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HYDROMAGNETIC CREEPING FLOW THROUGH A SLIT WITH EXPONENTIAL ABSORPTION

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Abstract. The present work is concerned with the study of the creeping flow of a magnetohydrodynamics (MHD) Newtonian fluid through a porous slit with exponential absorption across the walls. The governing equations of the considered problem are obtained as two dimensional partial differential equations (PDEs). Exact solutions of the flow equations lead to detailed expressions for velocity components, pressure difference and wall shear stress. It is shown that the magnetic and the exponential absorption parameter play a vital role in altering the flow properties. The expressions for fractional absorption and leakage flux are also obtained. The graphs for velocity components, pressure difference and wall shear stress have been shown. The velocity vectors are also shown to get a better insight of the flow.

2010 *Mathematics Subject Classification*: 76D27, 76D05, 76D08, 76D45.

Keywords: Exact solution; Creeping flow; Exponential absorption; Porous slit.

1. Introduction

The study of the flow of an electrically conducting fluid under the action of transverse magnetic field through a porous channel or tubes is of practical interest in biological and engineering problems, such as glomerular tubule ultrafiltration, proximal tubule reabsorption, artificial kidney, magnetic resonance imaging, giant magneto resistive technology, electromyography, impulse magnetic field therapy, gaseous diffusion, insulation of buildings, transpiration cooling, reverse osmosis desalination, geothermal energy extraction, nuclear reactor, plasma studies, purification of crude oil and electrostatics precipitation. Berman [1-2] investigated the steady laminar flow of an incompressible fluid through a porous, two-dimensional channel and tube. He obtained velocity profile and pressure drop in series form, assuming uniform wall suction. Later, Sellars [3], Yuan [4] and Terrill [5] extended Berman's work with a high suction Reynolds number. Thereafter, Mehta et al. [6] and Terrill et al. [7-8] studied the effect of uniform transverse magnetic field in the problem of Berman [1], and found significant effects of magnetic field parameter on velocity components, axial pressure and skin friction. Recently, Meena [9] and Rashidi et al [10-11] obtained approximate solutions for steady, laminar, incompressible, viscous and electrically conducting fluid through a semi porous channel in the presence of uniform magnetic field.

Fouling is the process of accumulation of unwanted material at an interface. In most studies, investigators demonstrated that microorganisms are able to produce inductive and inhibiting chemical compounds that can block the pores at the surfaces of the channels and tubes [12-13]. It is worth mentioning from a physical point of view that fouling can cause a variable absorption at the surface of the walls, which gives the evidence of the present study. The movement of particles at low Reynolds number is of practical interest in the field of chemistry, biomedical and environmental engineering, for example motion of organisms, sedimentation, coagulation, motion of cells in blood vessels and flow in earth's mantle. The theoretical study of Newtonian fluid at low Reynolds number is based on the classical work of Stokes [14]. Macey [15-16] studied the hydrodynamic of Stokes flow through porous cylinder, assuming linear and exponential wall absorption along the downstream distance. Later, Kozinski et al.[17] not only completed solutions of Macey's work in tube geometry, but also extended it for porous slit to obtain the expression for velocity and pressure fields. Recently, Haroon et al. [19] studied the behavior of Newtonian fluid through a slit with uniform reabsorption at the walls. Siddiqui et al. [20] discussed the hydrodynamics of viscous fluid through a porous slit with linear absorption at the walls. Later Haroon et al. [21] extended the work of Siddiqui et al. [20] and investigated the flow of Newtonian through a slit with periodic reabsorption at the walls. They believed in their work that the pore blockage phenomenon is random and periodic nature of pore blockage may be one of the case. Siddiqui et al. [22] further investigated the MHD flow of Newtonian fluid in a permeable tubule. They assumed that the fluid absorption at the tube walls is a function of the wall permeability and the pressure gradients across the tube wall.

The aim of the present paper is to study the hydrodynamics of the two dimensional creeping MHD flow through a porous slit with exponential absorption at the walls. Inverse method is used to to solve the PDEs with appropriate boundary conditions. Using inverse methods, exact solutions can be obtained by looking into the shape of the boundaries occupied by the fluid. Once the solution is obtained, the flow properties, like velocity components, pressure distribution and wall shear stress can also be easily calculated. Recently, Zeb et al. [23] applied the technique described above to analyze the flow of a viscous fluid induced by the motion of two parallel plates. Various authors [24-26] have obtained exact solutions of the Navier-Stokes and other equations using the inverse method. This paper is arranged as follows:

Basic equations governing the flow of the present problem and low Reynolds number hydrodynamics equations with boundary conditions relating to the present problem are set out in Section 2. Exact solutions are obtained by using an inverse method and expressions for stream function, velocity components, flow rate, fractional absorption, leakage flux, pressure distribution and wall shear stress are obtained in Section 3. Section 4 is concerned with results and discussion. The effects of magnetic and exponential absorption parameters are briefly discussed in this section. Finally, the conclusions of the present study are given in the last section.

