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RIEMANN SOLITONS ON GENERALIZED D-CONFORMALLY DEFORMED KENMOTSU MANIFOLD WITH A HYPER GENERALIZED PSEUDO SYMMETRIC STRUCTURE

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Abstract. The object of the present study is to investigate the nature of Riemann solitons for hyper generalized pseudo symmetric generelized D-conformally deformed Kenmotsu manifold.

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1. Introduction

Let the symbols ∇^d and ∇ stand for the generalized D-conformally deformed connection and the Riemann connection respectively. Also, let R^d, S^d, Q^d, r^d and R, S, Q, r respectively stand for curvature tensor, Ricci tensor, Ricci operator, scalar curvature with respect to ∇^d and ∇ respectively. In this study, we consider an almost contact metric manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ that consists of a $(1,1)$-tensor field ϕ, a vector field ξ and a 1-form η called respectively the structure endomorphism, the characteristic vector field and the contact form. recently, the authors ([14]) has introduced a new type of space named hyper generalized weaky symmetric manifold. Such type of manifold are also studied in ([11]).

Ricci flow was first introduced by R. S. Hamilton ([16]) in 1982. This concept generalized to the idea of Riemann flow ([19], [18]). In analogous with the definition of Ricci soliton, Hirica and Udrişte ([17]) introduced and studied Riemann soliton. The Riemann flow is an evolution equation for metrics on a Riemannian manifold defined as follows

$$\frac{\partial}{\partial t}G\left(t\right) = -2R\left(g\left(t\right)\right), \qquad t \in \left[0, I\right],$$

where $G = \frac{1}{2}g \circledast g$, $\circledast$ is the Kulkarni-Nomizu product and R is the Riemann curvature tensor associated to the metric g. For $(0,2)$-tensors A and B, the Kulkarni-Nomizu product $(A \circledast B)$ is given by

$$\begin{aligned}(A \circledast B)(Y,V,U,Z) &= A(Y,Z)B(V,U)+A(V,U)B(Y,Z)\\ &\quad -A(Y,U)B(V,Z)-A(V,Z)B(Y,U). \qquad (1)\end{aligned}$$

Recently, the present authors studied the Riemann solitons in the frame of $(LCS)_n$-manifolds ([7]) and α-cosymplectic manifolds ([8]). The Riemann soliton is a smooth manifold M together with Riemannian metric g that satisfies

$$2R + (g \circledast \pounds_W g) = 2\kappa(g \circledast g), \qquad (2)$$

where W is a potential vector field, $\pounds_W$ denotes the Lie-derivative along the vector field W and κ is a constant. The Riemann soliton also corresponds to the Riemann flow as a fixed point, and on the space of Riemannian metric modulo diffeomorphism they can be seen as a dynamic system. A Riemann soliton is called expanding, steady and shrinking when $\kappa < 0$, $\kappa = 0$ and $\kappa > 0$ respectively.

A $(2n+1)$-dimensional Kenmotsu manifold is said to be hyper generalized pseudo symmetric (which will be abbreviated hereafter as $[H(GPS)_n, \nabla]$) if it admits the equation

$$\begin{aligned}&(\nabla_X \bar{R})(Y,V,U,Z)\\ =\ & 2\alpha(X)\bar{R}(Y,V,U,Z)+\alpha(Y)\bar{R}(X,V,U,Z)\\ &+\alpha(V)\bar{R}(Y,X,U,Z)+\alpha(U)\bar{R}(Y,V,X,Z)\\ &+\alpha(Z)\bar{R}(Y,V,U,X)+2\beta(X)(g\wedge S)(Y,V,U,Z)\\ &+\beta(Y)(g\wedge S)(X,V,U,Z)+\beta(V)(g\wedge S)(Y,X,U,Z)\\ &+\beta(U)(g\wedge S)(Y,V,X,Z)+\beta(Z)(g\wedge S)(Y,V,U,X), \qquad (3)\end{aligned}$$

where

$$\begin{aligned}(g\wedge S)(Y,V,U,Z) &= g(Y,Z)S(V,U)+g(V,U)S(Y,Z)\\ &\quad -g(Y,U)S(V,Z)-g(V,Z)S(Y,U), \qquad (4)\end{aligned}$$

and α, β being non-zero 1-forms defined as $g(X,\theta_1)=\alpha(X)$ and $g(X,\theta_2)=\beta(X)$.

