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C-BOCHNER PSEUDOSYMMETRIC NULL HYPERSURFACES IN INDEFINITE KENMOTSU SPACE FORMS

S. KAZAN

Abstract. In this paper, firstly we study pseudosymmetric null hypersurfaces of an indefinite Kenmotsu space form, tangent to the structure vector field. We obtain sufficient conditions for null hypersurface to be pseudosymmetric in an indefinite Kenmotsu space form. Later, we investigate pseudosymmetric condition of null hypersurfaces with C-Bochner curvature tensor of an indefinite Kenmotsu space form and obtain sufficient condition for a null hypersurface to be C-Bochner pseudosymmetric in an indefinite Kenmotsu space form, give an example for these hypersurfaces. We give a result for the C-Bochner pseudosymmetric null hypersurfaces to be C-Bochner semi-symmetric and show that there is a close relationship of the C-Bochner semi-symmetry condition of a null hypersurface and its integrable screen distribution. Furthermore, we introduce Ricci-generalized C-Bochner pseudosymmetric null hypersurfaces.

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1. Introduction

Let (M, g) be a Riemannian manifold of dimension n and ∇ be the Levi-Civita connection. A Riemannian manifold is called locally symmetric if $\nabla R = 0$, where R is the Riemannian curvature tensor of M [3]. Locally symmetric Riemannian manifolds are a generalization of manifolds of constant curvature. As a generalization of locally symmetric Riemannian manifolds, semi-symmetric Riemannian manifolds have been defined by the condition $R \cdot R = 0$. It is known that locally symmetric manifolds are semi-symmetric manifolds but the converse is not true [21].

A semi-Riemannian manifold (M, g) is said to be a pseudosymmetric manifold if at every point of M the following condition is satisfied:

$$\textit{the tensor } R \cdot R \textit{ and } Q(g, R) \textit{ are linearly dependent.} \tag{1}$$

The condition (1) is equivalent to the fact that the equality $R \cdot R = LQ(g, R)$ holds on the set $U = \{x \in M \mid Q(g, R) \neq 0 \ \ at \ \ x\}$, where L is some function on U.

Also, pseudosymmetric manifolds have been discovered during the study of totally umbilical submanifolds of semi-symmetric manifolds [1]. It is clear that every semi-symmetric Riemannian manifold is pseudosymmetric manifold but the converse is not true. For pseudosymmetric manifolds, see also [4], [17], [18]. Besides, in [9], S. Haesen and L. Verstraelen gave geometrical meaning of the metrical endomorphism, pseudo-symmetry. According to this study, a geometrical symmetry of a Riemannian space (M^n, g) concerns the invariance of some geometrical quantity of (M^n, g) under the performance of some transformations defined on (M^n, g) . Various types of symmetries in Riemannian geometry can thus be considered, essentially depending on the kind of quantities and of transformations in question. And in complete analogy with such intrinsic symmetries, various types of extrinsic symmetries can be considered on submanifolds (M^n, g) in Riemannian ambient spaces $(\widetilde{M}^{m+n}, \widetilde{g})$. By natural geometrical symmetries, the authors say that we mean symmetries in the above sense for which the quantities and the transformations involved are the most natural indeed, at least in our opinion. Then one has the proposition; *(Schouten) The curvature operators of Riemannian manifolds measure the changes of directions at points under parallel transports fully around the infinitesimal parallelo-grams cornered at these points* and the theorem; *(Schouten) The locally Euclidean (or locally flat) Riemannian manifolds (the spaces* (M^n, g) *for which* $R \equiv 0$*), are precisely the Riemannian manifolds for which all directions are invariant under their parallel transports fully around all infinitesimal co-ordinate parallelograms.* The natural metrical endomorphism $X \wedge_g Y : TM \to TM$ associated with two vector fields X and Y on a Riemannian manifold (M^n, g) is defined by $(X \wedge_g Y)Z = g(Y, Z)X - g(X, Z)Y$. Let $\vec{x}$ and $\vec{y}$ be orthonormal vectors at p, and let $\vec{z} = \vec{z}_{\bar{\pi}} + \vec{z}_{\bar{\pi}^{\perp}}$ be the canonical orthogonal decomposition of any vector $\vec{z}$ at p in its components in $\bar{\pi} = \vec{x} \wedge \vec{y}$ and in the orthogonal complement $\bar{\pi}^{\perp}$ of $\bar{\pi}$ in $T_pM^n = \mathbb{R}^n$. Now, we rotate $\vec{z}_{\bar{\pi}}$ around p in the plane $\vec{z}_{\bar{\pi}}$ over an infinitesimal angle $\triangle\varphi$, thus obtaining a vector $(\vec{z}_{\bar{\pi}})_{\triangle\varphi}$, and define the vector $\vec{z}^{\wedge}_{\bar{\pi}} = (\vec{z}_{\bar{\pi}})_{\triangle\varphi} + \vec{z}_{\bar{\pi}^{\perp}}$. The procedure going from $\vec{z}$ to $\vec{z}^{\wedge}_{\bar{\pi}}$ is called the rotation of $\vec{z}$ at p with respect to the plane $\bar{\pi}$ over an angle $\triangle\varphi$, and one has the following: $\vec{z}_{\bar{\pi}^{\perp}} = \vec{z} + [(\vec{x} \wedge_g \vec{y})\vec{z}]\triangle_{\varphi} + \mathcal{O}^{>1}(\triangle_{\varphi})$. Thus the vector $(\vec{x} \wedge_g \vec{y})\vec{z}$ measures the first order change of the vector $\vec{z}$ after an infinitesimal rotation of $\vec{z}$ at p with respect to the plane $\bar{\pi} = \vec{x} \wedge_g \vec{y}$, or, formulated more loosely. Then it is said that the natural metrical endomorphisms $\wedge_g$ of Riemannian manifolds measure the changes of directions at points under infinitesimal rotations with respect to $2D$ planes at these points. Therefore, the Tachibana tensor $\wedge g \cdot R$ measures the changes of the sectional curvatures $K(p, \pi)$ of a Riemannian manifold (M^n, g) for all planes π at all points p under the infinitesimal rotations of these planes π at p with respect to all planes $\bar{\pi}$ at p [9].

In addition, a Riemannian manifold (M^n, g), $(n \geq 3)$, is said to be pseudo-symmetric in the sense of Deszcz or is called Deszcz symmetric if, for some function $L_R : M \to R, R \cdot R = L_R \wedge g \cdot R$. Then, a Riemannian manifold (M^n, g), $(n \geq 3)$, is Deszcz symmetric if and only if its sectional curvature of Deszcz $L(p, \pi, \bar{\pi})$ is isotropic, i.e., if $L(p, \pi, \bar{\pi})$ is independent of the planes π and $\bar{\pi}$, or, still, if the double sectional curvature function $L(p, \pi, \bar{\pi})$ actually is a function $L = L_R : M \to R$. Here, such theorem is given that (Sinyukov, Mikĕs, Venzi, Defever and Deszcz) *if a semi-symmetric Riemannian space admits a geodesic transformation onto some other Riemannian manifold, then this latter manifold must itself be pseudo-symmetric, and, if a pseudo-symmetric Riemannian space admits a geodesic transformation onto some other Riemannian manifold, then this latter manifold must itself also be pseudo-symmetric* [9].

Also, let M^n be a hypersurface in a Euclidean space E^{n+1}, $n \geq 3$. Amongst the simplest possible forms of the shape operator of M^n in E^{n+1}, one has those whereby the principal curvatures at every point are $(1) : (0, 0, ..., 0); (2) : (\lambda, \lambda, ..., \lambda), \lambda \neq 0; (3) : (\lambda, 0, ..., , 0), \lambda \neq 0; (4) : (\lambda, ..., \lambda, 0, ..., 0), \lambda \neq 0$ and λ appearing more than once; $(5) : (\lambda, \mu, 0, ..., 0), \lambda \neq 0 \neq \mu$ and $\lambda \neq \mu; (6) : (\lambda, \mu, ..., \mu), \lambda \neq 0 \neq \mu$ and $\lambda \neq \mu; (7) : (\lambda, ..., \lambda, \mu, ..., \mu), \lambda \neq 0 \neq \mu$ and $\lambda \neq \mu$ and both λ and μ appearing more than once. Then there are the following correspondences: $M^n \subset E^{n+1}$ is totally geodesic in case (1); (non-totally geodesic) totally umbilical in case (2); (non-totally geodesic) cylindrical in case (3); the cases (1), (2) and (3) together cover the hypersurfaces of constant sectional curvature, i.e. the M^n in E^{n+1} which are real space forms (which for these hypersurfaces is equivalent to being Einstein) and cases (1) and (3) deal with the locally flat hypersurfaces; semi-symmetric hypersurfaces which are not real space forms concern the cases (4) and (5), so that the semi-symmetric hypersurfaces M^n of E^{n+1} correspond to $(1) - (5)$, as shown by Nomizu; conformally flat M^n, for $n > 3$, which are not of constant curvature correspond to case (6); and the intrinsic pseudosymmetric M^n in E^{n+1} correspond to $(1) - (7)$. So, a hypersurface M^n in E^{n+1} is a non semi-symmetric, intrinsically pseudo-symmetric Riemannian manifold if and only if it has exactly two non-zero principal curvatures λ and μ, and then its double sectional curvature is given by $L = \lambda$ [9].