2. Equations of Motion and Formulation of the Problem

We consider the steady, laminar creeping flow of an incompressible electrically conducting Newtonian fluid through a porous slit of width $2H$ apart. It is assumed that the normal velocity decays exponentially along the length of the slit and fluid have small electrical conductivity with a magnetic Reynolds number much less than unity, so that the induced magnetic field is neglected. A rectangular Cartesian coordinate system (x, y) is chosen with the x-axis aligned with the center line of the slit and y- axis normal to it, Fig. (1) . It is assumed that the fluid has no slip at the walls and a magnetic field of constant strength H_0 is applied in a direction perpendicular to the flow of the fluid. Then, the equations governing the creeping flow of an incompressible fluid under the influence of a transverse magnetic field are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial p}{\partial x}-\mu\nabla^2 u+\sigma B_0^2 u = 0, \tag{2}$$

$$\frac{\partial p}{\partial y}-\mu\nabla^2 v = 0, \tag{3}$$

where u and v are the velocities in the x- and y-direction, respectively, p is the hydrodynamic pressure, ρ is the density, μ is the kinematic viscosity of the fluid, σ is the electrical conductivity of the fluid and $B_0 = \mu_0 H_0$ is the electromagnetic induction, in which μ_0 being the magnetic permeability.

The boundary conditions of the problem under consideration have the form:

$$\frac{\partial u}{\partial y} = 0, \qquad v = 0, \qquad \text{at} \qquad y = 0, \tag{4}$$

$$u = 0, \qquad v = V_0 \mathrm{e}^{-\alpha \mathrm{x}}, \qquad \text{at} \qquad \mathrm{y} = \mathrm{H}, \tag{5}$$

$$Q_0 = 2W\int_0^H u(0, y)dy, \tag{6}$$

where α is an exponential absorption parameter, W is the breath of the slit, V_0 and Q_0 are the uniform normal velocity and the flow rate, respectively at the entrance of the slit.

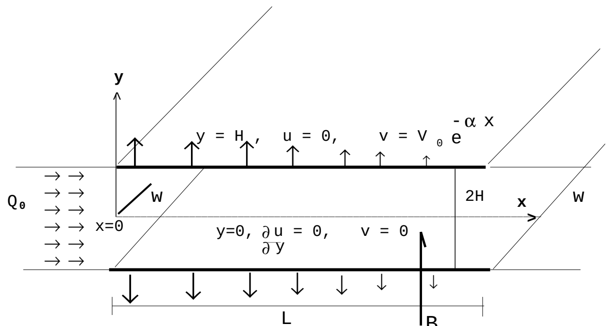


Fig (1): Geometry of the problem.

Introducing the stream function $\psi(x,y)$ in the following form

$$u = \frac{\partial \psi}{\partial y}, \qquad\qquad v = -\frac{\partial \psi}{\partial x}. \tag{7}$$

We find that equations (1) is identically satisfied and equations (2 -3) take the form

$$\frac{\partial p}{\partial x} = \mu \nabla^2 \left(\frac{\partial \psi}{\partial y}\right) - \sigma B_0^2 \left(\frac{\partial \psi}{\partial y}\right), \tag{8}$$

$$\frac{\partial p}{\partial y} = -\mu \nabla^2 \left(\frac{\partial \psi}{\partial x}\right). \tag{9}$$

Eliminating the pressure gradient from above equations, we obtain the following PDE

$$\nabla^4 \psi - M^2 \left(\frac{\partial^2 \psi}{\partial y^2}\right) = 0, \tag{10}$$

where $M^2 = \dfrac{\sigma B_0^2}{\mu}$ is magnetic parameter and $\nabla^4 = \nabla^2(\nabla^2)$ and $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ is a Laplacian operator. Further, the boundary conditions (4-6) in term of $\psi(x,y)$ become

$$\frac{\partial \psi}{\partial y} = 0, \quad -\frac{\partial \psi}{\partial x} = V_0 \mathrm{e}^{-\alpha \mathrm{x}}, \qquad \text{at} \quad y = H, \tag{11}$$

$$\frac{\partial^2 \psi}{\partial y^2} = 0, \quad \frac{\partial \psi}{\partial x} = 0, \qquad \text{at} \quad y = 0, \tag{12}$$

$$\frac{Q_0}{2W} = \psi(0,H) - \psi(0,0). \tag{13}$$

Conventionally, we take

$$\psi(0,0) = 0, \tag{14}$$

so

$$\psi(0,H) = \frac{Q_0}{2W}, \tag{15}$$

Equation (10) along with the boundary conditions (11-12) and (14-15) is two dimensional boundary value problem (BVP), describing the MHD flow of an incompressible Newtonian fluid flow through a porous slit with exponential absorption at the wall.

