. *If a weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M ($n > 1$) of non vanishing ξ -sectional curvature is locally Ricci φ symmetric ,then the relation (3.4) holds.*

Putting $Y = V = \xi$ in (3.3), we get

$$A(X)S(\xi, \xi) = -\{B(\xi) + C(\xi)\}S(X, \xi)$$

Similarly , we have

$$B(Y)S(\xi, \xi) = -\{A(\xi) + C(\xi)\}S(Y, \xi)$$
$$C(V)S(\xi, \xi) = -\{A(\xi) + B(\xi)\}S(V, \xi)$$

where $S(\xi,\xi) \neq 0$ on M is given by (2.7) and $S(X,\xi)$ is given by (2.6). Adding above equations by taking $X = Y = V$ and using (3.4),we get

$$A(X) + B(X) + C(X) = 0 \tag{3.5}$$

for any vector field X on M so that $A+B+C = 0$ provided $\alpha^2+\beta^2-\delta(\xi\beta) \neq 0$.Hence we state

Theorem 3.4. *If a weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M $(n > 1)$ of non vanishing ξ -sectional curvature is locally Ricci φ-symmetric ,then the sum of the associated 1-forms A ,B and C is zero everywhere.*

Corollary 3.5. *If a weakly Ricci φ-symmetric δ -Lorentzian β-Kenmotsu manifold M $(n > 1)$ with β non zero constant is locally Ricci φ-symmetric ,then the sum of the associated 1-forms A, B and C is zero everywhere..*

Proof. Follows from Theorem 3.4 □

Corollary 3.6. *If a weakly Ricci φ-symmetric δ -Lorentzian α-Sasakian manifold M $(n > 1)$ with α non zero constant is locally Ricci φ-symmetric ,then the sum of the associated 1-forms A, B and C is zero everywhere..*

Proof. Follows from Theorem 3.4 □

4. Recurrent Spaces

Definition 4.1. A weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M $(n > 1)$ is said to be Ricci φ-recurrent if it satisfies condition

$$\varphi^2(\nabla_X Q)(Y) = A(X)Q(Y),$$

where A is the nonzero associated 1-form and X,Y are any vector fields on M. From this equation, we have

$$g(\varphi^2(\nabla_X Q)(Y), V)) = A(X)S(Y,V),$$

where S is the Ricci tensor of type (0,2) is given by

$$S(Y,V) = g(Q(Y),V)$$

If a weakly Ricco φ-symmetric δ -Lorentzian trans-Sasakian manifold M is Ricci φ-recurrent , then (4.1) holds. From (3.2) , we find

$$B(Y)S(X,V) + S(X,Y)C(V) = 0 \tag{4.1}$$

for any vector fields X,Y on M, Next putting ,$X = Y = V = \xi$ in (4.2) and then using (2.3), we obtain

$$B(\xi) + C(\xi) = 0$$

provided $\alpha^2 + \beta^2 - \delta(\xi\beta) \neq 0$. Further proceeding as in the proof of Theorem3.4, using the fact that $B(\xi) + C(\xi) = 0$,obviously , one finds that

$$B(X) + C(X) = 0$$

for any vector field X on M,so that B+C=0. Hence we state

Theorem 4.2. *If a weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M $(n > 1)$ of non vanishing ξ -sectional curvature is Ricci φ-recurrent ,then the 1-forms B and C are in the opposite directions.*

Corollary 4.3. *If a weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M $(n>1)$ of type $(\alpha, 0)$ with α non zero constant is Ricci φ -recurrent ,then both the associated 1-forms B and C are in the opposite directions.*

Proof. Follows from Theorem5.2. □

Corollary 4.4. *If a weakly Ricci φ-symmetric δ -Lorentzian trans-Sasakian manifold M $(n>1)$ of type $(0, \beta)$ with β non zero constant is Ricci φ-recurrent,then both the associated 1-forms B and C are in the opposite directions.*

5. GENERALIZED RECURRENT SPACES

In this section , we study the characterizations of locally symmetric generalized recurrent spaces.Generalised recurrent spaces was introduced by U.C.De and Guha [2].

Definition 5.1. A non flat Riemannian manifold M is said to be the generalised recurrent manifold if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y,Z)V = A(X)R(Y,Z)V + B(X)\{g(Z,V)Y - g(Y,V)Z\} \tag{5.1}$$

where A and B are associated 1-forms and X,Y,Z.V are any vector fields on M.

