Acta Universitatis Apulensis No. 58/2019
ISSN: 1582-5329 pp. 1-11
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2019.58.01

ON THE CYCLIC DNA CODES OVER THE FINITE RING

Y. Cengellenmis and A. Dertli

Abstract. In this paper, the cyclic DNA codes over the finite ring $R = F_2 + uF_2 + vF_2 + wF_2 + uvF_2 + uwF_2 + vwF_2 + uvwF_2$, where $u^2 = 0, v^2 = v, w^2 = w, uv = vu, uw = wu, vw = wv$ are designed. A map from R to R_1^2, where $R_1 = F_2 + uF_2 + vF_2 + uvF_2$ with $u^2 = 0, v^2 = v, uv = vu$ is given. The cyclic codes of arbitrary length over R satisfy the reverse constraint and reverse complement constraint are studied. A one to one correspondence between the elements of the ring R and $S_{D_{256}}$ is established, where $S_{D_{256}} = \{AAAA, ..., GGGG\}$. The binary image of a cyclic DNA code over the finite ring R is determined.

2010 *Mathematics Subject Classification*: 94B05, 94B15, 92D10.

Keywords: cyclic codes, dna codes.

1. Introduction

First idea about computing DNA was given by Tom Head in 1987. In 1994, L.Adleman introduced an experiment involving to use of DNA molecules to solve a hard computational problem in test tube [2].

The cyclic DNA codes over the finite rings and finite fields play an important role in DNA computing. A lot of authors designed cyclic DNA codes over many finite rings in many papers [3,4,5,6,7,8,9]. Some DNA examples were obtained via the family cyclic codes.

In this paper, the cyclic DNA codes arbitrary length n over the finite ring $R = F_2 + uF_2 + vF_2 + wF_2 + uvF_2 + uwF_2 + vwF_2 + uvwF_2$, where $u^2 = 0, v^2 = v, w^2 = w, uv = vu, uw = wu, vw = wv$ are studied.

This paper is organized as follows. In section 2, some knowledges about the finite ring $R = F_2 + uF_2 + vF_2 + wF_2 + uvF_2 + uwF_2 + vwF_2 + uvwF_2$, where $u^2 = 0, v^2 = v, w^2 = w, uv = vu, uw = wu, vw = wv$ are given. A map from R to R_1^n, where $R_1 = F_2 + uF_2 + vF_2 + uvF_2$ with $u^2 = 0, v^2 = v, uv = vu$ is given. The structures of cyclic codes over the finite ring R are given. In section 3 and 4, the cyclic codes of arbitrary length over R satisfy reverse and reverse complement properties are studied. In section 5, the binary images of cyclic DNA codes over the finite R are investigated.

2. Preliminares

Let R be the commutative finite ring $F_2+uF_2+vF_2+wF_2+uvF_2+uwF_2+vwF_2+uvwF_2=\{a_1+a_2u+a_3v+a_4uv+wa_5+uwa_6+vwa_7+uvwa_8|a_i\in F_2, i=1,2,...,8\}$, where $u^2=0, v^2=v, w^2=w, uv=vu, uw=wu, vw=wv$ with characteristic 2.

$$R=F_2+uF_2+vF_2+wF_2+uvF_2+uwF_2+vwF_2+uvwF_2, u^2=0, v^2=v, w^2=w$$

$$=(F_2+uF_2+vF_2+uvF_2)+w(F_2+uF_2+vF_2+uvF_2), u^2=0, v^2=v, w^2=w$$

$$=R_1+wR_1, w^2=w$$

where $R_1=F_2+uF_2+vF_2+uvF_2$ with $u^2=0, v^2=v, uv=vu$.