In full analogy with the above mentioned studies concerning the parallel transport around infinitesimal co-ordinate parallelograms of the Riemann curvatures leading to the notions of pseudo-symmetry of Riemannian manifolds and of the sectional curvatures of Deszcz, the consideration of the Weyl conformal curvatures and of the Ricci curvatures instead lead to the notions of Weyl and Ricci pseudo-symmetry and of the Weyl and Ricci curvatures of Deszcz.

On the other than, in mathematical physics, lightlike hypersurfaces of semi-

Riemannian manifolds are important due to their physical applications. Moreover, in physics, lightlike hypersurfaces are interesting in general relativity since they produce models of different types of horizons. Lightlike hypersurfaces are also studied in the theory of electromagnetism. On the other hand, lightlike hypersurfaces of a semi-Riemannian manifold have been studied by Duggal-Bejancu and they obtain a transversal bundle for such hypersurfaces to overcome anomaly occured due to degenerate metric. After their book [7], many authors studied lightlike hypersurfaces by using their approach. In [20], Şahin has introduced the notion of semi-symmetric lightlike hypersurfaces of a semi-Riemannian manifold and obtained many new results. After Şahin's paper, many authors have studied such surfaces in various semi-Riemannian manifolds (see [11], [16], [19]). Moreover, in [12] Kazan and Şahin have studied pseudosymmetric lightlike hypersurfaces of semi-Riemannian manifolds and, in [13] have studied pseudosymmetric lightlike hypersurfaces in indefinite Sasakian space forms.

In this paper, considering the above statements, we study pseudosymmetric null hypersurfaces of an indefinite Kenmotsu space form such that its sectional curvature $c = -1$. In Section 3, firstly, we obtain integrability conditions for screen distribution of a null hypersurface and then we find sufficient condition for a null hypersurface to be pseudosymmetric under integrable screen distribution. We also give a characterization of a pseudosymmetric null hypersurface of indefinite Kenmotsu space form. In Section 4, we give sufficient condition for a null hypersurface to be C-Bochner pseudosymmetric in an indefinite Kenmotsu space form such that there are many papers deal with the C-Bochner tensor (see [10], [26]). And, we give an example for C-Bochner pseudosymmetric null hypersurfaces. Moreover, we give a result for the C-Bochner pseudosymmetric null hypersurfaces to be C-Bochner semi-symmetric and show that there is a close relationship of the C-Bochner semi-symmetry condition of a null hypersurface and its integrable screen distribution. Furthermore, we introduce Ricci-generalized C-Bochner pseudosymmetric null hypersurfaces.

2. Preliminaries

Let (M, g) be a connected n-dimensional, $n \geq 3$, semi-Riemannian manifold of class C^∞. For a $(0, k)$-tensor field T on M, $k \geq 1$, we define the $(0, k+2)$-tensors $R \cdot T$ and $Q(g, T)$ by

$$\begin{aligned}(R \cdot T)(X_1, ..., X_k; X, Y) &= -T(\tilde{R}(X, Y)X_1, X_2, ..., X_k) \\ &- ... - T(X_1, ..., X_{k-1}, \tilde{R}(X, Y)X_k), \qquad (2)\end{aligned}$$

and

$$\begin{aligned} Q(g,T)(X_1,...,X_k;X,Y) &= -T((X\wedge Y)X_1,X_2,...,X_k) \\ &- \quad ... - T(X_1,...,X_{k-1},(X\wedge Y)X_k) \end{aligned} \tag{3}$$

respectively, for $X_1,...,X_k,X,Y \in \Gamma(TM)$, where $\tilde{R}$ is the curvature tensor field of M and R is the Riemannian Christoffel tensor field given by $R(X_1,X_2,X_3,X_4) = g(\tilde{R}(X_1,X_2)X_3,X_4)$, the endomorphisms are defined by $\tilde{R}(X,Y)Z = [\nabla_X,\nabla_Y]Z - \nabla_{[X,Y]}Z, (X\wedge Y)Z = g(Y,Z)X - g(X,Z)Y$. Curvature conditions, involving the form $R\cdot T = 0$, are called curvature conditions of semi-symmetric type [5]. Then, a semi-Riemannian manifold (M,g) is said to be *semi-symmetric* if it satisfies the condition $R\cdot R = 0$. It is well known that the class of semisymmetric manifolds includes the set of locally symmetric manifolds ($\nabla R = 0$) as a proper subset [2], here, we suppose that (M,g) is a Riemmanian manifold. If M satisfies $\nabla R = 0$, then M is called *locally symmetric manifold* [17]. A semi-Riemannian manifold (M,g) is said to be a *pseudosymmetric manifold*, if at every point of M the tensor $R\cdot R$ and $Q(g,R)$ are linearly dependet. This is equivalent to the fact that the equality $R\cdot R = L_R Q(g,R)$ holds on $U_R = \{x \in M : Q(g,R) \neq 0\}$, for some function L_R on U_R [6].

Also, if the tensor $R\cdot R$ and $Q(S,R)$ are linearly dependent then M is called *Ricci − generalized pseudosymmetric*. This is equivalent to $R\cdot R = LQ(S,R)$ holding on the set $U = \{x \in M : Q(S,R) \neq 0\}$, for some function L on U, where S is the Ricci tensor [18].

Matsumoto and Chuman [25] have defined the C-Bochner curvature tensor in an almost contact metric manifold as follows:

$$\begin{aligned} &\tilde{B}(X,Y)Z = R(X,Y)Z + \frac{1}{2(n+2)}[S(X,Z)Y - S(Y,Z)X + g(X,Z)QY - g(Y,Z)QX \\ &+S(\phi X,Z)\phi Y - S(\phi Y,Z)\phi X + g(\phi X,Z)Q\phi Y - g(\phi Y,Z)Q\phi X + 2S(\phi X,Y)\phi Z \\ &+2g(\phi X,Y)Q\phi Z - S(X,Z)\eta(Y)\xi + S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY + \eta(Y)\eta(Z)QX] \\ &-\frac{\tau+2n}{2(n+2)}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X + 2g(\phi X,Y)\phi Z] - \frac{\tau-4}{2(n+2)}[g(X,Z)Y \\ &-g(Y,Z)X] + \frac{\tau}{2(n+2)}[g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi + \eta(X)\eta(Z)Y \\ &-\eta(Y)\eta(Z)X], \end{aligned} \tag{4}$$

where $\tau = \frac{r+2n}{2(n+2)}$, Q is the Ricci-operator, i.e., $g(QX,Y) = S(X,Y)$ for all X and Y and r is the scalar curvature of the manifold.

A Riemannian manifold M is said to be *C-Bochner pseudosymmetric* if $R\cdot\tilde{B} = L_{\tilde{B}}Q(g,\tilde{B})$ holds on the set $U_{\tilde{B}} = \{x \in M : \tilde{B} \neq 0 \ at \ x\}$, where $L_{\tilde{B}}$ is some function on $U_{\tilde{B}}$ and $\tilde{B}$ is the C-Bochner curvature tensor. If $L_{\tilde{B}} = 0$ on $U_{\tilde{B}}$, then a C-Bochner pseudosymmetric manifold is *C-Bochner semi-symmetric* [10].

Theorem 1. (Duggal-Bejancu) Let $(M, g, S(TM))$ be a lightlike hypersurface of $(\bar{M}, \bar{g})$. Then there exist a unique vector bundle $tr(TM)$ of rank 1 over M such that for any non-zero section ξ of $T^{\perp}M$ on a coordinate neighborhood $U \subset M$, there exists a unique section N of $tr(TM)$ on U

$$\bar{g}(\xi, N) = 1, \bar{g}(N, N) = \bar{g}(N, X) = 0, \forall X \in \Gamma(S(TM|_U)). \tag{5}$$

It follows from (5) that $tr(TM)$ is a lightlike vector bundle such that $tr(TM)_x \cap T_xM = \{0\}$ for any $x \in M$. Thus, from Theorem(1), we have

$$T\bar{M}|_M = S(TM) \oplus (TM^{\perp} \oplus tr(TM)) = TM \oplus tr(TM). \tag{6}$$

Here, the complementary (non-orthogonal) vector bundle $tr(TM)$ to the tangent bundle TM in $T\bar{M}|_M$ is called *the lightlike transversal bundle* of M with respect to screen distribution $S(TM)$ [7].

