Gheorghe Vrănceanu - successor of Gheorghe Tzitzeica at the Geometry chair of the University of Bucharest

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Abstract

Are presented the personality, the educational and scientific activity of the great Romanian scientist Gheorghe Vrănceanu - successor of Gheorghe Tzitzeica.

Professor Gh. Tzitzeica has taught at the University of Bucharest between 1900-1939. His scientific and educational activity was considered (and still continues to be considered) an exceptional one, both in Romania and abroad.

In 1939, a contest was organized for a tenure position at the full Professor level at the Chair of Analytical and Higher Geometry, position opened by the premature death of Professor Gh. Titeica. Four competitors applied for this position: Dan Barbilian, Grigore Moisil, Miron Nicolescu and Gheorghe Vrănceanu. The contest was won by Gheorghe Vrănceanu. In what follows, we shall present the activity done by Gh. Vrănceanu, as successor of Gh. Tzitzeica.

Worldwide known in the mathematics community as one of the founders of modern Differential Geometry, professor Gheorghe Vrănceanu brought through his work a remarkable brightness for the Romanian mathematics.

Born in 1900 near the city of Vaslui, professor Gheorghe Vrănceanu kept all his life the charme of a great storyteller and the sweet Moldavian accent.

After his studies at the Faculty of Mathematics in Iasi, graduated in 1922, professor Vrănceanu defended in 1924 a brilliant Ph.D. thesis at Rome, under the supervision of Tullio Levi-Civita, cellebrated for his research in Geometry and Mechanics. Under the influence of Levi-Civita's ideas, Gheorghe Vrănceanu discovered in 1926 the notion of non-holonomic space. This discovery brought him the celebrity very quickly.

In 1927-1928, he obtained a Rockefeller grant in USA and worked at Harvard and Princeton universities. With this occasion, he becomes known by the famous

American mathematiciens G. Birkhoff and O. Veblen, with whom he linked a long lasting frienship.

From 1921 to 1970, Gheorghe Vrănceanu belonged to the teaching staff at: the University of Iasi (1921-1929); the University of Cernauti (1929-1939); the University of Bucharest (1939-1970).

Countless lectures in Romania and abroad, colloquia and congresses organized in our country, a remarquable teaching activity and the creation of the Bucharest school of Geometry, are only but some features of his scientific life.

In his Geometry lectures, professor Gheorghe Vrănceanu was careful to eliminate the artefacts, to develop the natural suite of ideas. Professorul Solomon Marcus, referring to the teaching method of Academician Gh. Vrănceanu, said: "He was beginning with a problem, whose relevance was emphasazied, and was looking for the answer, through natural attempts, not always successful. He was not using any written notes. The theorem was crystallizing through the union of all the successive facts; so the theorem was not the starting point, but the end point."

The textbooks of Professor Gheorghe Vrănceanu and his Differential Geometry monography (appeared in 4 volumes and then translated in French and German) supported for a long time the training of students and researchers in Geometry.

The mathematical work of Gheorghe Vrănceanu, containing over 300 papers, is characterized by the deep notions he defined and by the importance of the solved problems. The scientific research of Gheorghe Vrănceanu was directed towards the following topics: nonholonomic spaces, the absolute differential calculus on congruences, Analytical Mechanics, the geometrization of equations with partial derivatives of second order, the nonholonomic Unitary theory, spaces with conformal connection, partial projective spaces, Lie groups, global geometry, transformation groups of spaces with affine connection, spaces with constant connection, harmonic tensors, Riemannian spaces with constant connection, the curvature of a differentiable manifold, Riemannian spaces in geodesic correspondence, the embedding of Riemannian spaces in the Euclidean space, submanifolds of the sphere, spaces with a nonlinear connection, the geometrization of mechanic systems.

This list of research domains in which he brought remarquable contributions, gives a quite large image of the mathematical work dimension of Gheorghe Vrănceanu.

It is very difficult to analize the whole scientific work of academician Gheorghe Vrănceanu. Hence, we will restrict ourselfs to a short review of three of his contributions on: spaces with constant affine connection, Riemannian spaces in geodesic correspondence and the Theory of Relativity.