In [9], Zhu and Chen introduced the finite ring $R_1=F_2+uF_2+vF_2+uvF_2$ with $u^2=0, v^2=v, uv=vu$. They gave a lot of properties of it. It is well known that the elements $0,1,u,1+u$ of F_2+uF_2 with $u^2=0$ are in one to one correspondence with the nucleotide DNA basis A,T,C,G respectively such that $0\mapsto A, 1\mapsto G, u\mapsto T, 1+u\mapsto C$. In [9], by using the DNA alphabet $S_{D_4}=\{A,T,G,C\}$, they defined a correspondence between the elements of the ring R_1 and DNA double pairs as in the following table, by means of Gray map from R_1 to $(F_2+uF_2)^2$ with $u^2=0$. For $a\in R_1$,

Elements a	DNA double pairs
0	AA
v	AG
uv	AT
$v+uv$	AC
1	GG
$1+v$	GA
$1+uv$	GC
$1+v+uv$	GT
u	TT
$u+v$	TC
$u+uvv$	TA
$u+v+uv$	TG
$1+u$	CC
$1+u+v$	CT
$1+u+uv$	CG
$1+u+v+uv$	CA

DNA has two strands that are governed by the rule called Watson Crick Complement (WCC) , that is A pairs with T, G pairs with C.

In [9], they denoted the WCC in their paper $\overline{A} = T, \overline{T} = A, \overline{G} = C, \overline{C} = G$. They used the same notation for the set $S_{D_{16}} = \{AA, TT, GG, CC, AT, AG, AC, TG, TC, TA, GC, GA, GT, CA, CT, CG\}$ and extended the Watson Crick Complement to the elements of $S_{D_{16}}$ such that $\overline{AA} = TT, ..., \overline{TG} = AC$.

Similarly, if the Gray map Φ from R to R_1^2 is defined as follows, we can define a γ correspondence between the elements of the ring R and DNA quartet.

$$
\begin{aligned}
\Phi \quad &: \quad R \longrightarrow R_1^2 \\
r = c + wd \quad &\longmapsto \quad \Phi(r) = (c, c+d)
\end{aligned}
$$

where $c, d \in R_1$, $w^2 = w$.

Elements r	Gray images in(R_1^2)	DNA quartet $\gamma(r)$
0	$(0,0)$	$AAAA$
v	(v,v)	$AGAG$
uv	(uv,uv)	$ATAT$
$v+uv$	$(v+uv, v+uv)$	$ACAC$
1	$(1,1)$	$GGGG$
$1+v$	$(1+v, 1+v)$	$GAGA$
$1+uv$	$(1+uv, 1+uv)$	$GCGC$
$1+v+uv$	$(1+v+uv, 1+u+uv)$	$GTGT$
u	(u,u)	$TTTT$
$u+v$	$(u+v, u+v)$	$TCTC$
$u+uv$	$(u+uv, u+uv)$	$TATA$
$u+v+uv$	$(u+v+uv, u+v+uv)$	$TGTG$
$1+u$	$(1+u, 1+u)$	$CCCC$
$1+u+v$	$(1+u+v, 1+u+v)$	$CTCT$
$1+u+uv$	$(1+u+uv, 1+u+uv)$	$CGCG$
$1+u+v+uv$	$(1+u+v+uv, 1+u+v+uv)$	$CACA$
$\vdots$	$\vdots$	$\vdots$

Naturally, we can extend the Watson Crick Complement to the elements of $S_{D_{256}} = \{AAAA, ..., GGGG\}$ such that $\overline{AAAA} = TTTT, ..., \overline{GGGG} = CCCC$. For any $r \in R$, we can define $\overline{r}$ as the complement r, where $\gamma(\overline{r}) = \overline{\gamma(r)}$.

A code of length n over S is a subset of S^n, where S is a finite ring. C is a linear iff C is an S-submodule of S^n. The elements of the code (linear code) is called codewords. The code C is said to be cyclic if $(c_0, ..., c_{n-1}) \in C$ for all $(c_{n-1}, c_0, ..., , c_{n-2}) \in C$.

In [1], the structure of a cyclic code over $F_2 + uF_2$ with $u^2 = 0$ was determined as follows.