Suppose ∇ and $\bar{\nabla}$ are the Levi-Civita connections of M lightlike hypersurface and $\bar{M}$ semi-Riemannian manifold, respectively. According to the (6), we have

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \text{ and } \bar{\nabla}_X N = -A_N X + \nabla^t_X N, \tag{7}$$

for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(tr(TM))$, where
$\nabla_X Y, A_N X \in \Gamma(TM)$ and $h(X, Y), \nabla^t_X N \in \Gamma(tr(TM))$. If we set $B(X, Y) = g(h(X, Y), \xi)$ and $\tau(X) = \bar{g}(\nabla^t_X N, \xi)$, then, from (7), we have

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N \text{ and } \bar{\nabla}_X N = -A_N X + \tau(X)N, \tag{8}$$

for any $X, Y \in \Gamma(TM)$, $N \in \Gamma(tr(TM))$, A_N and B are called *the shape operator* and *the second fundamental form* of the lightlike hypersurface M, respectively.

Let P be the projection of $\Gamma(TM)$ on $\Gamma(S(TM))$. Then, we have

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi \text{ and } \nabla_X \xi = -A^*_{\xi} X + \tau(X)\xi, \tag{9}$$

for any $X, Y \in \Gamma(TM)$, where $\nabla^*_X PY, A^*_{\xi} X \in \Gamma(S(TM))$ and C is a 1-form on U defined by $C(X, PY) = \bar{g}(\nabla_X PY, N)$. $C, A^*_{\xi} X$ and ∇^* are called *the local second fundamental form*, *the local shape operator* and *the induced connection* on $S(TM)$, respectively. Then, we have the following assertions,

$$g(A_N Y, PW) = C(Y, PW), g(A_N Y, N) = 0, B(X, \xi) = 0, \tag{10}$$

$$g(A^*_{\xi} X, PY) = B(X, PY), g(A^*_{\xi} X, N) = 0, \tag{11}$$

for $X, Y, W \in \Gamma(TM), \xi \in \Gamma(TM^{\perp})$ and $N \in \Gamma(tr(TM))$.

Let M be a lightlike hypersurface of a $\bar{M}$ semi-Euclidean space. Denote by $\bar{R}$ and R the Riemann curvature tensors of $\bar{M}$ and M, respectively. From Gauss-Codazzi equations [7], we have the following, for any $X, Y, Z \in \Gamma(TM_{|U})$,

$$\bar{R}(X,Y)Z = R(X,Y)Z + B(X,Z)A_N Y - B(Y,Z)A_N X$$
$$+ \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + \tau(X)B(Y,Z) - \tau(Y)B(X,Z)\}N. \qquad (12)$$

Let M be a lightlike hypersurface of a semi-Euclidean space. We say that M is a *semi-symmetric* if the condition $(R(X,Y) \cdot R)(X_1, X_2, X_3, X_4) = 0$ for any $X, Y, X_1, X_2, X_3, X_4 \in \Gamma(TM)$ [20] is satisfied, also a lightlike hypersurface M is called *Ricci semi-symmetric* lightlike hypersurface if the condition $(R(X,Y) \cdot Ric)(X_1, X_2) = 0$ for any $X, Y, X_1, X_2 \in \Gamma(TM)$ [20] is satisfied.

For the geometry of lightlike hypersurfaces, we refer to [7], [8], [20].

Also , let us recall some general notions about indefinite Kenmotsu space manifolds:

Let $\bar{M}$ be a (2m+1)-dimensional manifold endowed with an *almost contact structure* $(\bar{\phi}, \xi, \eta)$, i.e. $\bar{\phi}$ is a tensor field of type (1,1), ξ is a vector field and η is a 1-form satisfying

$$\bar{\phi}^2 = -I + \eta \otimes \xi, \;\; \eta(\xi) = 1, \;\; \eta \circ \phi = 0 \text{ and } \phi\xi = 0. \qquad (13)$$

Then $(\bar{\phi}, \xi, \eta, \bar{g})$ is called an *indefinite almost contact metric structure* on $\bar{M}$, if $(\bar{\phi}, \xi, \eta)$ is an almost contact structure on $\bar{M}$ and $\bar{g}$ is a semi-Riemannian metric on $\bar{M}$ such that, for any vector field $\bar{X}, \bar{Y}$ on $\bar{M}$,

$$\bar{g}(\bar{\phi}\bar{X}, \bar{\phi}\bar{Y}) = \bar{g}(\bar{X}, \bar{Y}) - \eta(\bar{X})\eta(\bar{Y}). \qquad (14)$$

If follows that, for any vector field $\bar{X}$ on $\bar{M}$, $\eta(\bar{X}) = \bar{g}(\xi, \bar{X})$. If, moreover, $(\bar{\nabla}_{\bar{X}}\bar{\phi})\bar{Y} = \bar{g}(\bar{\phi}\bar{X}, \bar{Y})\xi - \eta(\bar{Y})\bar{\phi}\bar{X}$, where $\bar{\nabla}$ is the Levi-Civita connection for the semi-Riemannian metric $\bar{g}$, we call $\bar{M}$ an *indefinite Kenmotsu manifold* [16].

Since Takahashi [22] shows that it suffices to consider indefinite almost contact manifolds with space-like ξ [11]. In this paper, we will restrict ourselves to the case of ξ a space-like unit vector (that is $\bar{g}(\xi, \xi) = 1$).

A plane section σ in $T_p\bar{M}$ is called a *$\bar{\phi}$-section* if it is spanned by $\bar{X}$ and $\bar{\phi}\bar{X}$, where $\bar{X}$ is a unit tangent vector field orthogonal to ξ. The sectional curvature of a $\bar{\phi}$-section σ is called a *$\bar{\phi}$-sectional curvature*. If an indefinite Kenmotsu manifold $\bar{M}$ has constant $\bar{\phi}$-sectional curvature c, then, by virtue of the Propositon 12 in [14], the curvature tensor $\bar{R}$ of $\bar{M}$ is given by

$$\bar{R}(\bar{X}, \bar{Y})\bar{Z} = \frac{c-3}{4}\{g(\bar{Y}, \bar{Z})\bar{X} - g(\bar{X}, \bar{Z})\bar{Y}\} + \frac{c+1}{4}\{\eta(\bar{X})\eta(\bar{Z})\bar{Y}$$
$$-\eta(\bar{Y})\eta(\bar{Z})\bar{X} + g(\bar{X}, \bar{Z})\eta(Y)\xi - g(\bar{Y}, \bar{Z})\eta(\bar{X})\xi + \bar{g}(\bar{\phi}\bar{Y}, \bar{Z})\bar{\phi}\bar{X}$$
$$-\bar{g}(\bar{\phi}\bar{X}, \bar{Z})\bar{\phi}Y - 2\bar{g}(\bar{\phi}\bar{X}, \bar{Y})\bar{\phi}\bar{Z}\}, \qquad (15)$$

where $\bar{X}, \bar{Y}, \bar{Z} \in \Gamma(T\bar{M})$. A Kenmotsu manifold $\bar{M}$ of constant $\bar{\phi}$-sectional curvature c will be called *Kenmotsu space form* and denote by $\bar{M}(c)$. If an indefinite Kenmotsu manifold $\bar{M}$ has a constant $\bar{\phi}$-sectional curvature c, then, $\bar{M}$ is an *Einstein* one and $c = -1$. This means that, it is locally isometric to the pseudo hyperbolic space $H_s^{2n+1}(-1)$, s being the index of its metric [16].

Example 1. (Example 1-3 of [15] and [16]) Let $\bar{M} = \{x = (x_1, x_2, ..., x_7) \in \mathbb{R}^7 : x_7 > 0\}$ be 7-dimensional indefinite Kenmotsu manifold. The vector fields, $e_p = x_7 \frac{\partial}{\partial x_p}$, $e_q = -x_7 \frac{\partial}{\partial x_q}$, for any $p = 1, 2, 3, 4,\ , q = 5, 6, 7$ are linearly independent at each point of $\bar{M}^7$. Also let M be a hypersurface of $(\bar{M}, \bar{\phi}, \xi, \eta, \bar{g})$ given by $x_5 = \sqrt{2}(x_2 + x_3)$. Thus, the tangent space TM is spanned by $\{U_i\}_{1 \leq i \leq 6}$, where $U_1 = e_1$, $U_2 = e_2 - e_3$, $U_3 = \frac{1}{\sqrt{2}}(e_2 + e_3) - e_5$, $U_4 = e_4$, $U_5 = e_6$, $U_6 = \xi$ and the 1- dimensional distribution $TM^{\perp}$ of rank 1 is spanned by E, where $E = U_3$. It follows that $TM^{\perp} \subset TM$. Then M is a 6- dimensional lightlike hypersurface of $\bar{M}$. $N(TM)$ is spanned by $N = \frac{1}{2}\{\frac{1}{\sqrt{2}}(e_2 + e_3) + e_5\}$ and the distribution D_0, $< \xi >$, $\bar{\phi}(TM^{\perp})$ and $\bar{\phi}(N(TM))$ are spanned, respectively, by $\{F = U_2, \bar{\phi}F = U_1 + U_4\}$, ξ, $\bar{\phi}E = \frac{1}{\sqrt{2}}(U_1 - U_4) + U_5$ and $\bar{\phi}N = \frac{1}{2}\{\frac{1}{\sqrt{2}}(U_1 - U_4) - U_5\}$. Denote by $\bar{\nabla}$ the Levi-Civita connection on $\bar{M}$. Then, we obtain

$$\bar{\nabla}_{U_3} N = -\xi \text{ and } \bar{\nabla}_X N,\ \forall X \in \Gamma(TM),\ X \neq U_3.$$