We organize our present paper as follows: After Introduction, in Section 2, we briefly recall some known results for Kenmotsu manifolds and generelized D-conformally deformed of a Kenmotsu manifold and claimed some properties of the deformed Kenmotsu manifold. In Section 3, we discuss the properties of a generelized D-conformally deformed Kenmotsu manifold under hyper generalized pseudo symmetric curvature condition equipped with Riemann solitions. We determine a necessary condition for shrinking, steady and expanding of the soliton.

2. Preliminaries

According to the definition of Blair ([1]), an *almost contact structure* (ϕ, ξ, η) on a $(2n+1)$-dimensional Riemannian manifold satisfies the following conditions

$$\phi^2 = -I + \eta \otimes \xi, \tag{5}$$

$$g(X, \xi) = \eta(X), \tag{6}$$

$$\eta(\xi) = 1, \tag{7}$$

$$\phi\xi = 0,\ \eta \circ \phi = 0,\ \text{rank}\ \phi = n - 1. \tag{8}$$

Moreover, if g is a Riemannian metric on M^{2n+1} satisfying

$$g(\phi V, \phi U) = g(V, U) - \eta(V)\eta(U), \tag{9}$$

$$g(\phi V, U) = -g(V, \phi U), \tag{10}$$

for any vector fields X, Y on M^{2n+1}, then the manifold M^{2n+1} ([1]) is said to admit an *almost contact metric structure* (ϕ, ξ, η, g).

Definition 1. *([5]) If in an almost contact metric structure (ϕ, ξ, η, g) on M^{2n+1}, the Riemann connection ∇ of g satisfies $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y on M^{2n+1}, then the structure is called Kenmotsu.*

Proposition 1. *([5], [12]) If $(M^{2n+1}, \phi, \xi, \eta, g)$ is a Kenmotsu manifold, then for any vector fields X, Y, Z on M^{2n+1}, the following relations hold*

$$\nabla_X \xi = X - \eta(X)\xi, \tag{11}$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{12}$$

$$\eta(R(X, Y)Z) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{13}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{14}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X. \tag{15}$$

$$S(X, \xi) = -2n\eta(X), \tag{16}$$

Definition 2. *([6]) If a contact metric manifold M^{2n+1} with the almost contact metric structure (ϕ, ξ, η, g) is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, where*

$$\phi^d = \phi,\ \ \xi^d = \frac{1}{p}\xi,\ \ \eta^d = p\eta,\ \ g^d = qg + (p^2 - q)\eta \otimes \eta, \tag{17}$$

where p and q are constants such that $p \neq 0$ and $q > 0$, then the transformation is called a generalized D-conformal deformation.

Note that the generalized D-conformal deformation give rise to conformal deformation (for $p^2 = q$) and D-homothetic deformation (for $p = q =$ constant) ([15], [9], [13]). The generalized D-conformal deformation are studied by various authors in ([21], [22], [20], [23]).

The relation between the Levi-Civita connections ∇ of g and ∇^d of g^d is given by ([6])

$$\nabla^d_X Y = \nabla_X Y + \frac{(p^2 - q)}{p^2} g(\phi X, \phi Y)\xi, \tag{18}$$

for any vector fields X, Y on M^{2n+1}.

In view of (17), (18) and definition of Riemannian curvature tensor, Ricci tensor, scalar curvature, we get the following:

Proposition 2. *([6]) If a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$ under a generalized D-conformal deformation, then R, R^d, S, S^d, r and r^d are related by*

$$R^d(X,Y)Z = R(X,Y)Z - \frac{(p^2 - q)}{p^2}[g(\phi X, \phi Z)Y - g(\phi Y, \phi Z)X], \tag{19}$$

$$S^d(X,Y) = S(X,Y) + \frac{2n(p^2 - q)}{p^2} g(\phi X, \phi Y), \tag{20}$$

$$r^d = \frac{r}{q} + \frac{2n(2n+1)(p^2 - q)}{p^2}, \tag{21}$$

for any vector fields X, Y, Z on M^{2n+1}.