3. Method of Solution

To obtain the exact solutions of the above BVP, let us choose a particular form of stream function, in view of boundary conditions

$$\psi = V_0 \mathrm{e}^{-\alpha x} F(y) + K(y), \tag{16}$$

where $F(y)$ and $K(y)$ are some arbitrary function of the y. Using equation (16) in equation (10), yields

$$V_0 \mathrm{e}^{-\alpha \mathrm{x}} \left[\frac{d^4 F}{dy^4} + (2\alpha^2 - M^2) \frac{d^2 F}{dy^2} + \alpha^4 F \right] + \frac{d^4 K}{dy^4} - M^2 \frac{d^2 K}{dy^2} = 0. \tag{17}$$

One of the possibility to hold above equality is

$$V_0 \mathrm{e}^{-\alpha \mathrm{x}} \left[\frac{d^4 F}{dy^4} + (2\alpha^2 - M^2) \frac{d^2 F}{dy^2} + \alpha^4 F \right] = 0, \tag{18}$$

and

$$\frac{d^4 K}{dy^4} - M^2 \frac{d^2 K}{dy^2} \quad = \quad 0, \tag{19}$$

Consider equality (18), since $V_0 \mathrm{e}^{-\alpha \mathrm{x}} \neq 0$, then

$$\left[\frac{d^4 F}{dy^4} + (2\alpha^2 - M^2) \frac{d^2 F}{dy^2} + \alpha^4 F \right] = 0. \tag{20}$$

Both equations (19-20) are fourth order ordinary differential equations. With the help of (16), boundary conditions (11 -12) and (14 -15) reduce to

$$\begin{aligned} F(0) &= 0, \quad \frac{d^2F(0)}{dy^2} = 0, && (21)\\ F(H) &= \frac{1}{\alpha}, \quad \frac{dF(H)}{dy} = 0, && (22) \end{aligned}$$

and

$$\begin{aligned} K(0) &= 0, \quad \frac{d^2K(0)}{dy^2} = 0, && (23)\\ K(H) &= \frac{\alpha Q_0 - 2V_0W}{2\alpha W}, \quad \frac{dK(H)}{dy} = 0. && (24) \end{aligned}$$

The solution of equation (20) along with the boundary condition (21- 22) is obtained as

$$F(y) = \Delta_1 \sinh\left(\frac{My}{2}\right)\cos\left(\frac{\Delta y}{2}\right) + \Delta_2 \cosh\left(\frac{My}{2}\right)\sin\left(\frac{\Delta y}{2}\right), \tag{25}$$

where

$$\begin{aligned} \Delta &= \sqrt{4\alpha^2 - M^2},\\ \Delta_1 &= -\frac{M\sinh\left(\frac{MH}{2}\right)\sin\left(\frac{\Delta H}{2}\right) + \Delta\cosh\left(\frac{MH}{2}\right)\cos\left(\frac{\Delta H}{2}\right)}{\alpha\left[M\sin\left(\frac{\Delta H}{2}\right)\cos\left(\frac{\Delta H}{2}\right) - \Delta\cosh\left(\frac{MH}{2}\right)\sinh\left(\frac{MH}{2}\right)\right]},\\ \Delta_2 &= \frac{M\cosh\left(\frac{MH}{2}\right)\cos\left(\frac{\Delta H}{2}\right) - \Delta\sinh\left(\frac{MH}{2}\right)\sin\left(\frac{\Delta H}{2}\right)}{\alpha\left[M\sin\left(\frac{\Delta H}{2}\right)\cos\left(\frac{\Delta H}{2}\right) - \Delta\cosh\left(\frac{MH}{2}\right)\sinh\left(\frac{MH}{2}\right)\right]}. \end{aligned}$$

The solution of equation (19) along with the boundary condition (23- 24) becomes

$$K(y) = \frac{(\alpha Q_0 - 2V_0W)(M\cosh(MH)y - \sinh(My))}{2\alpha W(MH\cosh(MH) - \sinh(MH))}. \tag{26}$$

Using solutions (25) and (26) in equation (16), $\psi(x,y)$ becomes

$$\begin{aligned} \psi(x,y) &= V_0 e^{-\alpha x}\left[\Delta_1 \sinh\left(\frac{My}{2}\right)\cos\left(\frac{\Delta y}{2}\right) + \Delta_2\cosh\left(\frac{My}{2}\right)\sin\left(\frac{\Delta y}{2}\right)\right]\\ &+ \frac{(\alpha Q_0 - 2V_0W)(M\cosh(MH)y - \sinh(My))}{2\alpha W(MH\cosh(MH)) - \sinh(MH)}, \qquad (27) \end{aligned}$$

which strongly depends upon the magnetic parameter and the exponential absorption parameter.

3.1. Components of Velocity

The velocity components are obtained with the help of relation (7) :

$$\begin{aligned} u(x,y) &= \frac{1}{2}V_0 \mathrm{e}^{-\alpha x}\left[(\Delta_1 M + \Delta_2\Delta)\cosh\left(\frac{My}{2}\right)\cos\left(\frac{\Delta y}{2}\right)\right. \\ &+ \left.(\Delta_2 M - \Delta_1\Delta)\sinh\left(\frac{My}{2}\right)\sin\left(\frac{\Delta y}{2}\right)\right] \\ &+ \frac{(\alpha Q_0 - 2V_0 W)[M\cosh(MH) - M\cosh(My)]}{2\alpha W(MH\cosh(MH)) - \sinh(MH)}, \qquad (28) \\ v(x,y) &= \alpha V_0 \mathrm{e}^{-\alpha x}\left[\Delta_1\sinh\left(\frac{My}{2}\right)\cos\left(\frac{\Delta y}{2}\right) + \Delta_2\cosh\left(\frac{My}{2}\right)\sin\left(\frac{\Delta y}{2}\right)\right] \qquad (29) \end{aligned}$$