Theorem 1. *Let B be a cyclic code over $F_2 + uF_2$ with $u^2 = 0$. Then,*

(1) *If n is odd, then $B = (g(x), ua(x)) = (g(x) + ua(x))$ where $g(x), a(x)$ are binary polynomials with $a(x)|g(x)|x^n - 1(mod2)$,*

(2) *If n is not odd, then,*

(2.1) *$B = (g(x) + ua(x))$ where $g(x)|x^n - 1(mod2)$, $g(x) + ua(x)|x^n - 1(mod2)$ and $g(x)|p(x)(x^n - 1/g(x))$ or*

(2.2) *$B = (g(x) + up(x), ua(x))$ where $g(x), a(x)$ and $p(x)$ are binary polynomials with $a(x)|g(x)|x^n - 1(mod2)$ and $a(x)|p(x)(x^n - 1/g(x))$ and deg $p(x)$ <deg $a(x)$.*

In [9], Zhu and Chen presented the linear code C over R_1 as

$$C = vC_1 \oplus (1 + v)C_2$$

where

$$C_1 = \{(x + y) \in (F_2 + uF_2)^n | (x + y)v + x(1 + v) \in C, \text{ for some } x, y \in (F_2 + uF_2)^n\}$$

and

$$C_2 = \{x \in (F_2 + uF_2)^n | (x + y)v + x(1 + v) \in C, \text{for some } y \in (F_2 + uF_2)^n\}$$

are linear codes over $F_2 + uF_2$, with $u^2 = 0$.

They also shown that C is a linear code over $F_2 + uF_2$ with $u^2 = 0$ such that $C = vC_1 \oplus (1+v)C_2$, then C is a cyclic code if and only if C_1 and C_2 are both cyclic codes over $F_2 + uF_2$ with $u^2 = 0$ in [9].

Let D be a linear code over R. So it can be similarly written as follows;

$$D = wD_1 \oplus (1 + w)D_2$$

where

$$D_1 = \{(x+y) \in R_1^n | (x+y)w + x(1+w) \in D, for\ some\ x, y \in R_1^n\}$$

and

$$D_2 = \{x \in R_1^n | (x+y)w + x(1+w) \in D, for\ some\ y \in R_1^n\}$$

are linear codes over $R_1 = F_2 + uF_2 + vF_2 + uvF_2$, with $u^2 = 0, v^2 = v, uv = vu$.

Theorem 2. *Let D be a linear code of odd length n over R such that $D = wD_1 \oplus (1+w)D_2$. Then D is a cyclic code if and only if $D_1 = v(g_1(x) + ua_1(x)) \oplus (1+v)(g_2(x) + ua_2(x))$ and $D_2 = v(g_3(x) + ua_3(x)) \oplus (1+v)(g_4(x) + ua_4(x))$, where $g_i(x), a_i(x)$ are binary polynomials with $a_i(x)|g_i(x)|x^n - 1(mod2)$ for $i = 1, 2, 3, 4$.*

Theorem 3. *Let D be a linear code of even length n over R such that $D = wD_1 \oplus (1+w)D_2$. Then D is a cyclic code if and only if*

$$D_1 = v(g_1(x) + up_1(x)) \oplus (1+v)(g_2(x) + up_2(x))$$

(or $D_1 = v(g_1(x) + up_1(x), ua_1(x)) \oplus (1+v)(g_2(x) + up_2(x), ua_2(x))$

and

$$D_2 = v(g_3(x) + up_3(x)) \oplus (1+v)(g_4(x) + up_4(x))$$

(or $D_2 = v(g_3(x) + up_3(x), ua_3(x)) \oplus (1+v)(g_4(x) + up_4(x), ua_4(x))$

where $g_i(x)|x^n - 1(mod2)$ and $g_i(x) + up_i(x)|x^n - 1(mod2)$ and $g_i(x)|p_i(x)(x^n - 1/g_i(x))$ for $i = 1, 2, 3, 4$. (or $g_i(x), a_i(x)$ and $p_i(x)$ are binary polynomials with $a_i(x)|g_i(x)|x^n - 1(mod2)$ and $a_i(x)|p_i(x)(x^n - 1/g_i(x))$ and deg $p_i(x) <$deg $a_i(x)$ for $i = 1, 2, 3, 4$.)