Using these equations above, the differential 1-form τ vanishes i.e. $\tau(X) = 0$, for any $X \in \Gamma(TM)$. So, from Gauss and Weingarten formulas we have

$$A_N U_2 = \xi,\ A_N X = 0,\ \forall X \in \Gamma(TM),\ X \neq U_2 \tag{16}$$

$$A_E^* X = 0,\ \nabla_X E = 0,\ \forall X \in \Gamma(TM). \tag{17}$$

From (16) and (17), $C(U_2, \xi) = 1$ and $trA_E^* = 0$. Therefore, the hypersurface M of $\bar{M}$ is totally geodesic and its screen distribution is not parallel. Using Lemma 3.2 of [15], $\bar{M}$, endowed with the structure $(\bar{\phi}, \xi, \eta, \bar{g})$ defined in Example1 of [15], is of constant curvature $c = -1$.

Theorem 2. Let M be a null hypersurface of an indefinite Kenmotsu space form $\bar{M}(c)$, with $\xi \in \Gamma(TM)$. If the second fundamental form h of M is parallel, then M is totally geodesic [16].

3. Pseudosymmetric Null Hypersurfaces in Indefinite Kenmotsu Space Forms

In this section, we investigate pseudosymmetric null hypersurfaces in an indefinite Kenmotsu space form. Firstly, let us recall some general notions about null hypersurfaces of indefinite Kenmotsu manifolds:

Let$(\bar{M}, \bar{\phi}, \xi, \eta, \bar{g})$ be an indefinite Kenmotsu manifold and let (M, g) be a null hypersurface of $(\bar{M}, \bar{g})$, tangent to the structure vector field $\xi \in \Gamma(TM)$. If E is a local section of $TM^{\perp}$, it is easy to check that $\bar{\phi}E \neq 0$ and $\bar{g}(\bar{\phi}E, E) = 0$, then $\bar{\phi}E$ is tangent to M. Thus $\bar{\phi}(TM^{\perp})$ is a distribution on M of rank 1 such that $\bar{\phi}(TM^{\perp}) \cap TM^{\perp} = \{0\}$. This enables us to choose a screen distribution $S(TM)$ such that it contains $\bar{\phi}(TM^{\perp})$ as a vector subbundle. If we consider a local section N of $N(TM)$, we have $\bar{\phi}N \neq 0$. Since $\bar{g}(\bar{\phi}N, E) = -\bar{g}(N, \bar{\phi}E) = 0$, we deduce that $\bar{\phi}E \in \Gamma(S(TM))$ and $\bar{\phi}N$ is also tangent to M. At the same time, $\bar{g}(\bar{\phi}N, N) = 0$ i.e. $\bar{\phi}N$ has no component with respect to E. Thus $\bar{\phi}N \in \Gamma(S(TM))$, that is, $\bar{\phi}(N(TM))$ is also a vector subbundle of $S(TM)$ of rank 1. From (13), we have $\bar{g}(\bar{\phi}N, \bar{\phi}E) = 1$. Therefore, $\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(tr(TM))$ is a non-degenerate vector subbundle of $S(TM)$ of rank 2. If $\xi \in TM$, we may choose $S(TM)$ so that ξ belogns to $S(TM)$. Using this and since $\bar{g}(\bar{\phi}E, \xi) = \bar{g}(\bar{\phi}N, \xi) = 0$, there exists a non-degenerate distribution D_0 of rank $2n-4$ on M such that

$$S(TM) = \{\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(tr(TM))\} \perp D_0 \perp < \xi > \tag{18}$$

where $< \xi >$ is the distribution spanned by ξ. The distribution D_0 is invariant under $\bar{\phi}$, i.e. $\bar{\phi}(D_0) = D_0$. Moreover, from (6) and (18) , we obtain the decompositions

$$TM = \{\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(tr(TM))\} \perp D_0 \perp < \xi > \perp TM^{\perp}, \tag{19}$$

$$T\bar{M}|_M = \{\bar{\phi}(TM^{\perp}) \oplus \bar{\phi}(tr(TM))\} \perp D_0 \perp < \xi > \perp (TM^{\perp} \oplus tr(TM)). \tag{20}$$

Now, we consider the distributions on M, $D := TM^{\perp} \perp \bar{\phi}(TM^{\perp}) \perp D_0$, $D' := \bar{\phi}(tr(TM))$. Then D is invariant under $\bar{\phi}$ and

$$TM = (D \oplus D') \perp < \xi > . \tag{21}$$

Let us consider the local null vector fields $U := -\bar{\phi}N$, $V := -\bar{\phi}E$. Then, from (21), any $X \in \Gamma(TM)$ is written as $X = RX + QX + \eta(X)\xi$, $QX = u(X)U$, where R and Q are the projection morphisms of TM into D and D', respectively, and u is a differential 1-form locally defined on M by $u(\cdot) = g(V, \cdot)$. Applying $\bar{\phi}$ and (13), one obtain $\bar{\phi}X = \phi X + u(X)N$, where $\bar{\phi}$ is a tensor field of type $(1, 1)$ defined on M by $\phi X := \bar{\phi}RX$. In addition, we obtain, $\phi^2 X = -X + \eta(X)\xi + u(X)U$ and $\nabla_X \xi = X - \eta(X)\xi$. We have the following identities, for any $X \in \Gamma(TM)$, $\nabla_X \xi = X - \eta(X)\xi$ and

$$B(X, \xi) = 0, \ C(X, \xi) = \theta(X) \tag{22}$$

Define the induced Ricci type tensor $R^{(0,2)}$ of M as

$$R^{(0,2)}(X, Y) = trace(Z \to R(Z, X)Y), \ \forall X, Y \in \Gamma(TM). \tag{23}$$

Since the induced connection ∇ on M is not a Levi-Civita connection, in general, $R^{(0,2)}$ is not symmetric. Therefore, in general, it is just a tensor quantity and has no geometric or physical meaning similar to the symmetric Ricci tensor of $\bar{M}$. If $\bar{M}$ is an indefinite Kenmotsu space form $(\bar{M}(c), \bar{g})$, then, the relation (15) becomes, for any $X, Y, Z \in \Gamma(TM)$,

$$\bar{R}(X,Y)Z = g(X,Z)Y - g(Y,Z)X. \tag{24}$$

Using (12), a direct calculation gives

$$R^{(0,2)}(X,Y) = -(2n-1)g(X,Y) + B(X,Y)trA_N - B(A_N X, Y), \tag{25}$$

where tracetr is written with respect to g restricted to $S(TM)$. Note that the Ricci tensor does not depend on the choice of the vector field E of the distribution $TM^{\perp}$. The tensor field $R^{(0,2)}$ of a null hypersurface M of an indefinite Kenmotsu manifold $\bar{M}$ is called induced Ricci tensor if it is symmetric. Let (M, g) be a null hypersurface of an indefinite Kenmotsu space form $(\bar{M}(c), \bar{g})$ with $\xi \in TM$. Let us consider the pair $\{E, N\}$ on $U \subset M$. From (12), comparing the tangential and transversal part, we get, for any $X, Y, Z \in \Gamma(TM)$

$$R(X,Y)Z = g(X,Z)Y - g(Y,Z)X + B(Y,Z)A_N X - B(X,Z)A_N Y \tag{26}$$

[16]. And, let $(M, g, S(TM))$ be a screen integrable null hypersurface of an indefinite Kenmotsu space form $\bar{M}(c)$ with $\xi \in \Gamma(TM)$. Using Gauss and Weingarten equations, we have

$$\begin{aligned} R(X,Y)Z &= R^*(X,Y)Zg(X,Z)Y + C(X,Z)A^*_E Y - C(Y,Z)A^*_E X \\ &+\{(\nabla_X C)(Y,Z) - (\nabla_Y C)(X,Z) + \tau(Y)C(X,Z) - \tau(X)C(Y,Z)\}E \end{aligned} \tag{27}$$

where $X, Y, Z \in \Gamma(S(TM))$ and $(\nabla_X C)(Y,Z) = X(C(Y,Z)) - C(\nabla^*_X Y, Z) - C(Y, \nabla^*_X Z)$ [16].