Equations (28) and (29) give a complete description of the fluid velocities at all the points in the porous slit. We observed that by making use of Δ, Δ_1 and Δ_2 and taking $M \to 0$, velocity components of Kozinski et al. [17] are recovered:

$$\begin{aligned} u(x,y) &= \frac{\mathrm{e}^{-\alpha x}V_0\,\alpha\,[H\sin(\alpha H)\cos(\alpha y) - \cos(\alpha H)\sin(\alpha y)\,y]}{\alpha H - \cos(\alpha H)\sin(\alpha H)} \\ &+ \frac{3}{4}\frac{H^2 Q_0\,\alpha - 2H^2 V_0 W - Q_0\,\alpha\,y^2 + 2V_0 W y^2}{\alpha W H^3}, \\ v(x,y) &= \frac{V_0\,[H\sin(\alpha H)\sin(\alpha y)\,\alpha + \alpha\cos(\alpha H)\cos(\alpha y)\,y - \cos(\alpha H)\sin(\alpha y)]\,\mathrm{e}^{-\alpha x}}{\alpha H - \cos(\alpha H)\sin(\alpha H)}, \end{aligned}$$

providing a mathematical verification of the model. We also noted that if $M \to 0$, $\alpha \to 0$ and $V_0 \to 0$ the classical result of Poiseuille flow are recovered [27]:

$$u = \frac{3Q_0}{4WH}\left[1 - \left(\frac{y}{H}\right)^2\right].$$

The volume flow rate can be obtained by using the relation

$$Q(x) = 2W\int_0^H u(x,y)dy, \qquad (30)$$

which becomes

$$Q(x) = \frac{2WV_0\mathrm{e}^{-\alpha x} + \alpha Q_0 - 2V_0 W}{\alpha}, \qquad (31)$$

which shows that flow rate is independent of magnetic parameter and indicates that the bulk flow decreases inside the slit. The fractional absorption, FA is defined as:

$$FA = \frac{Q(0) - Q(L)}{Q(0)}. \qquad (32)$$

Using (31) in above equation, we get

$$FA = \frac{2V_0 W(1 - \mathrm{e}^{-\alpha \mathrm{L}})}{\alpha Q_0}. \tag{33}$$

Fractional absorption strongly depends upon exponential absorption parameter, α and having inverse relation with volume flow rate, Q_0. The leakage flux $q(x)$ is defined as

$$q(x) = -\frac{dQ(x)}{dx}. \tag{34}$$

Substituting (31) in above equation, we arrive at

$$q(x) = 2WV_0\mathrm{e}^{-\alpha x}, \tag{35}$$

which shows that leakage flux has the direct relation with the product of breath and exponential absorption. The maximum leakage is observed at the entrance of the slit which goes decreasing downstream.

3.2. Pressure Distribution

To get an expression of pressure we use equation (16) in equation (8-9) and get

$$\frac{\partial p}{\partial x} = \mu\left[V_0\mathrm{e}^{-\alpha x}\left\{\alpha^2\frac{dF}{dy} + \frac{d^3F}{dy^3} - M^2\frac{dF}{dy}\right\} + \frac{d^3K}{dy^3} - M^2\frac{dF}{dy}\right], \tag{36}$$

$$\frac{\partial p}{\partial y} = \mu\left[\alpha V_0\mathrm{e}^{-\alpha x}\left\{\alpha^2 F + \frac{d^2F}{dy^2}\right\}\right]. \tag{37}$$

To find $p(x,y)$, we integrate equation (36) with respect to x to get

$$p(x,y) = \mu\left[-\frac{V_0\mathrm{e}^{-\alpha x}}{\alpha}\left\{\alpha^2\frac{dF}{dy} + \frac{d^3F}{dy^3} - M^2\frac{dF}{dy}\right\} + \left\{\frac{d^3K}{dy^3} - M^2\frac{dF}{dy}\right\}x\right] + R(y), \tag{38}$$

where $R(y)$ is unknown function need to be determined. By differentiating equation (38) with respect to y and comparing with (37), along with the use of (19-20), we obtain

$$\frac{dR}{dy} = 0. \tag{39}$$

Integrating above equation, we get

$$R(y) = C, \tag{40}$$

where C is unknown constant of integration. Using equation (40) in equation (38), we arrive at

$$p(x,y) = \mu\left[-\frac{V_0 e^{-\alpha x}}{\alpha}\left\{\alpha^2\frac{dF}{dy}+\frac{d^3F}{dy^3}-M^2\frac{dF}{dy}\right\}+\left\{\frac{d^3K}{dy^3}-M^2\frac{dF}{dy}\right\}x\right]+C. \quad (41)$$