3. Reversible codes over R

Let $d = (d_0, ..., d_{n-1}) \in R^n$ be a vector. The reverse of d is defined as $d^r = (d_{n-1}, ..., d_0)$. A linear code D of length n over R is said to be reversible if $d^r \in D$, for all $d \in D$.

Let $f(x) = a_0 + a_1x + ... + a_sx^s$ be a polynomial of s with $a_s \neq 0$. The reciprocal of $f(x)$ is defined as $f^*(x) = x^s f(1/x)$. The polynomial $f(x)$ is called self reciprocal polynomial if and only if $f^*(x) = f(x)$.

In [5] and [6], necessary and sufficient conditions for a cyclic code of either odd or even length over F_2+uF_2 with $u^2=0$ to be reversible were determined as follows, respectively.

Lemma 4 (5)**.** *Let $B=(g(x),ua(x))=(g(x)+ua(x))$ be a cyclic code of odd length n over F_2+uF_2 with $u^2=0$. Then B is reversible if and only if $g(x)$ and $a(x)$ are self-reciprocal.*

Lemma 5 (6)**.** *Let $B=(g(x)+ua(x))$ be a cyclic code of even length n over F_2+uF_2 with $u^2=0$. Then B is reversible if and only if*
1. $g(x)$ is self-reciprocal.
2. (a) $x^i p^*(x)=p(x)$ or
(b) $g(x)=x^i p^*(x)+p(x)$, where i=deg $g(x)$-deg $p(x)$.

Lemma 6 (6)**.** *Let $B=(g(x)+up(x),ua(x))$ with $a(x)|g(x)|x^n-1(mod2)$, $a(x)|p(x)$ $(x^n-1/g(x))$ and $degp(x)\leq dega(x)$ be a cyclic code of even length n over F_2+uF_2 with $u^2=0$. Then B is reversible if and only if*
1. $g(x)$ and $a(x)$ are self-reciprocal.
2. $a(x)|(x^i p^*(x)+p(x))$, where i=deg $g(x)$-deg $p(x)$.

Theorem 7. *Let $D=wD_1\oplus(1+w)D_2$ be a cyclic code of odd length over R, where $D_1=v(g_1(x)+ua_1(x))\oplus(1+v(g_2(x)+ua_2(x))$ and $D_2=v(g_3(x)+ua_3(x))\oplus(1+v)(g_4(x)+ua_4(x))$, where $g_i(x),a_i(x)$ are binary polynomials with $a_i(x)|g_i(x)|x^n-1(mod2)$ for $i=1,2,3,4$. Then D is reversible code if and only if the polynomials $g_i(x),a_i(x)$ are self reciprocal for $i=1,2,3,4$.*

Theorem 8. *Let $D=wD_1\oplus(1+w)D_2$ be a cyclic code of even length over R, where $D_1=v(g_1(x)+up_1(x))\oplus(1+v)(g_2(x)+up_2(x))$ and $D_2=v(g_3(x)+up_3(x))\oplus(1+v)(g_4(x)+up_4(x))$, with $g_i(x)|x^n-1(mod2)$ and $g_i(x)+up_i(x)|x^n-1(mod2)$ and $g_i(x)|p_i(x)(x^n-1/g_i(x))$ for $i=1,2,3,4$. Then D is reversible code if and only if the polynomials $g_i(x)$ are self reciprocal for $i=1,2,3,4$ and $x^j p_i^*(x)=p_i(x)$ or $g_i(x)=x^j p_i^*(x)+p_i(x)$, where j=deg $g_i(x)$-deg $p_i(x)$ for $i=1,2,3,4$.*

Theorem 9. *Let $D=wD_1\oplus(1+w)D_2$ be a cyclic code of even length over R, where $D_1=v(g_1(x)+up_1(x),ua_1(x))\oplus(1+v)(g_2(x)+up_2(x),ua_2(x))$ and $D_2=v(g_3(x)+up_3(x),ua_3(x))\oplus(1+v)(g_4(x)+up_4(x),ua_4(x))$ with $g_i(x),a_i(x)$, $p_i(x)$ are binary polynomials with $a_i(x)|g_i(x)|x^n-1(mod2)$ and $a_i(x)|p_i(x).(x^n-1/g_i(x))$ and deg $p_i(x)$ <deg $a_i(x)$ for $i=1,2,3,4$. Then D is reversible code if and only if the polynomials $g_i(x)$ and $a_i(x)$ are self reciprocal for $i=1,2,3,4$ and $a_i(x)|x^j p_i^*(x)+p_i(x)$, where j=deg $g_i(x)$-deg $p_i(x)$ for $i=1,2,3,4$.*