Suppose a generalized recurrent weakly symmetric δ- Lorentzian trans-Sasakian manifold is locally symmetric.Then $\nabla R = 0$ so that from (5.1), we have

$$A(X)R(Y,Z)V + B(X)\{g(Z,V)Y - g(Y,V)Z\} = 0, \tag{5.2}$$

Now (5.2) can be written as

$$A(X)R(Y,Z,V,U) + B(X)\{g(Z,V)g(Y,U) - g(Y,V)g(Z,U)\} = 0 \tag{5.3}$$

where $R(X,Y,V,U) = g(R(X,Y)V,U)$. Now contracting Y and U,in (5.3),we get

$$A(X)S(Z,V) + 2nB(X)g(Z,V) = 0 \tag{5.4}$$

Next put $Z = V = \xi$ in (5.4), we find

$$(\alpha^2 + \beta^2 - \delta(\xi\beta))A(X) - 2n\delta B(X) = 0$$

for any vector field X so that

$$(\alpha^2 + \beta^2 - \delta(\xi\beta))A - 2n\delta B = 0 \tag{5.5}$$

Theorem 5.2. *If a generalized recurrent weakly symmetric δ -Lorentzian trans-Sasakian manifold M $(n>1)$ of non vanishing ξ -sectional curvature is locally symmetric ,then the 1-forms A and B are related by (5.5).*

Corollary 5.3. *If a generalized recurrent weakly symmetric δ -Lorentzian Sasakian manifold M $(n>1)$ with α non zero constant is locally symmetric ,then the relation $A - 2n\delta B = 0$ holds.*

Theorem 5.4. *If a generalized recurrent weakly symmetric δ -Lorentzian trans-Sasakian manifold M $(n>1)$ of zero ξ -sectional curvature is locally symmetric if and only if both A and B vanish.*

Proof. For a Locally symmetric space, (5.2) holds. If the ξ -sectional curvature vanishes, then from (5.5), B=0. Again from(5.2), it follows that A=0. Second part is obvious from (5.1). □

Corollary 5.5. *If a generalized recurrent weakly symmetric δ -Lorentzian Sasakian manifold M $(n > 1)$ of zero ξ -sectional curvature is locally symmetric if and only if both A and B vanish.*

References

[1] K.L. Duggal, Space time manifold and contact structures, *International. J. of Math. and Mathematical Sciences,*13(3)(1990), 545-554.
[2] U.C. De and N. Guha, On generalised recurrent manifolds, *Proceedings of Math.Soc,* **7**(1991),7-11.
[3] A. Gray and L.M. Harvella, The sixteen classes of almost Hermitian manifolds and their linear invariants,*Ann.Math.Pura Appl,* **123**(4)(1980),35-58.
[4] J.C. Morrero and D. Chinea, On trans-Sasakian manifolds, *Proceedings of the XIV in Spanish -Portugues Conference, Vol.*-**I-III** *(Spanish)(Puerto de la Craz,* (1989)655-659 , Uni.La Laguna,(1990).
[5] K. Motsumoto,On Lorentzian para contact manifolds, *Bull.of the Yamagata Uni,* **12**(2)(1989),151-156.
[6] J.A. Oubina, New classes og almost contact metric structure, *Publ. Math.Debercen,* **32** (1987), 187-193.
[7] S.S.Pujar and V.J.Khairnar, On Lorentzian trans-Sasakian manifold-I, *Int.J.of Ultra Sciences of Physical Sciences,* **23**(1)(2011),53-66.
[8] S.S. Pujar and V.J.Khairnar, On weak symmetries of Lorentzian trans-Sasakian manifolds,*Acta Ciencia,* **38**(2) (2012),287-300.
[9] S.S. Pujar, On δ Lorentzian α Sasakian manifolds, to appear in Antactica J. of Mathematics **8**(2012).
[10] R. Ralph and Gomez, Lorentzian Sasakian-Einstein metrics on connected sums of S^2XS^3, *Geom.Dedicata,* **59**(2011),249-255.
[11] A.A. Shaikh and S. Hui, On weak symmetries of trans-Sasakian manifolds, *Proceedings of Estonian Acad.of Sciences,* **58**(2009), 213-223.
[12] S.H. Shukla and D.D. Singh, On (ϵ)-trans-sasakian manifolds, *Int.J.of math analysis,* **4**(2010),2401-2414.
[13] G.T. Srinivas, Venatesh and C.S. Bagewadi, On Lorentzian β- Kenmotsu manifolds, *General Mathematics,* **18**(4)(2010),61-69.
[14] Mine Turan A.Y. and Eftal Acet A,On three dimensional Lorentzian α- Sasakian manifolds, *Bulletin of Mathematical Analysis and Applications,* ISSN 1821-1291 (2009),90-98.
[15] L.Tamassy and T.Q.Binha, On weak symmetries of Einstein and Sasakian manifolds, *Tensor N.S.* **53**(1993),140-148.
[16] T.Takahashi, Sasakian manifolds with Pseudo -Riemannian metric,*Tohoku Math.J.* **21** (1969),271-290.
[17] M.M. Thripathi, Erole Kilic and S.Y.Perktas, Indefinite almost metric manifolds, *Int.J. of Math. and Mathematical Sciences,* (2010) Article ID 846195,doi.10,1155/846195.
[18] S. Tanno, The automorphism groups of almost contact Riemannian manifolds,*Tohoku Math.J.* **21** (1969),21-38.
[19] Ahmed Yildiz, On Lorentzian α- Sasakian manifolds,*Kyungpock Math.J.* **45** (2005),95-103.

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