Using equations (25) and (26) in above expression, we get

$$\begin{aligned} p(x,y)-p(0,0) &= \frac{\mu V_0 e^{-\alpha x}}{\alpha}\left[\Delta_3\cosh\left(\frac{My}{2}\right)\cos\left(\frac{\Delta y}{2}\right)-\Delta_4\sinh\left(\frac{My}{2}\right)\sin\left(\frac{\Delta y}{2}\right)\right] \\ &- \frac{\mu(\alpha Q_0-2V_0W)M^3\cosh(MH)x}{2\alpha W(MH\cosh(MH)-\sinh(MH))}-\frac{\mu V_0\Delta_3}{\alpha}. \end{aligned} \quad (42)$$

where

$$\begin{aligned} \Delta_3 &= \frac{1}{8}\left[\Delta^3\Delta_2+3M\Delta^2\Delta_1+\left(\Delta_2M^2-4\Delta_2\alpha^2\right)\Delta+3M^3\Delta_1-4M\Delta_1\alpha^2\right], \\ \Delta_4 &= \frac{1}{8}\left[\Delta^3\Delta_1-3M\Delta^2\Delta_2+\left(\Delta_1M^2-4\Delta_1\alpha^2\right)\Delta-3M^3\Delta_2+4M\Delta_2\alpha^2\right], \end{aligned}$$

and $p(0,0)$ is the value of the pressure at the entrance of the slit at $y=0$. It is checked that by making use of Δ, Δ_1, Δ_2, Δ_3 and Δ_4 in equation (42) and taking $M\to 0$, previous result of Kozinski et al [17] are successfully recovered:

$$p(x,y)-p(0,0) = -\frac{3}{2}\left[\frac{\mu\,(Q_0\,\alpha-2\,V_0\,W)\,x}{WH^3\alpha}\right]+\frac{2\,V_0\,\mu\,\alpha e^{-\alpha\,x}\,\cos(\alpha\,H)\cos(\alpha\,y)}{\alpha\,H-\sin(\alpha\,H)\cos(\alpha\,H)}$$

3.3. Wall Shear Stress

The wall shear stress is defined as

$$\tau_w\Big|_{y=H} = -\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)_{y=H}. \quad (43)$$

Using (28 - 29) in above formula, we get

$$\begin{aligned} \tau_w\Big|_{y=H} &= -\frac{\mu V_0 e^{-\alpha x}}{4}\left[\Delta_5\sinh\left(\frac{MH}{2}\right)\cos\left(\frac{\Delta H}{2}\right)+\Delta_6\cosh\left(\frac{MH}{2}\right)\sin\left(\frac{\Delta H}{2}\right)\right] \\ &+ \frac{\mu(\alpha Q_0-2WV_0)M^2\sinh(MH)}{2\alpha W(MH\cosh(MH)-\sinh(MH))}, \end{aligned} \quad (44)$$

where

$$\begin{aligned} \Delta_5 &= 2\Delta\Delta_2M+\Delta_1\Delta_7, \\ \Delta_6 &= -2\Delta\Delta_1M+\Delta_2\Delta_7, \\ \Delta_7 &= (M^2-\Delta^2-4\alpha^2). \end{aligned}$$

If $M \to 0$, one can get the expression for wall shear stress of Kozinski et al.[17]:

$$\tau_w\Big|_{y=H} = \frac{\alpha\ (H\alpha\ \cos(\alpha\, H)\cos(\alpha) + \sin(\alpha)\sin(\alpha\, H)\,\alpha + \cos(\alpha)\sin(\alpha\, H))}{\sin(\alpha)\cos(\alpha) - \alpha}.$$

4. Results and Discussion

The purpose of the present study is to analyze the effect of magnetic parameter M and exponential absorption α on velocity components at different positions $x = 0.1$ (entrance), $x = 0.5$ (mid) and $x = 0.9$ (exit) of the slit, pressure difference and wall shear stress. The expressions for the velocity components (28-29), pressure difference (42) and wall shear stress (44) are normalized by introducing the following dimensionless quantities

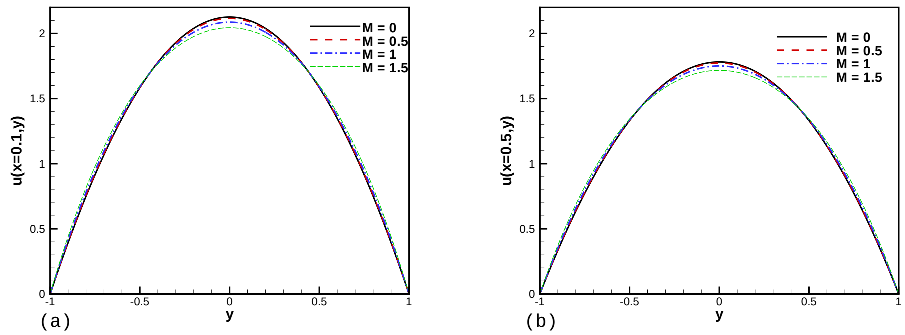
$$x^* = \frac{x}{H}, \qquad y^* = \frac{y}{H}, \qquad \psi^* = \frac{\psi}{V_0 H}, \qquad Q_0^* = \frac{Q_0}{V_0 W H}, \quad M^* = MH^2,$$
$$p^* = \frac{p}{\mu V_0/H}, \qquad \tau_w^* = \frac{\tau_w}{\mu V_0/H}, \tag{45}$$