Corollary 10. *Let $D=wD_1\oplus(1+w)D_2$ be a cyclic code of arbitrary length n over R. Then D is reversible if and only if D_1 and D_2 are reversible with D_1 and D_2 are cyclic codes over R_1.*

Proof. Let D_1, D_2 be reversible codes. For any $b \in D$, $b = wb_1 + (1+w)b_2$, where $b_1 \in D_1, b_2 \in D_2$. As D_1, D_2 are reversible codes, $b_1^r \in D_1$,$b_2^r \in D_2$, so $b^r = wb_1^r + (1+w)b_2^r \in D$. Hence D is reversible codes.

On the other hand, let D be a reversible code over R. So for any $b = wb_1 + (1+w)b_2 \in D$, where $b_1 \in D_1$, $b_2 \in D_2$, we get $b^r = wb_1^r + (1+w)b_2^r \in D$. Let $b^r = wb_1^r + (1+w)b_2^r = ws_1 + (1+w)s_2$,where $s_1 \in D_1, s_2 \in D_2$. Therefore D_1 and D_2 are reversible codes over R_1.

Example 1. *Let $x^8 - 1 = (x+1)^8 = g^8$ over F_2. Let $D_1 = v(g_1 + up_1) \oplus (1+v)(g_1 + up_1)$ where $g_1 = g^6, p_1 = x^5 + x$ and $D_2 = v(g_2 + up_2) \oplus (1+v)(g_2 + up_2)$ where $g_2 = g^4, p_2 = x^3 + x$. As $(f) = D = wD_1 \oplus (1+w)D_2$, we get $f = wx^6 + uwx^5 + x^4 + (u+uw)x^3 + wx^2 + ux + 1 \in D$ and $f^r = wx + uwx^2 + x^3 + (u+uw)x^4 + wx^5 + ux^6 + x^7$. As $(wx + (1+w)x^3)f = f^r$, we get that D is a reversible code over R.*

4. Reversible complement codes over R

Let $x = (x_0, ..., x_{n-1}) \in R^n$ be a vector. The reverse complement is defined as $x^{rc} = (\overline{x_{n-1}}, ..., \overline{x_0})$, where $\overline{y}$ represents complement of any element y of R.

A linear code C of length n over R is said to be reversible complement if $x^{rc} \in C$, for all $x \in C$.

In [5,6], the reverse and reverse complement constraint on cyclic codes of odd and even length over $F_2 + uF_2$ with $u^2 = 0$ was determined, respectively as follows.

Theorem 11 (5,6)**.** *Let B be a cyclic code of length n over $F_2 + uF_2$ with $u^2 = 0$. Then,*
1.*If n is odd, then $B = (g(x) + ua(x)) = (g(x) + ua(x))$ is reversible complement if and only if $(u, ..., u) \in B$, $g(x)$ and $a(x)$ are self-reciprocal polynomials*
2.*If n is even, then*
(i) *$B = (g(x) + ua(x))$ is reversible complement if and only if*
a) $g(x)$ is self-reciprocal and $(u, ..., u) \in B$
b) $x^i p^*(x) = p(x)$ and $g(x) = x^i p^*(x) + p(x)$, where i=deg g(x)-deg p(x).
(ii) *$B = (g(x) + up(x), ua(x))$ with $a(x)|g(x)|x^n - 1(mod2), a(x)|p(x)(x^n - 1/g(x))$ and $degp(x) \leq dega(x)$ is reversible complement if and only if*
a) $(u, ..., u) \in B$, $g(x)$ and $a(x)$ are self-reciprocal,
b) $a(x)|(x^i p^*(x) + p(x))$, where i=deg g(x)-deg p(x).