I. SPACES WITH CONSTANT AFFINE CONNECTION

Academician Gh. Vrănceanu defined a space with constant affine connection A_n as a differentiable manifold with an affine connection, with the following property: each point has a coordinate system x^1, ..., x^n, such that the components Γ^i_{jk} of the connection are constant. With respect to an affine coordinate transformation

$$(*) \qquad\qquad \tilde{x}^i = a^i_j x^j + a^i$$

(where a_j^i and a^i are constant and $det(aij) \neq 0$), the connection components change following the rule

$$\tilde{\Gamma}_{rs}^i a_j^r a_k^s = \Gamma_{jk}^t a_t^i$$

So, for this transformations Γ_{jk}^i behave as the components of a (1,2)-type tensor field. It follows that a space A_n with constant connection is determined, with respect to an adapted coordinate transformation (*), by a constant tensor field of type (1,2).

The spaces A_n with constant connection were called *Vrănceanu spaces* ([10]). The following theorem holds true ([25]): a space A_n with affine connection is a Vrănceanu space if and only if A_n admits an abelian and simply transitive group of transformations. The affine connection of the space A_n becames constant with respect to those coordinates in which this group is the group of translations.

Studying the spaces A_n with constant affine connection, Gh. Vrănceanu investigated the global equivalence of such (locally Euclidian) spaces with the Euclidean space E_n. He gave sufficient conditions for the space A_n to be globally equivalent with E_n ([25]).

In the sixtees, during a discussion between acad. Gh. Vrănceanu and acad. Gr. Moisil, was suggested a natural method ([23]), developped soon after by acad. Gh. Vrănceanu ([24]), for associating to each finite real algebra, a space with constant affine connection, by defining the structure constants of the algebra as the connection coefficients. Conversely, the coefficients of each affine connection may be considered as the structure constants (with respect to some basis), of some finite real algebra. In this manner, one may establish a parallelism between some algebraic and some geometric properties. So, saying that a Vrănceanu space A_n is locally Euclidian is equivalent to say that its algebra is commutative and associative.

Another problem related to Vrănceanu spaces is the classification of locally Euclidean spaces A_n with constant affine connection. This problem was solved for $n = 2$ by acad. Gh. Vrănceanu ([25]). For $n = 3$, the problem was solved (in the case of the null contracted connection), by P. Mocanu ([4]). Expressed in the algebraic setting, this result is the following ([18]): A Vrănceanu space A_n is globally equivalent with the Euclidian space if and only if its associated algebra is commutative, associative and nilpotent.

K. Teleman gave ([18]) a method which allows the classification, by recurrence with respect to the dimension, of all the commutative and associative algebras of finite dimension, so of all locally Euclidean Vrănceanu spaces. The theorem of K. Teleman shows the following assertions are equivalent:

(i) Finding all the commutative and associative algebras of finite dimension.

(ii) Finding all the abelian linear groups, locally transitive in the complex affine space.

(iii) Finding all the abelian linear groups, locally transitive in the complex projective space.

(iv) Finding all the locally Euclidean Vrănceanu spaces.

(v) Finding all the spaces with affine connection, projective Euclidean, with constant associated Thomas connection.

The results obtained by Gh. Vrănceanu in the theory of spaces with constant connection are quoted and used by many Romanian and foreign mathematicians ([11]-[14], [18]-[20]).

We mention the ideas of Gh. Vrănceanu from the study of spaces with constant connection proved to be very useful in the introduction and the study of the deformation algebras of two linear connections, domain of active research, with many written papers (see [7]).

II. RIEMANNIAN SPACES IN GEODESIC CORRESPONDENCE

In Riemannian geometry, the well-known theorem of Beltrami ([25]) says that if a Riemannian space V_n admits a geodesic representation on a constant curvature space V_n', then V_n has also constant curvature.

Siniukov proved ([26]) that if a Riemannian space V_n $(n > 3)$ admits a nontrivial geodesic representation on a locally symmetric V_n', (in E. Cartan sense), then V_n and V_n' are spaces with constant curvature.

Gh. Vrănceanu gave a new (and simpler) proof to the theorem of Siniukov, by establishing first the following theorem ([26]):

Let V_n and V_n' two Riemannian spaces in nontrivial geodesic correspondence ($n > 3$). Suppose V_n is an Einstein space and V_n' is a symmetric space (Cartan). Then V_n and V_n' are spaces with constant curvature.