and the variations have been explained through Figs. (2-11), after skipping *. By inserting $M = 0$ in equations (28-29) and (42), we achieve the results of Kozinski et al [18]. Fig. 2 demonstrates the effects of M on transverse component of velocity. It is observed that with increase in M, the velocity $u(x, y)$ increases near the wall and decreases at the center. Parabolic profile at the entrance of the slit is higher than at the mid and the exit of the sit. Effect of M on the normal component of velocity $v(x, y)$ can be analyzed through Figs. (3b-4). From Fig. (3b) it is found that $v(x, y)$ increases from the center to the boundary of the slit and its magnitude decreases with increasing M, also the profile of $v(x, y)$ is higher at the entrance than at the mid and the exit of the slit, see Figs. (3b-4). The pressure difference decreases down stream and with increasing M, it decreases, Figs.(5a). It is noted that for $M = 0$, the pressure difference profile is higher in magnitude as compared to $M = 0.5, 1, 1.5$. Wall shear stress is demonstrated in Fig. (5b), it is observed that pressure difference and wall shear stress have the same effects.

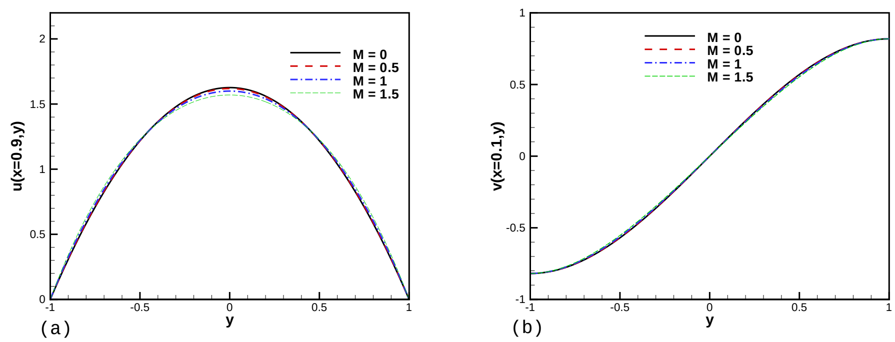
Effects of α on velocity components, pressure difference and wall shear stress in the presence of a magnetic field are depicted in Figs. (6-9). Fig. (6a) demonstrates the effects of α on $u(x, y)$ at the entrance of the slit. For higher value of $\alpha = 3$, $u(x, y)$ looks flatten while for smaller values of α, a parabolic profile is observed. It is noticed that with increasing α, $u(x, y)$ increases near the walls and decreases in the center. In Fig. (6b), effects of α in the mid of the slit is observed, with increasing α, $u(x, y)$ decreases near the wall and remain uniform at the center of the slit. At the exit of the slit a parabolic profile is depicted for all values of α and with increasing α, it decreases, see Fig. (7a). In Fig. (7b) effects of α on normal velocity $v(x, y)$ are shown at the entrance of the slit, and with increasing α, $v(x, y)$ decreases from the center to the upper wall of the slit. Similar effects are observed at the mid and the exit of the slit, see Fig. (8). Pressure difference and wall shear stress increases with increasing α, see Fig. (9). Figs. (10-11) demonstrate the velocity vectors for different values of M and α. From these figures it is observed that show that for fixed values of inlet flow rate and the absorption parameter, the fluid absorption may be increased or decreased by varying M. Thus, we can say that magnetic field can play significant role in controlling the absorption of fluid at the walls of the slit. Similarly, effect of α are depicted in Fig. (11), for large values of α, fluid absorption near the wall increases near the entrance while it goes decrease at the mid and vanish at the exit of the slit walls.

5. Conclusion

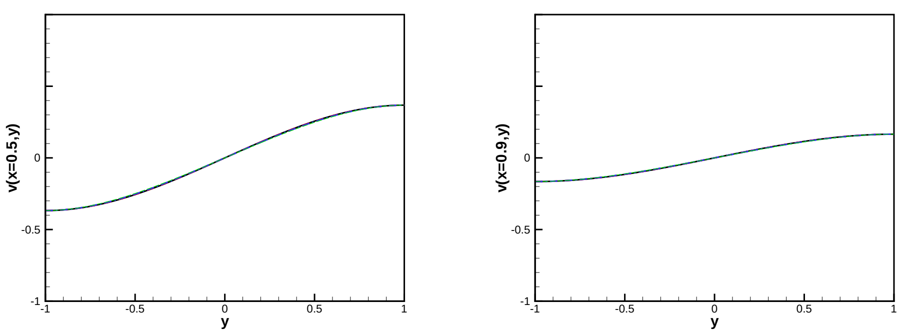
The problem of MHD creeping flow of Newtonian fluid through a uniform porous slit was investigated. Exact solutions were obtained by using an inverse method. Expression for components of velocity, volume flow rate, pressure difference and wall shear stress were derived. We observed that the volume flow rate is independent of the magnetic field which decreases down stream. It was found that pressure difference decreases considerably and wall shear stress increases with the increase in the strength of the magnetic field. The longitudinal velocity increases at all the position with increasing α, while normal velocity decreases near the slit walls with increasing exponential suction. The leakage flux is maximum at the entrance of the slit. Thus in the last we can say that magnetic field and exponential suction parameters can play significant role in controlling the absorption of fluid at the walls of the slit and the obtained results coincide well with those existing in the literature for the study of fluid flow through a slit when $M \to 0$ and α approaches to zero.