Now we shall bring out some properties of a generalized D-conformally deformed structure $(\phi^d, \xi^d, \eta^d, g^d)$ of a Kenmotsu manifold M^{2n+1} as follows:

Proposition 3. *Under a generalized D-conformal deformation of a Kenmotsu structure (ϕ, ξ, η, g) on M^{2n+1} is transformed into $(\phi^d, \xi^d, \eta^d, g^d)$, then for any vector fields X, Y, Z on M^{2n+1}, we have*

$$\phi^d = -I + \eta^d \otimes \xi^d, \tag{22}$$

$$\eta^d(\xi^d) = 1, \tag{23}$$

$$\phi^d \xi^d = 0, \ \eta^d \circ \phi^d = 0, \tag{24}$$

$$g^d(\phi^d X, \phi^d Y) = g^d(X,Y) - \eta^d(X)\eta^d(Y), \tag{25}$$

$$g^d(X, \xi^d) = \eta^d(X), \tag{26}$$

$$\nabla^d_X \xi^d = \frac{1}{p}[X - \eta^d(X)\xi^d], \tag{27}$$

$$(\nabla^d_X \eta^d)Y = \frac{1}{p}[g^d(X,Y) - \eta^d(X)\eta^d(Y)], \tag{28}$$

$$S^d(X,\xi^d) = -\frac{2n}{p^2}\eta^d(X), \tag{29}$$

$$\eta^d(R^d(X,Y)Z) = \frac{1}{p^2}[g^d(X,Z)\eta^d(Y) - g^d(Y,Z)\eta^d(X)], \tag{30}$$

$$R^d(\xi^d,X)Y = \frac{1}{p^2}[\eta^d(Y)X - g^d(X,Y)\xi^d], \tag{31}$$

$$R^d(X,Y)\xi^d = \frac{1}{p^2}[\eta^d(X)Y - \eta^d(Y)X]. \tag{32}$$

Now using (28) and (29), we obtain

$$(\nabla^d_X S^d)(Y,\xi^d) = -\frac{1}{p}[\frac{2n}{p^2}g^d(X,Y) + S^d(X,Y)], \tag{33}$$

for any vector fields X ,Y and Z on M^{2n+1}.

3. Main results

We shall first define a hyper generalized pseudo symmetric space on a generalized D-conformally deformed Kenmotsu manifold $(M^{2n+1}, \phi^d, \xi^d, \eta^d, g^d)$.

Definition 3. *A generalized D-conformally deformed Kenmotsu manifold $(M^{2n+1}, \phi^d, \xi^d, \eta^d, g^d)$ is said to be hyper generalized pseudo symmetric if it satisfies the condition*

$$\begin{aligned}
& (\nabla^d_X \bar{R}^d)(Y,V,U,Z) \\
= \ & 2\alpha^d(X)\bar{R}^d(Y,V,U,\ Z) + \alpha^d(Y)\bar{R}^d(X,V,U,\ Z) \\
& +\alpha^d(V)\bar{R}^d(Y,X,U,\ Z) + \alpha^d(U)\bar{R}^d(Y,V,X,\ Z) \\
& +\alpha^d(Z)\bar{R}^d(Y,V,U,\ X) + 2\beta^d(X)(g^d \wedge S^d)(Y,V,U,Z) \\
& +\beta^d(Y)(g^d \wedge S^d)(X,V,U,Z) + \beta^d(V)\ (g^d \wedge S^d)(Y,X,U,Z) \\
& +\beta^d(U)\ (g^d \wedge S^d)(Y,V,X,Z) + \beta^d(Z)\ (g^d \wedge S^d)(Y,V,U,X).
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
(g^d \wedge S^d)(Y,V,U,Z) \ = \ & g^d(Y,Z)S^d(V,U) + g^d(V,U)S^d(Y,Z) \\
& -g^d(Y,U)S^d(V,Z) - g^d(V,Z)S^d(Y,U),
\end{aligned} \tag{35}$$

and A^d_i are non-zero 1-forms defined by $A^d_i(X) = g^d(X,\sigma_i)$, for $i = 1,2$.