For symmetry properties of null hypersurfaces of indefinite Kenmotsu manifolds, we refer to [15], [16], [24].

Now, we can give main definition:

Definition 1. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. We say that M is a pseudosymmetric null hypersurface, if the tensors of $R \cdot R$ and $Q(g, R)$ are linearly dependent at $\forall p \in M$. This is equivalent to $R \cdot R = L_R Q(g, R)$ on $U_R = \{p \in M | Q(g, R) \neq 0\}$, where L_R is some function on U_R.

A condition for integrable of screen distribution of M to be a null hypersurface in indefinite Kenmotsu space form is given by following theorem:

Lemma 3. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. Then $S(TM)$ integrable if and only if

$$g(\nabla^*_X \phi Y - \nabla^*_Y \phi X, \phi N) = g(u(Y)A_N X - u(X)A_N Y, \phi N).$$

Proof. For $X, Y \in \Gamma S(TM)$, using (13) and (14) , we have

$$\bar{g}([X,Y],N) = \bar{g}(\bar{\nabla}_X \bar{\phi} Y, \bar{\phi} N) + \eta(Y) g(\bar{\phi} X, \bar{\phi} N) - \bar{g}(\bar{\nabla}_Y \bar{\phi} X, \bar{\phi} N) - \eta(X) g(\bar{\phi} Y, \bar{\phi} N).$$

On the other hand, using $\bar{\phi} Y = \phi Y + u(Y) N$ and Gauss formulas (8) and (9), we get

$$\bar{g}([X,Y],N) = g(\nabla^*_X \phi Y - u(Y) g(A_N X, \phi N) - g(\nabla^*_Y \phi X + u(X) g(A_N Y, \phi N).$$

Thus, proof is complete.

Now, we can give the following theorem for a condition to be pseudosymmetric of a null hypersurface in indefinite Kenmotsu space form:

Theorem 4. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with integrable screen distribution and $\xi \in \Gamma(TM)$. If $B(X,Y)A_N^2 Z = -g(X,Y)A_N Z$, $B(X,Y)A_E^* A_N Z = -g(X,Y)A_E^* Z$ and $C(X,Y)Z = C(X,Z)Y$, then M is a pseudosymmetric null hypersurface such that $L_R = -2$, where $X, Y, Z \in \Gamma(TM)$, $E \in \Gamma(RadTM)$.

Proof. From the hypothesis, for $X, Y, Z, W, U \in \Gamma(TM)$, we get

$$\begin{aligned} B(X,Y)A_N^2 Z &= -g(X,Y)A_N Z \Rightarrow g(B(X,Y)A_N^2 Z, W) = -g(g(X,Y)A_N Z, W) \\ &\Rightarrow B(X,Y) g(A_N^2 Z, W) = -g(X,Y) g(A_N Z, W) \end{aligned} \tag{28}$$

and

$$\begin{aligned} B(X,Y)A_\xi^* A_N Z &= -g(X,Y)A_\xi^* Z \Rightarrow g(B(X,Y)A_\xi^* A_N Z, U) = -g(g(X,Y)A_\xi^* Z, U) \\ &\Rightarrow B(X,Y) B(A_N Z, U) = -g(X,Y) B(Z,U). \end{aligned} \tag{29}$$

Since M is a Kenmotsu space form, $c = -1$. Using $c = -1$ and from (26), we have

$$\begin{aligned} &(R \cdot R)(X_1, X_2, X_3, X_4; X, Y) \\ &= -g(X,X_1)g(Y,X_3)g(X_2,X_4) + g(X,X_1)g(X_2,X_3)g(Y,X_4) \\ &+ g(X,X_1)B(Y,X_3)g(A_N X_2, X_4) - g(X,X_1)B(X_2,X_3)g(A_N Y, X_4) \\ &+ g(Y,X_1)g(X,X_3)g(X_2,X_4) - g(Y,X_1)g(X_2,X_3)g(X,X_4) \\ &- g(Y,X_1)B(X,X_3)g(A_N X_2, X_4) + g(Y,X_1)B(X_2,X_3)g(A_N X, X_4) \\ &+ B(X,X_1)g(A_N Y, X_3)g(X_2,X_4) - B(X,X_1)g(X_2,X_3)g(A_N Y, X_4) \\ &- B(X,X_1)B(A_N Y, X_3)g(A_N X_2, X_4) + B(X,X_1)B(X_2,X_3)g(A_N^2 Y, X_4) \end{aligned}$$

$$
\begin{aligned}
&- B(Y,X_1)g(A_NX,X_3)g(X_2,X_4) + B(Y,X_1)g(X_2,X_3)g(A_NX,X_4)\\
&+ B(Y,X_1)B(A_NX,X_3)g(A_NX_2,X_4) - B(Y,X_1)B(X_2,X_3)g(A_N^2X,X_4)\\
&- g(X,X_2)g(X_1,X_3)g(Y,X_4) + g(X,X_2)g(Y,X_3)g(X_1,X_4)\\
&+ g(X,X_2)B(X_1,X_3)g(A_NY,X_4) - g(X,X_2)B(Y,X_3)g(A_NX_1,X_4)\\
&+ g(Y,X_2)g(X_1,X_3)g(X,X_4) - g(Y,X_2)g(X,X_3)g(X_1,X_4)\\
&- g(Y,X_2)B(X_1,X_3)g(A_NX,X_4) + g(Y,X_2)B(X,X_3)g(A_NX_1,X_4)\\
&+ B(X,X_2)g(X_1,X_3)g(A_NY,X_4) - B(X,X_2)g(A_NY,X_3)g(X_1,X_4)\\
&- B(X,X_2)B(X_1,X_3)g(A_N^2Y,X_4) + B(X,X_2)B(A_NY,X_3)g(A_NX_1,X_4)\\
&- B(Y,X_2)g(X_1,X_3)g(A_NX,X_4) + B(Y,X_2)g(A_NX,X_3)g(X_1,X_4)\\
&+ B(Y,X_2)B(X_1,X_3)g(A_N^2X,X_4 - B(Y,X_2)B(A_NX,X_3)g(A_NX_1,X_4))\\
&- g(X,X_3)g(X_1,Y)g(X_2,X_4) + g(X,X_3)g(X_2,Y)g(X_1,X_4)\\
&+ g(X,X_3)B(X_1,Y)g(A_NX_2,X_4) - g(X,X_3)B(X_2,Y)g(A_NX_1,X_4)\\
&+ g(Y,X_3)g(X_1,X)g(X_2,X_4 - g(Y,X_3)g(X_2,X)g(X_1,X_4))\\
&- g(Y,X_3)B(X_1,X)g(A_NX_2,X_4) + g(Y,X_3)B(X_2,X)g(A_NX_1,X_4)\\
&+ B(X,X_3)g(X_1,A_NY)g(X_2,X_4) - B(X,X_3)g(X_2,A_NY)g(X_1,X_4)\\
&- B(X,X_3)B(X_1,A_NY)g(A_NX_2,X_4) + B(X,X_3)B(X_2,A_NY)g(A_NX_1,X_4)\\
&- B(Y,X_3)g(X_1,A_NX)g(X_2,X_4) + B(Y,X_3)g(X_2,A_NX)g(X_1,X_4)\\
&+ B(Y,X_3)B(X_1,A_NX)g(A_NX_2,X_4) - B(Y,X_3)B(X_2,A_NX)g(A_NX_1,X_4)\\
&- g(X,X_4)g(X_1,X_3)g(X_2,Y) + g(X,X_4)g(X_2,X_3)g(X_1,Y)\\
&+ g(X,X_4)B(X_1,X_3)g(A_NX_2,Y) - g(X,X_4)B(X_2,X_3)g(A_NX_1,Y)\\
&+ g(Y,X_4)g(X_1,X_3)g(X_2,X) - g(Y,X_4)g(X_2,X_3)g(X_1,X)\\
&- g(Y,X_4)B(X_1,X_3)g(A_NX_2,X) + g(Y,X_4)B(X_2,X_3)g(A_NX_1,X)\\
&+ B(X,X_4)g(X_1,X_3)g(X_2,A_NY) - B(X,X_4)g(X_2,X_3)g(X_1,A_NY)\\
&- B(X,X_4)B(X_1,X_3)g(A_NX_2,A_NY) + B(X,X_4)B(X_2,X_3)g(A_NX_1,A_NY)\\
&- B(Y,X_4)g(X_1,X_3)g(X_2,A_NX) + B(Y,X_4)g(X_2,X_3)g(X_1,A_NX)\\
&+ B(Y,X_4)B(X_1,X_3)g(A_NX_2,A_NX) - B(Y,X_4)B(X_2,X_3)g(A_NX_1,A_NX).
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
&(R\cdot R)(X_1,X_2,X_3,X_4;X,Y)\\
&= -Q(g,R)(X_1,X_2,X_3,X_4;X,Y)\\
&+ B(X,X_1)g(A_NY,X_3)g(X_2,X_4) - B(X,X_1)g(X_2,X_3)g(A_NY,X_4)\\
&- B(X,X_1)B(A_NY,X_3)g(A_NX_2,X_4) + B(X,X_1)B(X_2,X_3)g(A_N^2Y,X_4)
\end{aligned}
$$