Lemma 12. *For any $s \in R$, we have $s + \overline{s} = u$.*

Theorem 13. *Let $D = wD_1 \oplus (1+w)D_2$ be a cyclic code of arbitrary length n over R. Then D is reversible complement over R iff D is reversible over R and $(u, u, ..., u) \in D$.*

Proof. Since d is reversible complement, for any $d = (d_0, ...d_{n-1}) \in d, d^{rc} = (\overline{d}_{n-1}, ..., \overline{d}_0) \in D$. Since D is a linear code, so $(0, 0, ..., 0) \in D$. Since D is reversible complement, so $(\overline{0}, \overline{0}, ...\overline{0}) \in C$. By using Lemma 12, we get

$$d^r = (d_{n-1}, ..., d_0) = (\overline{d}_{n-1}, ..., \overline{d}_0) + (u, u, u, ..., u) \in D$$

Hence for any $d \in D$, we have $d^r \in D$.

On the other hand, let D be reversible code over R. So, for any $d = (d_0, ..., d_{n-1}) \in D$, then $d^r = (d_{n-1}, ..., d_0) \in D$. For any $d \in D$,

$$d^{rc} = (\overline{d}_{n-1}, ..., \overline{d}_0) = (d_{n-1}, ...d_0) + (u, ..., u) \in D$$

So, D is reversible complement code over R.

Example 2. *Let $x^8 - 1 = (x+1)^8$ over F_2. Let $D = \langle h(x) \rangle$, where $h(x) = w(p(x) + uq(x)) + (1+w)(p(x) + uq(x))$, $p(x) = x^6 + x^4 + x^2 + 1$ and $q(x) = x^5 + x$. The code D is a cyclic DNA code of length 32 and minimum Hamming distance 4. This code has 256 codewords. These codewords are given as follows;*

$AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA$
$GGGGTTTTGGGGAAAAGGGGTTTTGGGGAAAA$
$TTTTAAAATTTTAAAATTTTAAAATTTTAAAA$
$CCCCTTTTCCCCAAAACCCCTTTTCCCCAAAA$
$AAAAGGGGTTTTGGGGAAAAGGGGTTTTGGGG$
$AAAATTTTAAAATTTTAAAATTTTAAAATTTT$
$\ldots$
$\vdots$
$\ldots$
$AAAACCCCTTTTCCCCAAAACCCCTTTTCCCC$
$AGAGATATAGAGAAAAAGAGATATAGAGAAAA$
$GAGATATAGAGAAAAAGAGATATAGAGAAAAA$
$AAAAGCGCTTTTGCGCAAAAGCGCTTTTGCGC$
$AAGGAATTAAGGAAAAAAGGAATTAAGGAAAA$

5. Binary images of cyclic DNA codes over R

Thanks to a Grap map from R to F_2^8, we can convert the properties of DNA codes to the binary codes.

We define the Gray map as follows

$$\begin{aligned} \breve{O} \quad &: \quad R \longrightarrow F_2^8 \\ a \quad &\longmapsto \quad \breve{O}(a) = (a_2, a_1 + a_2, a_2 + a_4, a_1 + a_2 + a_3 + a_4, a_2 + a_6, \\ &\qquad a_1 + a_5 + a_2 + a_6, a_2 + a_4 + a_6 + a_8, \\ &\qquad a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8) \end{aligned}$$

where $a = a_1 + ua_2 + va_3 + uva_4 + wa_5 + uwa_6 + vwa_7 + uvwa_8$ with $a_i \in F_2$ for $i = 1, ..., 8$.

The Hamming weight of codeword $c = (c_0, ..., c_{n-1})$ denoted by $w_H(c)$ is the number of non zero entires in c. The Hamming distance $d_H(c_1, c_2)$ between two codewords c_1 and c_2 is the Hamming weight of the codewords $c_1 - c_2$.

The Gray weight is defined over the ring R as $w_G(a) = w_H(\breve{O}(a))$ and the Gray distance d_G is given by $d_G(c_1, c_2) = w_G(c_1 - c_2)$.

It is noted that the image of a linear code over R is a binary linear code.