Now, making use of (35) in (34) we obtained

$$\begin{aligned}
&(\nabla_X \bar{R}^d)(Y,V,U,\ Z)\\
=\ &2\alpha^d(X)\bar{R}^d(Y,V,U,\ Z)+\alpha^d(Y)\bar{R}^d(X,V,U,\ Z)\\
&+\alpha^d(V)\bar{R}^d(Y,X,U,\ Z)+\alpha^d(U)\bar{R}^d(Y,V,X,\ Z)\\
&+\alpha^d(Z)\bar{R}^d(Y,V,U,\ X)+2\beta^d(X)[g^d(Y,Z)S^d(V,U)\\
&+g^d(V,U)S^d(Y,Z)-g^d(Y,U)S^d(V,Z)\\
&-g^d(V,Z)S^d(Y,U)]+\beta^d(Y)[g^d(X,Z)S^d(V,U)\\
&+g^d(V,U)S^d(X,Z)-g^d(X,U)S^d(V,Z)\\
&-g^d(V,Z)S^d(X,U)]+\beta^d(V)\ [g^d(Y,Z)S^d(X,U)\\
&+g^d(X,U)S^d(Y,Z)-g^d(Y,U)S^d(X,Z)\\
&-g^d(X,Z)S^d(Y,U)]+\beta^d(U)\ [g^d(Y,Z)S^d(V,X)\\
&+g^d(V,X)S^d(Y,Z)-g^d(Y,X)S^d(V,Z)-g^d(V,Z)S^d(Y,X)]\\
&+\beta^d(Z)[g^d(Y,X)S^d(V,U)+g^d(V,U)S^d(Y,X)\\
&-g^d(Y,U)S^d(V,X)-g^d(V,X)S^d(Y,U)].
\end{aligned}\tag{36}$$

Now, contracting over Y and Z in (36), we get

$$\begin{aligned}
&(\nabla^d_X S^d)(V,U)\\
=\ &2\alpha^d(X)S^d(V,U)+\alpha^d(V)S^d(X,U)\\
&+\alpha^d(R^d(X,V)U)+2\beta^d(Q^dX)g^d(V,U)\\
&+\alpha^d(R^d(X,U)V)+\alpha^d(U)S^d(X,V)\\
&+2\beta^d(X)[2nS^d(V,U)+r^dg^d(V,U)]\\
&+\beta^d(V)\ [(2n-2)S^d(X,U)+r^dg^d(X,U)]\\
&+\beta^d(U)\ [(2n-2)S^d(X,V)+r^dg^d(X,V)]\\
&-\beta^d(Q^dV)g^d(X,U)-\beta^d(Q^dU)g^d(X,V).
\end{aligned}\tag{37}$$

Now setting $U = \xi^d$ and using (29), (31), (32) in the foregoing equation, we obtain

$$\begin{aligned}
&(\nabla^d_X S^d)(V, \xi^d) \\
= &-\frac{2n}{p^2}[2\alpha^d(X)\eta^d(V) + \alpha^d(V)\eta^d(X)] + \alpha^d(\xi^d)S^d(X, V) \\
&+\frac{1}{p^2}[g^d(X, V)\alpha^d(\xi^d) - 2\eta^d(V)\alpha^d(X) + \eta^d(X)\alpha^d(V)] \\
&+2\beta^d(X)(r^d - \frac{4n^2}{p^2})\eta^d(V) + \beta^d(V)(r^d - \frac{4n(n-1)}{p^2})\eta^d(X) \\
&+\beta^d(\xi^d)\ [2(n-1)S^d(V, X) + r^d g^d(V, X)] \\
&+2\beta^d(Q^d X)\eta^d(V) - \beta^d(Q^d V)\eta^d(X) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(V, X) \qquad (38)
\end{aligned}$$

which yields by using (33)

$$\begin{aligned}
&-\frac{1}{p}[\frac{2n}{p^2}g^d\,(X, V) + S^d\,(X, V)] \\
= &[-\frac{2(2n+1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X)]\eta^d(V) \\
&+[-\frac{(2n-1)}{p^2}\alpha^d(V) + (r^d - \frac{4n(n-1)}{p^2})\beta^d(V) - \beta^d(Q^d V)]\eta^d(X) \\
&+\frac{1}{p^2}\alpha^d(\xi^d)g^d(X, V) + \alpha^d(\xi^d)S^d(X, V) + \frac{2n}{p^2}\beta^d(\xi^d)g^d(V, X) \\
&+\beta^d(\xi^d)\ [2(n-1)S^d(V, X) + r^d g^d(V, X)]. \qquad (39)
\end{aligned}$$