$$
\begin{aligned}
&- B(Y,X_1)g(A_NX,X_3)g(X_2,X_4) + B(Y,X_1)g(X_2,X_3)g(A_NX,X_4)\\
&+ B(Y,X_1)B(A_NX,X_3)g(A_NX_2,X_4) - B(Y,X_1)B(X_2,X_3)g(A_N^2X,X_4)\\
&+ B(X,X_2)g(X_1,X_3)g(A_NY,X_4) - B(X,X_2)g(A_NY,X_3)g(X_1,X_4)\\
&- B(X,X_2)B(X_1,X_3)g(A_N^2Y,X_4) + B(X,X_2)B(A_NY,X_3)g(A_NX_1,X_4)\\
&- B(Y,X_2)g(X_1,X_3)g(A_NX,X_4) + B(Y,X_2)g(A_NX,X_3)g(X_1,X_4)\\
&+ B(Y,X_2)B(X_1,X_3)g(A_N^2X,X_4) - B(Y,X_2)B(A_NX,X_3)g(A_NX_1,X_4)\\
&+ B(X,X_3)g(X_1,A_NY)g(X_2,X_4) - B(X,X_3)g(X_2,A_NY)g(X_1,X_4)\\
&- B(X,X_3)B(X_1,A_NY)g(A_NX_2,X_4) + B(X,X_3)B(X_2,A_NY)g(A_NX_1,X_4)\\
&- B(Y,X_3)g(X_1,A_NX)g(X_2,X_4) + B(Y,X_3)g(X_2,A_NX)g(X_1,X_4)\\
&+ B(Y,X_3)B(X_1,A_NX)g(A_NX_2,X_4) - B(Y,X_3)B(X_2,A_NX)g(A_NX_1,X_4)\\
&+ B(X,X_4)g(X_1,X_3)g(X_2,A_NY) - B(X,X_4)g(X_2,X_3)g(X_1,A_NY)\\
&- B(X,X_4)B(X_1,X_3)g(A_NX_2,A_NY) + B(X,X_4)B(X_2,X_3)g(A_NX_1,A_NY)\\
&- B(Y,X_4)g(X_1,X_3)g(X_2,A_NX) + B(Y,X_4)g(X_2,X_3)g(X_1,A_NX)\\
&+ B(Y,X_4)B(X_1,X_3)g(A_NX_2,A_NX) - B(Y,X_4)B(X_2,X_3)g(A_NX_1,A_NX), \quad (30)
\end{aligned}
$$

where $X,Y,X_1,X_2,X_3,X_4 \in \Gamma(TM)$. Here, from the hypothesis and using (28), (29) and $C(X,Y)Z = C(X,Z)Y$, we obtain

$$(R \cdot R)(X_1,X_2,X_3,X_4;X,Y) = -2Q(g,R)(X_1,X_2,X_3,X_4;X,Y).$$

Thus, the proof is complete.

Lemma 5. Let M be a null hypersurface of an indefinite Kenmotsu manifold $\bar{M}$. For an orthonormal basis $\{z_1,...,z_{m-2},...,z_{2m-4},\xi,E,\phi E,\phi N\}$ of $\Gamma(TM)$ such that $\phi z_i = z_{m-2+i}, \phi z_{m-2+i} = -z_i$ and $\phi = 0$ for every $i = 1,...,m-2$ and $j = 1,...,n$,

$$A_NE = \sum_{i=1}^{2m-4} \frac{C(E,z_i)}{\varepsilon_i} z_i + C(E,\xi)\xi + C(E,U)V \tag{31}$$

where $\{\varepsilon_i\}$ is the signature of the basis $\{z_i\}$ [23].

Here, we give sufficient conditions for a null hypersurface to be totally geodesic in indefinite Kenmotsu space form:

Theorem 6. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a pseudosymmetric $(L_R = -1)$ null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$ such that $B(X,V) \neq 0$, $\forall X \in \Gamma(TM)$. Then either M is totally geodesic or

$$g(g(A_E^*X,V)A_NY - g(A_E^*Y,V)A_NX, A_NE) = 0,$$

where $X, Y \in \Gamma(TM)$, $N \in \Gamma(tr(TM))$, $E \in \Gamma(Rad(TM))$.

Proof. Suppose that M is a pseudosymmetric null hypersurface of an indefinite Kenmotsu space form. Then, we have $c = -1$. So, for $X_1 \in \Gamma(Rad(TM))$ and $X_4 = V = -\bar{\phi}E$, we have

$$(R \cdot R)(E, X_2, X_3, -\bar{\phi}E; X, Y) = L_R Q(g, R)(E, X_2, X_3, -\bar{\phi}E; X, Y).$$

Thus, we get

$$\begin{aligned} -Q(g,R)(E, X_2, X_3, -\bar{\phi}E; X, Y) - B(X, X_2)B(A_N Y, X_3)g(A_N E, \bar{\phi}E) \\ +B(Y, X_2)B(A_N X, X_3)g(A_N E, \bar{\phi}E) - B(X, X_3)B(X_2, A_N Y)g(A_N E, \bar{\phi}E) \\ +B(Y, X_3)B(X_2, A_N X)g(A_N E, \bar{\phi}E) - B(X, \bar{\phi}E)B(X_2, X_3)g(A_N E, A_N Y) \\ +B(Y, \bar{\phi}E)B(X_2, X_3)g(A_N E, A_N X) - L_R Q(g,R)(E, X_2, X_3, -\bar{\phi}E; X, Y) = 0. \end{aligned}$$

Since ($L_R = -1$) and by using (31) in the above equation, from the hypothesis, we obtain

$$B(X_2, X_3)g(B(X, V)A_N Y - B(Y, V)A_N X, A_N E) = 0,$$

where $X, Y, X_1, X_2, X_3, X_4 \in \Gamma(TM)$. This completes the proof.

For totally geodesic pseudosimetric null hypersurface, we can give the following result:

Corollary 7. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a pseudosymmetric null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form. If M is totally geodesic, then M is semi-symmetric.

Proof. The proof is obvious from (30).

4. C-Bochner Pseudosymmetric Null Hypersurfaces in Indefinite Kenmotsu Space Forms

In this section, we investigate C-Bochner pseudosymmetric null hypersurfaces in an indefinite Kenmotsu space form.

Definition 2. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. We say that M is a *C-Bochner pseudosymmetric null hypersurface*, if the tensors of $R \cdot \tilde{B}$ and $Q(g, \tilde{B})$ are linearly dependent at $\forall p \in M$. This is equivalent to $R \cdot \tilde{B} = L_{\tilde{B}} Q(g, \tilde{B})$ on $U_{\tilde{B}} = \{p \in M | Q(g, \tilde{B}) \neq 0\}$, where $L_{\tilde{B}}$ is some function on $U_{\tilde{B}}$.

Now we give main result in the following:

Theorem 8. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. If M is totally geodesic, then M is a C-Bochner pseudosymmetric null hypersurface such that $L_{\tilde{B}} = -1$.