In [26], Gh. Vrănceanu generalized the results of Levi- Civita referring to the geodesic correspondence of Riemannian spaces. In this sense, he determined all the Riemannian spaces V_n which admit a nontrivial geodesic correspondence on another Riemannian space V_n' and proved these spaces belong to n families, following the properties of the vector which realize the correspondence between V_n and V_n' (to have or not vanishing components on orthogonal congruences, common to both V_n and V_n'). In the same paper, Gh. Vrănceanu proves that if V_n and V_n' are in trivial geodesic correspondence (and V_n is irreducible), then the metrics of V_n and V_n' are homothetic. On this line of research we find a theorem by K. Teleman ([17]) which proves the curvature tensor of an irreducible Riemannian space determines the metric of the space, modulo a constant factor.

The ideas of Vrănceanu in the study of Riemannian spaces geodesically corellated are quoted and used world widely. In this sense, we recall the fact that in the eightees was studied (by the Vrănceanu's method) the geodesic applicability of a Riemannian space V_n on another Riemannian space V_n', imposing conditions on the curvature tensor, on the concircular curvature tensor or on the conharmonic curvature tensor of the V_n' space ([1], [9],[21]).

An interesting Ph.D. thesis of P. Venzi uses results of Gh. Vrănceanu related to the geodesic applicability of Riemannian spaces. Another Ph.D. thesis on the same subject belongs to A.V. Aminova (1993) at the University of Kazan.

The investigating methods of Gh. Vrănceanu in the study of Riemannian spaces geodesically correlated proved to be extremely fertile in the study of the subgeodesic correspondence of Riemannian spaces; this study was realized in Romania, beginning with 1980 ([8]).

III. CONTRIBUTIONS TO THE THEORY OF RELATIVITY

There exist in History some special coincidences, when events and persons apparently without causal connections mark an epoch. From this speculative viewpoint, the biological existence of the great Romanian geometer Gh. Vrănceanu (1900-1979) coincide with the period of the most fascinating paradigm change in the human thought: this period begins with the appearance of the Theory of Relativity (Lorentz, Poincare, Minkowski, Einstein 1900-1905) and of Quantum Mechanics (Planck 1900) and ends with the consacration of the Standard Model of Theoretical Physics (Glashow, Salam, Weinberg - Nobel Prize for Physics in 1979).

But academician Gh. Vrănceanu was not only a passive contemporary of this gnoseological adventure; its mathematical work lies under the sign of deep original discovery and creation, of the search for solutions for many major problems of the 20-th Century Geometry and Physics. In the following, we shall try to re-discover the sinuous but unitary path of his interest in the Theory of Relativity.

In the research of acad. Gh. Vrănceanu, referring to the Theory of Relativity, we distinguish three stages:

1. Construction of a unitary theory for gravitation and electromagnetism (1936-1937)

This theory was exposed for the first time in June 3, 1935, in a lecture at the Institute H. Poincare, then published in two notes in C.R.A.S. Paris ([27], [29]) and in a detailed paper in J. Physique Radium ([28]). Some elements of this theory are exposed also in Romanian, in two other papers ([30], [31]).

Considered the most important contribution of acad. Gh. Vrănceanu to the Theory of Relativity, the non-holonomic Unitary Theory belongs to the search for the simultaneous modelling of the four forces in Nature (weak, electromagnetic, strong and the gravity), which represents also one of the fundamental open problems in today Physics.

The second and the third decades of the 20-th Century lead to the validation of the Theory of Relativity through the well-known classical tests. The deep impact in the scientific world of that time made many great mathematiciens and physicists to be attracted by the next gnoseological challenge: finding a new theory to unify gravitation and electromagnetism. Hundreds of such ("unitary") theories were proposed, competed and still compete today, waiting for some crucial experiments able to select and to embed them into a Grand Unifying Theory (the Standard Model together with the Theory of Relativity).

Between the first unitary theories which influenced the young (at that time) geometer Gh. Vrănceanu, we recall those of H. Weyl (1918), Th. Kaluza (1921), O. Klein

(1926), T. Levi-Civita (1929), E. Cartan (1931-35), A. Einstein and W. Mayer (1931), O. Veblen (1933) and J.Schouten (1935).