We get now putting successively $X = V = \xi^d$, $V = \xi^d$ and $X = \xi^d$ in (39), we get respectively that

$$[r^d - \frac{2n(2n-1)}{p^2}]\beta^d(\xi^d) = \frac{2n}{p^2}\alpha^d(\xi^d). \qquad (40)$$

$$\begin{aligned}
&-\frac{2(2n+1)}{p^2}\alpha^d(X) + 2\beta^d(X)(r^d - \frac{4n^2}{p^2}) + 2\beta^d(Q^d X) \\
= &[\frac{2(2n-1)}{p^2}\alpha^d(\xi^d) - 2\beta^d(\xi^d)(r^d - \frac{4n(n-1)}{p^2}) + 2\beta^d(Q^d\xi^d)]\eta^d(X). \qquad (41)
\end{aligned}$$

and

$$\begin{aligned}
&-\frac{(2n-1)}{p^2}\alpha^d(V) + (r^d - \frac{4n(n-1)}{p^2})\beta^d(V) - \beta^d(Q^d V) \\
= &[\frac{(6n+1)}{p^2}\alpha^d(\xi^d) - (3r^d - \frac{12n^2 - 2n}{p^2})\beta^d(\xi^d)]\eta^d(V). \qquad (42)
\end{aligned}$$

By virtue of (40), (41) and (42), the equation (39) yields

$$\begin{aligned} & S^d(X,V) \\ = & -\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+(r^d+\frac{2n}{p^2})\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(V,X) \\ & -\left(\frac{\frac{(10n-1)}{p^2}\alpha^d(\xi^d)-(5r^d-\frac{20n^2-14n}{p^2})\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(V)\eta^d(X). \end{aligned} \tag{43}$$

and (40) gives

$$r^d=\frac{2n}{p^2}[\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}+(2n-1)]. \tag{44}$$

Next using (44) in (43) we have

$$\begin{aligned} & S^d(X,V) \\ = & -\left(\frac{\frac{(2n+1)}{p^2}\alpha^d(\xi^d)+\frac{4n^2}{p^2}\beta^d(\xi^d)+\frac{2n}{p^3}}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)g^d(V,X) \\ & +\left(\frac{\frac{1}{p^2}\alpha^d(\xi^d)+\frac{4n}{p^2}\beta^d(\xi^d)}{\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)}\right)\eta^d(V)\eta^d(X). \end{aligned} \tag{45}$$

Thus we can state the following:

Theorem 1. *Let $(M^{2n+1}, \phi^d, \xi^d, \eta^d, g^d)$ be a hyper generalized pseudo symmetric generalized D-conformally deformed Kenmotsu manifold. Then such a manifold is always an η-Einstein provided $\frac{1}{p}+\alpha^d(\xi^d)+2(n-1)\beta^d(\xi^d)\neq 0$.*

Theorem 2. *The scalar curvature of a hyper generalized pseudo symmetric generalized D-conformally deformed Kenmotsu manifold is $\frac{2n}{p^2}[2n-1+\frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)}]$.*

4. Riemann solitons in the deformed manifold with potential vector field ξ^d

In this section we consider a generalized D-conformally deformed Kenmotsu manifold $(\phi^d, \xi^d, \eta^d, g^d)$ admitting a Riemann soliton. Then taking (1) and (2) into account, we obtain

$$\begin{aligned}
& 2R^d(Y,V,U,Z) + g^d(Y,Z)(\pounds_{\xi^d} g^d)(V,U) \\
& + g^d(V,U)(\pounds_{\xi^d} g^d)(Y,Z) \\
& - g^d(Y,U)(\pounds_{\xi^d} g^d)(V,Z) - g^d(V,Z)(\pounds_{\xi^d} g^d)(Y,U) \\
= & \; 2\kappa\left[g^d(Y,Z)\,g^d(V,U) - g^d(Y,U)\,g^d(V,Z)\right]. \qquad (46)
\end{aligned}$$

Now by contrction over Y and Z we get

$$\frac{1}{2}(\pounds_{\xi^d} g^d)(V,U) + \frac{1}{2n-1} S^d(V,U) = \frac{2n\kappa - div(\xi^d)}{2n-1} g^d(V,U). \qquad (47)$$

and then

$$r^d = 2n[(2n+1)\kappa - \frac{4n}{p}]. \qquad (48)$$

Comparing (44) with (48) we have

$$p^2\kappa(2n+1) = \frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} + 4np + (2n-1). \qquad (49)$$

This leads to the following:

Theorem 3. *Assume that a Kenmotsu structure* $(\phi,\ \xi,\ \eta,\ g)$ *on* M^{2n+1} *is transformed into* $(\phi^d,\ \xi^d,\ \eta^d,\ g^d)$ *under a generalized* D*-conformally deformation which is a hyper generalized pseudo symmetric space. Then the Riemann soliton is expanding, steady and shrinking as* $2n(2p+1) + \frac{\alpha^d(\xi^d)}{\beta^d(\xi^d)} <=> 1$.

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