Proof. Using the curvature tensor of M, we get

$$\begin{aligned}
&(R(X,Y)\cdot\tilde{B})(U,V,W)\\
&= -\tilde{B}(R(X,Y)U,V)W - \tilde{B}(U,R(X,Y)V)W - \tilde{B}(U,V)R(X,Y)W\\
&= -g(X,U)\tilde{B}(Y,V)W + g(Y,U)\tilde{B}(X,V)W + B(X,U)\tilde{B}(A_NY,V)W - B(Y,U)\tilde{B}(A_NX,V)W\\
&- g(X,V)\tilde{B}(U,Y)W + g(Y,V)\tilde{B}(U,X)W + B(X,V)\tilde{B}(U,A_NY)W - B(Y,V)\tilde{B}(U,A_NX)W\\
&- g(X,W)\tilde{B}(U,V)Y + g(Y,W)\tilde{B}(U,V)X + B(X,W)\tilde{B}(U,V)A_NY - B(Y,W)\tilde{B}(U,V)A_NX,
\end{aligned}$$

where B is the second fundamental form of null hypersurface M. Since M is totally geodesic, then M is a Einstein null hypersurface of the Kenmotsu space form. Thus, using (4), we obtain

$$\begin{aligned}
&(R(X,Y)\cdot\tilde{B})(U,V,W) = (1-\frac{\tau-4}{2(n+2)})\{g(X,U)g(V,W)Y - g(Y,U)g(V,W)X\\
&- g(X,V)g(U,W)Y + g(Y,V)g(U,W)X\}\\
&+ \frac{1}{2(n+2)}\{\alpha g(X,U)g(V,W)Y + g(X,U)g(V,W)QY\\
&- \alpha g(X,U)g(\phi Y,W)\phi V + \alpha g(X,U)g(\phi V,W)\phi Y - g(X,U)g(\phi Y,W)Q\phi V\\
&+ g(X,U)g(\phi V,W)Q\phi Y - 2\alpha g(X,U)g(\phi Y,V)\phi W - 2g(X,U)g(\phi Y,V)Q\phi W\\
&- \alpha g(X,U)g(V,W)\eta(Y)\xi + g(X,U)\eta(Y)\eta(W)QV - g(X,U)\eta(V)\eta(W)QY\\
&- \alpha g(Y,U)g(V,W)X - g(Y,U)g(V,W)QX + \alpha g(Y,U)g(\phi X,W)\phi V\\
&- \alpha g(Y,U)g(\phi V,W)\phi X + g(Y,U)g(\phi X,W)Q\phi V - g(Y,U)g(\phi V,W)Q\phi X\\
&+ 2\alpha g(Y,U)g(\phi X,V)\phi W + 2g(Y,U)g(\phi X,V)Q\phi W + \alpha g(Y,U)g(V,W)\eta(X)\xi\\
&- g(Y,U)\eta(X)\eta(W)QV + g(Y,U)\eta(V)\eta(W)QX - \alpha g(X,V)g(U,W)Y\\
&- g(X,V)g(U,W)QY - \alpha g(X,V)g(\phi U,W)\phi Y + \alpha g(X,V)g(\phi Y,W)\phi U\\
&- g(X,V)g(\phi U,W)Q\phi Y + g(X,V)g(\phi Y,W)Q\phi U - 2\alpha g(X,V)g(\phi U,Y)\phi W\\
&- 2g(X,V)g(\phi U,Y)Q\phi W + \alpha g(X,V)g(U,W)\eta(Y)\xi + g(X,V)\eta(U)\eta(W)QY\\
&- g(X,V)\eta(Y)\eta(W)QU + \alpha g(Y,V)g(U,W)X + g(Y,V)g(U,W)QX\\
&+ \alpha g(Y,V)g(\phi U,W)\phi X - \alpha g(Y,V)g(\phi X,W)\phi U + g(Y,V)g(\phi U,W)Q\phi X
\end{aligned}$$

$$
\begin{aligned}
&- g(Y,V)g(\phi X,W)Q\phi U + 2\alpha g(Y,V)g(\phi U,X)\phi W + 2g(Y,V)g(\phi U,X)Q\phi W \\
&- \alpha g(Y,V)g(U,W)\eta(X)\xi + \alpha g(Y,V)g(X,W)\eta(U)\xi - g(Y,V)\eta(U)\eta(W)QX \\
&+ g(Y,V)\eta(X)\eta(W)QU - \alpha g(X,W)g(\phi U,Y)\phi V + \alpha g(X,W)g(\phi V,Y)\phi U \\
&- g(X,W)g(\phi U,Y)Q\phi V + g(X,W)g(\phi V,Y)Q\phi U - 2\alpha g(X,W)g(\phi U,V)\phi Y \\
&- 2g(X,W)g(\phi U,V)Q\phi Y - \alpha g(X,W)g(V,Y)\eta(U)\xi + g(X,W)\eta(U)\eta(Y)QV \\
&- g(X,W)\eta(V)\eta(Y)QU + \alpha g(Y,W)g(\phi U,X)\phi V - \alpha g(Y,W)g(\phi V,X)\phi U \\
&+ g(Y,W)g(\phi U,X)Q\phi V - g(Y,W)g(\phi V,X)Q\phi U + 2\alpha g(Y,W)g(\phi U,V)\phi X \\
&+ 2g(Y,W)g(\phi U,V)Q\phi X - g(Y,W)\eta(U)\eta(X)QV + g(Y,W)\eta(V)\eta(X)QU\} \\
&- \frac{\tau+2n}{2(n+2)}\{-g(X,U)g(\phi Y,W)\phi V + g(X,U)g(\phi V,W)\phi Y \\
&- 2g(X,U)g(\phi Y,V)\phi W + g(Y,U)g(\phi X,W)\phi V - g(Y,U)g(\phi V,W)\phi X \\
&+ 2g(Y,U)g(\phi X,V)\phi W - g(X,V)g(\phi U,W)\phi Y + g(X,V)g(\phi Y,W)\phi U \\
&- 2g(X,V)g(\phi U,Y)\phi W + g(Y,V)g(\phi U,W)\phi X - g(Y,V)g(\phi X,W)\phi U \\
&+ 2g(Y,V)g(\phi U,X)\phi W - g(X,W)g(\phi U,Y)\phi V + g(X,W)g(\phi V,W)\phi U \\
&- 2g(X,W)g(\phi U,V)\phi Y + g(Y,W)g(\phi U,X)\phi V - g(Y,W)g(\phi V,X)\phi U \\
&+ 2g(Y,W)g(\phi U,V)\phi X\} + \frac{\tau}{2(n+2)}\{g(X,U)g(V,W)\eta(Y)\xi \\
&- g(X,U)\eta(Y)\eta(W)V + g(X,U)\eta(V)\eta(W)Y - g(Y,U)g(V,W)\eta(X)\xi \\
&+ g(Y,U)\eta(X)\eta(W)V - g(Y,U)\eta(V)\eta(W)X - g(X,V)g(U,W)\eta(Y)\xi \\
&- g(X,V)\eta(U)\eta(W)Y + g(X,V)\eta(Y)\eta(W)U + g(Y,V)g(U,W)\eta(X)\xi \\
&+ g(Y,V)\eta(U)\eta(W)X - g(Y,V)\eta(X)\eta(W)U - g(X,W)\eta(U)\eta(Y)V \\
&+ g(X,W)\eta(V)\eta(Y)U + g(Y,W)\eta(U)\eta(X)V - g(Y,W)\eta(V)\eta(X)U\} \\
&= -Q(g,\tilde{B})(U,V,W;X,Y),
\end{aligned}
$$

where any $U,V,W,X,Y \in \Gamma(TM)$, $\alpha = -(2n-1)$ and $Q(g,\tilde{B})(U,V,W;X,Y) \neq 0$. This completes proof.

Not that when the null hypersurface M is totally geodesic, by relation (25), M is Einstein. This also occur when M is parallel or totally umbilical [16]

Now, we give an example:

Example 2. Let M be a null hypersurface of $\bar{M}^7$, of Example 1 (i.e. Example 1-3 of [15]) such that totally geodesic, by $x_5 = \sqrt{2}(x_2 + x_3)$, where $(x_1, ..., x_7)$ is a local coordinate system for $\bar{M}^7$. As explaned in Example 1, M is a null hypersurface of $\bar{M}^7$ having a local quasi-orthogonal field of frames $U_1 = e_1,\ U_2 = e_2 - e_3,\ U_3 = E = \frac{1}{\sqrt{2}}(e_2 + e_3) - e_5,\ U_4 = e_4,\ U_5 = e_6,\ U_6 = \xi, N = \frac{1}{2}\{\frac{1}{\sqrt{2}}(U_1 - U_4) - U_5\}$ along M.

The non-zero components of the curvature tensor are given by $R(e_i, e_j)e_i$, $\forall i, j, i \neq j$, $R(e_i, e_m)e_m = e_i$, $\forall i, m = 5, 6$, $R(e_i, e_l)e_l = -e_i$, $\forall i \neq l, l = 1, 2, 3, 4, 7$ and the Ricci tensor are $Ric(e_l, e_l) = -5$, $\forall l = 1, 2, 3, 4, 7$, $Ric(e_m, e_m) = 5$, $\forall m = 5, 6$. Using these relations, it is easy to see that M is Einstein with $\alpha = -5$ [16]. Then, for $\forall p = 1, 2, 3, 4$ and $\forall q = 5, 6$, we obtain

$$\begin{aligned}(R \cdot \tilde{B})(e_p, e_q, e_p; e_p, e_q) &= -\frac{\lambda e_p + Qe_p}{2(n+2)} \\ &= -Q(g, \tilde{B})(e_p, e_q, e_p; e_p, e_q),\end{aligned}$$

where $\lambda = 2n + 3 - \tau$ and $\tau = \frac{r+2n}{2(n+2)}$. Also, for $\forall p = 1, 2, 3, 4$ and $q = 7$, we obtain

$$\begin{aligned}(R \cdot \tilde{B})(e_p, e_q, e_p; e_p, e_q) &= -\frac{\mu e_p - Qe_p}{2(n+2)} \\ &= -Q(g, \tilde{B})(e_p, e_q, e_p; e_p, e_q),\end{aligned}$$

where $\mu = 2n + 5$. Thus, for $\forall p, q$, we say that

$$(R \cdot \tilde{B})(e_p, e_q, e_p; e_p, e_q) = -Q(g, \tilde{B})(e_p, e_q, e_p; e_p, e_q)$$

Therefore, we show that the null hypersurface M of $\bar{M}^7$ is a C-Bochner pseudosymmetric null hypersurface.