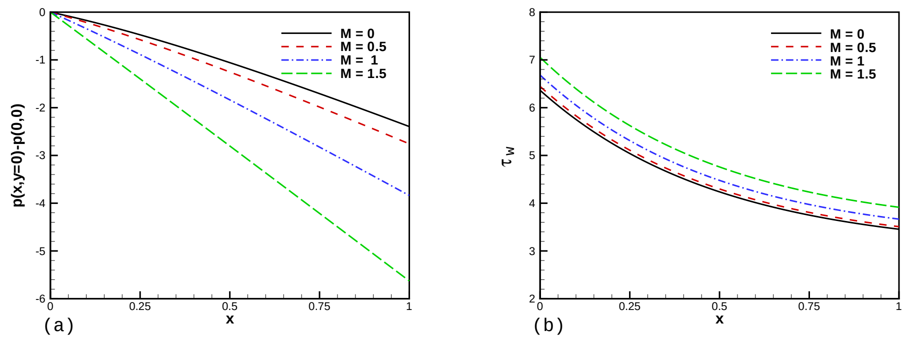
Fig(2): Effect of M on longitudinal velocity at the (a) entrance ($x = 0.1$) and (b) mid ($x = 0.5$) of the slit when $Q_0 = 3$ and $\alpha = 2$.



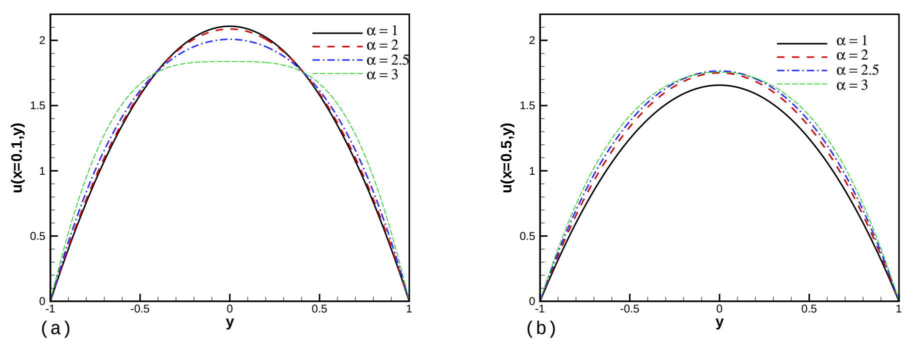
Fig(3): Effect of M on (a) longitudinal velocity at the exit ($x = 0.9$) and (b) normal velocity at the entrance ($x = 0.1$) of the slit when $Q_0 = 3$ and $\alpha = 2$.



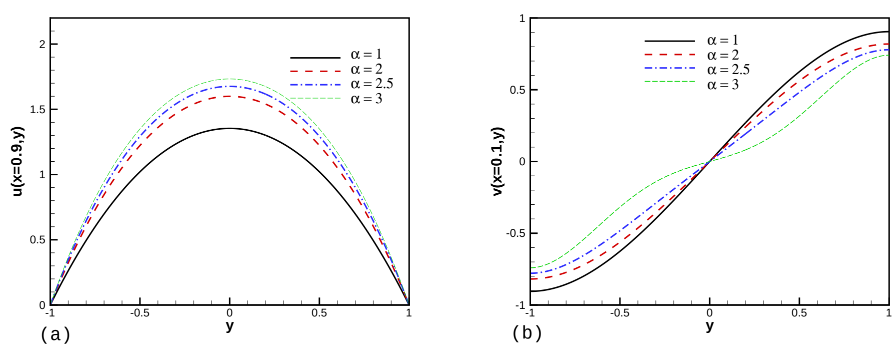
Fig(4): Effect of M on normal velocity at the (a) mid ($x = 0.5$) and (b) exit ($x = 0.9$) of the slit when $Q_0 = 3$ and $\alpha = 2$.



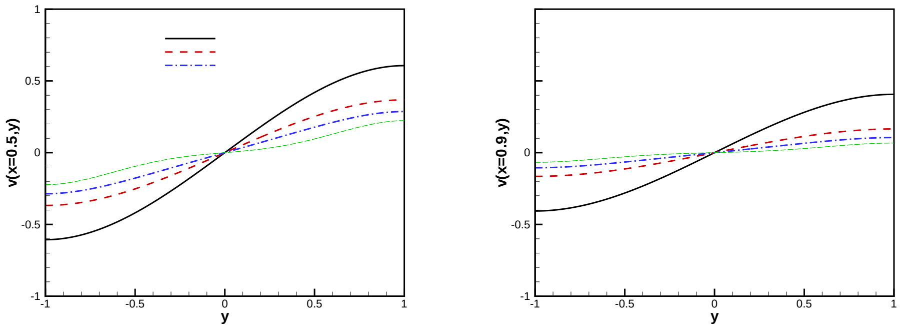
Fig(5): Effect of M on (a) pressure difference at the center line and (b) wall shear stress when $Q_0 = 3$ and $\alpha = 2$.