The ideea of academician Gh. Vrănceanu was to model the Universe as a 5-dimensional spacetime, doubly "directed" by a 1-dimensional space-like distribution (containing the "electromagnetic" information) and by a time-like distribution, orthogonal to the first one (and, in general, non-integrable), whose values be some 4-dimensional analogues of the "gravitational observable Universe". By considering the 1-form whose kernel define the 4-dimensional time-like distribution, one obtains the Maxwell's equations (by anullating the exterior derivative and the divergence). The Einstein's equations follow by means of the curvature tensor field from the ambient space. In the first papers ([27]-[29]), the 4-dimensional time-like distribution is supposed to be totally geodesic. Later on ([35], [42]), Vrănceanu abandon this hypothesis, increasing the degree of generality of the theory; in exchange, the curvature tensor field of the ambient space does not invariate the 4-dimensional distribution anymore; this fact requests additional precautions for writting the Einstein's equations and for their physical interpretation.

The geometrization of the non-holonomic spaces, by introducing a parallel transport which contains (in a subtle way) information about the distribution, offered at that time an appropriate setting, extremely original but less exploited. We remark the modernity of this unitary theory, 70 years old by now; today, many "in fashion" unitary theories postulate the existence of Universes with more than 4 dimensions, from which we observe only 4-dimensional shadows, as the cave idols invented by Platon.

2. A synthesis of the foundations of the Theory of Relativity. (1959-1964)

At the end of the fiftees and the beginning of the sixtees, one remark in academician Gh. Vrănceanu's concerns an invigoration of the research in the domain of the Theory of Relativity, in the larger framework of non- Euclidean geometries. This interest (catalized by the collaboration with Andrei Popovici, the titular of the course of the Theory of Relativity at the Faculty of Mathematics and Physics in Bucharest) becames stronger (maybe) as a consequence of the participation in 1962 at The Congress of Relativistic Physics in Varsovia. For sure is that his activity in this period materialized through 4 monographic papers, with detalied reviews in Geometry and Relativistic Physics.

3. Construction of a system of invariants for the classification of Einstein's equations (1962-1979)

Using the pseudo-orthogonal congruences, Gh. Vrănceanu developped a special formalism for simplifying Einstein's equations. He considered some local frames (which, in general, are not coordinate systems associated to some maps), in which the space-time metric takes a diagonal form and the Ricci tensor (hence also the electromagnetic one) write in a simplified Jordan canonical form. In particular, academician Vrănceanu applied this technique for the spacetimes with spherical symmetry. The respective papers were published beginning with 1962; later on, the author included these results in two monographs devoted to the Theory of Relativity.

IV. THE SCIENTIFIC IMPACT OF GH. VRĂNCEANU'S ACTIVITY

The results established by Professor Gh. Vrănceanu belong for a long time to the international fundamental core of Science. They influenced directly or indirectly many Romanian and foreign geometers, being sometimes starting points for new researches made by: Thomas, Wagner, K. Yano, Walker, Nomizu, Kobayashi, Egorov, Rasevsky, Petrov, Blaschke, Helgason, E. Cartan. We are sure that the mathematical work of Professor Gh. Vrănceanu deeply influenced, a half century long, the geometric research in the whole world.

The importance of the studied problems, their diversity, their smart solutions, the original methods, the new style and the permanent effort to impose the modern ideas, characterize Gheorghe Vrănceanu as one of the greatest geometers of the 20-th century. So, the school of geometry from Bucharest, created by Gheorghe Titeica, found in Gheorghe Vrănceanu a dignified follower.

The strong scientific personality of academician Gh. Vrănceanu atracted around a kernel of researchers (colegues, teachers, Ph.D. students). We recall the collaboration with professors N. Mihaileanu, A. Popovici, R. Rosca and K. Teleman (which produced some pioneering monographs), and also the prolific emulation between the younger (at that time) geometers from the team of Bucharest University (S. Ianus, R. Iordanescu, L. Nicolescu, I. Popovici, D. Smaranda, I.D. Teodorescu, A. Turtoi, C. Udriste and so on), all having many results in Differential Geometry and reseach interests commun with those of Professor Gh. Vrănceanu.

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