As a result the following corollary:

Corollary 9. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. If the second fundamental form h of M is parallel, then M is a C-Bochner pseudosymmetric null hypersurface such that $L_{\tilde{B}} = -1$.

Proof. The proof is obvious from Theorem 2.

Theorem 10. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a C-Bochner pseudosymmetric ($L_{\tilde{B}} = -1$) null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. Then either M is totally geodesic or $\tilde{B}(\xi, A_N E)V = -\tilde{B}(\xi, V)A_N E$, where $V \in \Gamma(TM)$, $N \in \Gamma(tr(TM))$.

Proof. Suppose that M is a C-Bochner pseudosymmetric null hypersurface of an indefinite Kenmotsu space form. Then, for $X \in (Rad(TM))$ and $U = \xi$, we have

$$(R \cdot \tilde{B})(\xi, V, W; E, Y) = L_{\tilde{B}} Q(g, \tilde{B})(\xi, V, W; E, Y).$$

Therefore, we get

$$(1+L_{\tilde{B}})Q(g,\tilde{B})(\xi,V,W;E,Y)-B(E,\xi)\tilde{B}(A_NY,V)W+B(Y,\xi)\tilde{B}(A_NE,V)W$$
$$-B(E,V)\tilde{B}(\xi,A_NY)W+B(Y,V)\tilde{B}(\xi,A_NE)W-B(E,W)\tilde{B}(\xi,V)A_NY$$
$$+B(Y,W)\tilde{B}(\xi,V)A_NE=0$$

and we obtain

$$(1+L_{\tilde{B}})Q(g,\tilde{B})(\xi,V,W;E,Y)+B(Y,V)\tilde{B}(\xi,A_NE)W+B(Y,W)\tilde{B}(\xi,V)A_NE=0.$$

Thus, from the hypothesis, we get

$$B(Y,V)\tilde{B}(\xi,A_NE)W+B(Y,W)\tilde{B}(\xi,V)A_NE=0. \tag{32}$$

Here, if we get $V=W$, then (32) is equivalent to

$$B(Y,V)[\tilde{B}(\xi,A_NE)V+\tilde{B}(\xi,V)A_NE]=0.$$

Thus, the proof is complete.

Corollary 11. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a C-Bochner pseudosymmetric null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form. If M is C-Bochner flat, then M is C-Bochner semi-symmetric.

Proof. The proof is obvious.

Theorem 12. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a screen integrable of C-Bochner pseudosymmetric null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi\in\Gamma(TM)$. If $C=0$, then M is C-Bochner semi-symmetric if and only if the integral manifold of screen distribution is C-Bochner semi-symmetric, where C is the second fundamental form of screen distribution of M.

Proof. For $U,V,W,X,Y\in\Gamma(TM)$, we have

$$(R(X,Y)\cdot\tilde{B})(U,V,W)=-\tilde{B}(R(X,Y)U,V)W-\tilde{B}(U,R(X,Y)V)W-\tilde{B}(U,V)R(X,Y)W.$$

Here, using (27), we get

$$(R(X,Y)\cdot\tilde{B})(U,V,W)$$
$$=-\tilde{B}(R^*(X,Y)U,V)W-C(X,U)\tilde{B}(A^*_EY,V)W+C(Y,U)\tilde{B}(A^*_EX,V)W$$
$$-\{(\nabla_XC)(Y,U)-(\nabla_YC)(X,U)+\tau(Y)C(X,U)-\tau(X)C(Y,U)\}\tilde{B}(E,V)W$$
$$-\tilde{B}(U,R^*(X,Y)V)W-C(X,V)\tilde{B}(U,A^*_EY)W+C(Y,V)\tilde{B}(U,A^*_EX)W$$
$$-\{(\nabla_XC)(Y,V)-(\nabla_YC)(X,V)+\tau(Y)C(X,V)-\tau(X)C(Y,V)\}\tilde{B}(U,E)W$$

$$
\begin{aligned}
&- \tilde{B}(U,V)R^*(X,Y)W - C(X,W)\tilde{B}(U,V)A^*_E Y + C(Y,W)\tilde{B}(U,V)A^*_E X \\
&- \{(\nabla_X C)(Y,W) - (\nabla_Y C)(X,W) + \tau(Y)C(X,W) - \tau(X)C(Y,W)\}\tilde{B}(U,V)E \\
&= (R^*(X,Y)\cdot\tilde{B})(U,V,W) - C(X,U)\tilde{B}(A^*_E Y,V)W + C(Y,U)\tilde{B}(A^*_E X,V)W \\
&- \{(\nabla_X C)(Y,U) - (\nabla_Y C)(X,U) + \tau(Y)C(X,U) - \tau(X)C(Y,U)\}\tilde{B}(E,V)W \\
&- C(X,V)\tilde{B}(U,A^*_E Y)W + C(Y,V)\tilde{B}(U,A^*_E X)W \\
&- \{(\nabla_X C)(Y,V) - (\nabla_Y C)(X,V) + \tau(Y)C(X,V) - \tau(X)C(Y,V)\}\tilde{B}(U,E)W \\
&- C(X,W)\tilde{B}(U,V)A^*_E Y + C(Y,W)\tilde{B}(U,V)A^*_E X \\
&- \{(\nabla_X C)(Y,W) - (\nabla_Y C)(X,W) + \tau(Y)C(X,W) - \tau(X)C(Y,W)\}\tilde{B}(U,V)E,
\end{aligned}
$$

where $(\nabla_X C)(Y,U) = X(C(Y,U)) - C(\nabla^*_X Y,U) - C(Y,\nabla^*_X U)$. Thus, from the hypothesis, the proof is complete.

Theorem 13. Let $\bar{M}(c)$ be an indefinite Kenmotsu space form and M be a null hypersurface of $\bar{M}(c)$ indefinite Kenmotsu space form with $\xi \in \Gamma(TM)$. If M is totally geodesic, then M is also Ricci-generalized C-Bochner pseudosymmetric null hypersurface such that $L_{\tilde{B}} = -\alpha$, where S is the Ricci tensor of M.

Proof. For $U,V,W,X,Y \in \Gamma(TM)$, we obtain

$$
\begin{aligned}
&Q(S,\tilde{B})(U,V,W;X,Y) \\
&= -\tilde{B}((X\wedge_S Y)U,V)W - \tilde{B}(U,(X\wedge_S)V)W - \tilde{B}(U,V)(X\wedge_S Y)W \\
&= -\tilde{B}(S(Y,U)X - S(X,U)Y,V)W - \tilde{B}(U,S(Y,V)X - S(X,V)Y)W \\
&- \tilde{B}(U,V)(S(Y,W)X - S(X,W)Y).
\end{aligned}
$$

Here, from the hypothesis, we have $S(X,Y) = \alpha g(X,Y)$. Then, from the equation in above, we have

$$
\begin{aligned}
Q(S,\tilde{B})(U,V,W;X,Y) &= -S(Y,U)\tilde{B}(X,V)W + S(X,U)\tilde{B}(Y,V)W - S(Y,V)\tilde{B}(U,X)W \\
&+ S(X,V)\tilde{B}(U,Y)W - S(Y,W)\tilde{B}(U,V)X + S(X,W)\tilde{B}(U,V)Y.
\end{aligned}
$$

Thus, if M is totally geodesic, we obtain

$$(R\cdot\tilde{B})(U,V,W;X,Y) = -\alpha Q(S,\tilde{B})(U,V,W;X,Y),$$

where $\alpha = -(2n-1)$. Then, the proof is complete.

Example 3. Let M be a null hypersurface of $\bar{M}^7$, of Example 1 and Example 3 such that totally geodesic, by $x_5 = \sqrt{2}(x_2+x_3)$, where $(x_1,...,x_7)$ is a local coordinate system for $\bar{M}^7$. Then we obtain $(R(e_p,e_q)\cdot\tilde{B})(e_p,e_q,e_p) = -5Q(S,\tilde{B})(e_p,e_q,e_p;e_p,e_q)$, $\forall p,q$. Hence, we show that the null hypersurface M of $\bar{M}^7$ is also a Ricci-generalized C-Bochner pseudosymmetric null hypersurface.

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Sema KAZAN
Department of Mathematics, Faculty of Science and Arts,
Inonu University, 44280
Malatya, Turkey
email: *sema.bulut@inonu.edu.tr*