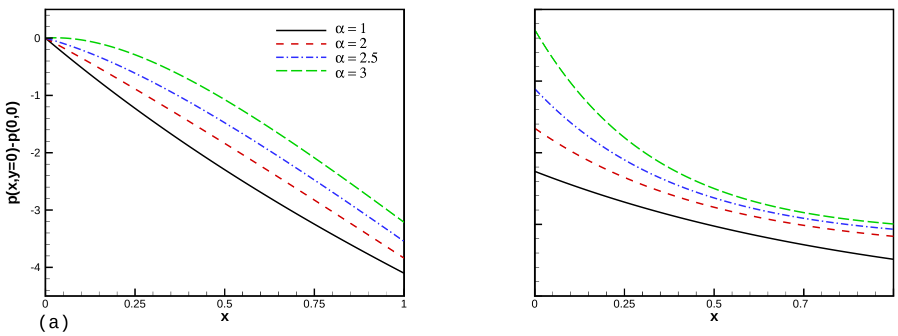
Fig(6): Effect of α on longitudinal velocity at the (a) entrance ($x = 0.1$) and (b) mid ($x = 0.5$) of the slit when $Q_0 = 3$ and $M = 1$.



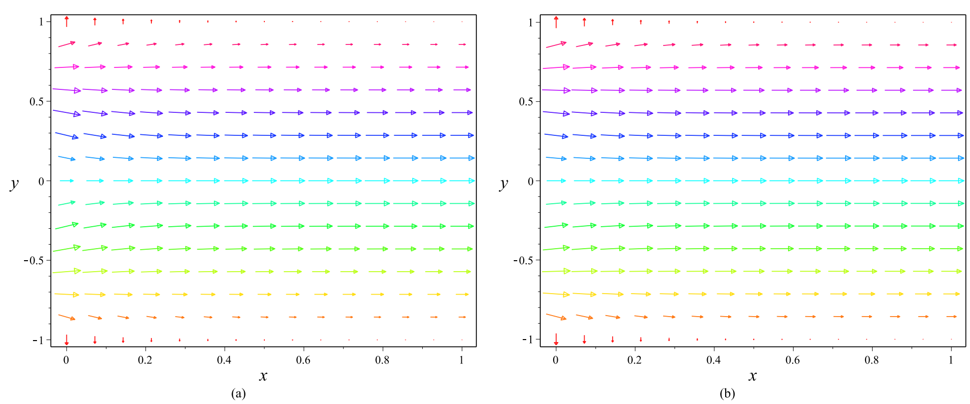
Fig(7): Effect of α on (a) longitudinal velocity at the exit ($x = 0.9$) and (b) normal velocity at the entrance ($x = 0.1$) of the slit when $Q_0 = 3$ and $M = 1$.



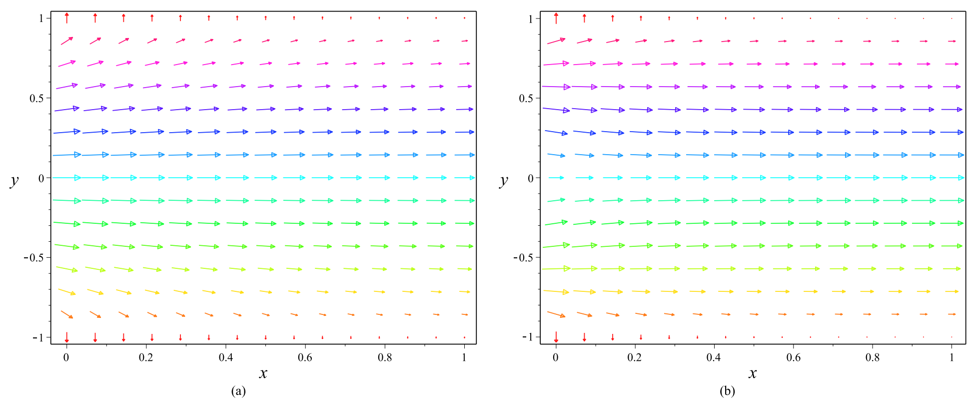
Fig(8): Effect of α on normal velocity at the (a) mid ($x = 0.5$) and (b) exit ($x = 0.9$) of the slit when $Q_0 = 3$ and $M = 1$.



Fig(9): Effect of α on (a) pressure difference at the center line and (b) wall shear stress when $Q_0 = 3$ and $M = 1$.



Fig(10): The velocity vectors when (a) $M = 0$ and (b) $M = 3$ when $Q_0 = 3$ and $\alpha = 5$.



Fig(11): The velocity vectors when (a) $\alpha = 2$ and (b) $\alpha = 4$ when $Q_0 = 3$ and $M = 1$.

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