DNA Quartet	Binary images
AAAA	$(0,0,0,0,0,0,0,0)$
AGAG	$(0,0,0,1,0,0,0,1)$
ATAT	$(0,0,1,1,0,0,1,1)$
......	.....

Theorem 14. *The Gray map $\breve{O}$ is a distance preserving map from(R^n,Gray distance) to (F_2^{8n}, Hamming distance). It is also linear.*

Proof. For $c_1, c_2 \in R^n$, we have $\breve{O}(c_1 - c_2) = \breve{O}(c_1) - \breve{O}(c_2)$. So, $d_G(c_1, c_2) = w_G(c_1 - c_2) = w_H(\breve{O}(c_1 - c_2)) = w_H(\breve{O}(c_1) - \breve{O}(c_2)) = d_H(\breve{O}(c_1), \breve{O}(c_2))$. So, the Gray map $\breve{O}$ is distance preserving map.

For any $c_1, c_2 \in R^n, k_1, k_2 \in F_2$,we have $\breve{O}(k_1c_1 + k_2c_2) = k_1\breve{O}(c_1) + k_2\breve{O}(c_2)$. Thus, $\breve{O}$ is linear.

Proposition 1. *Let σ be the cyclic shift of R^n and υ be the 8-quasi-cyclic shift of F_2^{8n}. Let $\breve{O}$ be the Gray map from R^n to F_2^{8n}. Then $\breve{O}\sigma = \upsilon\breve{O}$.*

Proof. For any $c = (c_0, c_1, ..., c_{n-1}) \in R^n$, it is easily seen that $\breve{O}\sigma(c) = \upsilon\breve{O}(c)$. So we have expected result.

Theorem 15. *If C is a cyclic DNA code of length n over R then $\breve{O}(C)$ is binary quasi-cyclic DNA code of length $8n$ with index 8.*

6. Conclusion

The cyclic DNA codes over the ring R are introduced and some properties of them are investigated. Morever the binary images of them are determined.

References

[1] Abualrub T., Siap I., *Cyclic codes over the rings Z_2+uZ_2 and $Z_2+uZ_2+u^2Z_2$*, Des. Codes. Cryp., 42 (2007), 273-287.

[2] Adleman L., *Molecular computation of the solutions to combinatorial problems*, Science, 266 (1994), 1021-1024.

[3] Bennenni N., Guenda K., and Mesnager S., *DNA cyclic codes over rings*, Advances in Mathematics of Communications 11 (2017).

[4] Dertli A. , Cengellenmis Y., *On cyclic DNA codes over the rings Z_4+wZ_4 and $Z_4+wZ_4+vZ_4+wvZ_4$*, arXiv preprint arXiv:1605.02968 (2016).

[5] Guenda K.,Aaron Gulliver, T., *Construction of cyclic codes over F_2+uF_2 for DNA computing*, AAECC, 24 (2013), 445-459.

[6] Liang J., Wang L., *On cyclic DNA codes over F_2+uF_2*, J.Appl Math Comput., DOI 10.1007/s12190-015-0892-8.

[7] Limbachiya D. , Rao B., and Gupta Manish K. ,*The Art of DNA Strings: Sixteen Years of DNA Coding Theory*, arXiv preprint arXiv:1607.00266 (2016).

[8] Pattanayak S., Singh A.K., *Construction of cyclic DNA codes over the Ring $Z_4[u]/<u^2-1>$ Based on the deletion distance*, arXiv preprint arXiv:1603.04055 (2016).

[9] Zhu S., Chen X., *Cyclic DNA codes over $F_2+uF_2+vF_2+uvF_2$ and their applications*, J. Appl.Math Comput, DOI 10.1007/s12190-016-1046-3.

Yasemin Cengellenmis
Department of Mathematics, Faculty of Science,
University of Trakya,
Edirne, Turkey
email: *ycengellenmis@gmail.com*

Abdullah Dertli
Department of Mathematics, Faculty of Art and Science,
University of Ondokuz Mayıs,
Samsun, Turkey
email: *abdullah.dertli@gmail.com*