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# ON THE CERTAIN SUBCLASS OF ANALYTIC AND BI-UNIVALENT FUNCTIONS DEFINED BY CONVOLUTION

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Abstract. In this paper, we introduce and investigate an interesting subclass $\mathcal{B}_{\Sigma}^{p,q}(h,\lambda)$ of bi-univalent functions in the open unit disk $\mathbb{U}$. Furthermore, we find estimates on the $|a_2|$ and $|a_3|$ coefficients for functions in this subclass. The results presented in this paper would generalize and improve those in related works of several earlier authors.

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## 1. Introduction

Let $\mathcal{A}$ denote the class of analytic functions in the unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, that have the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Further, by $\mathcal{S}$ we shall denote the class of functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$ (for details, see [2, 3, 5]).

It is well known that every functions $f \in \mathcal{S}$ has an inverse $f^{-1}$, defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots.$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$.

Let $\Sigma$ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). Brannan and Taha [2] (see also[11]) introduced certain subclasses of the bi-univalent function class $\Sigma$ similar to the familiar subclasses $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ of starlike and convex functions of order $\alpha$ $(0 < \alpha \leq 1)$, respectively (see [1]).
Determination of the bounds for the coefficients $a_n$ is an important problem in geometric function theory as they give information about the geometric properties of these functions. Recently there interest to study the bi-univalent functions class $\Sigma$ (see [3, 6, 7, 9, 10, 12]) and obtain non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. The coefficient estimate problem i.e. bound of $|a_n|$ $(n \in \mathbb{N} - \{1, 2\})$ for each $f \in \Sigma$ is still an open problem.

Srivastava et al. [10] introduced the following two subclasses of the bi-univalent function class $\Sigma$ and obtained non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

**Definition 1.** *[10] A function $f(z)$ given by* (1) *is said to be in the* $\mathcal{H}_\Sigma^\alpha$ $(0 < \alpha \leq 1)$, *if the following conditions are satisfied:*

$$f \in \Sigma, \quad |arg(f'(z))| < \frac{\alpha\pi}{2} \quad (z \in \mathbb{U}), \qquad |arg(g'(w))| < \frac{\alpha\pi}{2} \quad (w \in \mathbb{U}),$$

*where $g$ is the extension of $f^{-1}$ to* $\mathbb{U}$.

**Theorem 1.** *[10] Let the function $f(z)$ given by* (1) *be in the* $\mathcal{H}_\Sigma^\alpha$ $(0 < \alpha \leq 1)$. *Then*

$$|a_2| \leq \alpha\sqrt{\frac{2}{\alpha+2}}, \qquad |a_3| \leq \frac{\alpha(3\alpha+2)}{3}.$$

**Definition 2** ([10])**.** *A function $f(z)$ given by* (1) *is said to be in the* $\mathcal{H}_\Sigma(\beta)$ $(0 \leq \beta < 1)$, *if the following conditions are satisfied:*

$$f \in \Sigma, \quad Re(f'(z)) > \beta \quad (z \in \mathbb{U}), \qquad Re(g'(w)) > \beta \quad (w \in \mathbb{U}),$$

*where $g$ is the extension of $f^{-1}$ to* $\mathbb{U}$.

**Theorem 2.** *[10] Let the function $f(z)$ given by* (1) *be in the* $\mathcal{H}_\Sigma(\beta)$ $(0 \leq \beta < 1)$. *Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{3}}, \qquad |a_3| \leq \frac{(1-\beta)(5-3\beta)}{3}.$$

As a generalization of two subclasses $\mathcal{H}_{\Sigma}^{\alpha}$ and $\mathcal{H}_{\Sigma}(\beta)$, Frasin [7] introduced the following two subclasses of the bi-univalent function class $\Sigma$ and obtained non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

**Definition 3.** *[7] A function $f(z) \in \Sigma$ given by (1) is said to be in the $\mathcal{B}_{\Sigma}(\alpha, \lambda)$ $(0 < \alpha \leq 1, \lambda \geq 1)$, if the following conditions are satisfied:*

$$|arg((1-\lambda)\frac{f(z)}{z} + \lambda f'(z))| < \frac{\alpha\pi}{2} \ (z \in \mathbb{U}), \quad |arg((1-\lambda)\frac{g(w)}{w} + \lambda g'(w))| < \frac{\alpha\pi}{2} \ (w \in \mathbb{U}),$$

*where $g$ is the extension of $f^{-1}$ to $\mathbb{U}$.*

**Theorem 3.** *[7] Let the function $f(z)$ given by (1) be in the $\mathcal{B}_{\Sigma}(\alpha, \lambda)$ $(0 < \alpha \leq 1, \lambda \geq 1)$. Then*

$$|a_2| \leq \frac{2\alpha}{\sqrt{(\lambda+1)^2 + \alpha(1+2\lambda-\lambda^2)}}, \qquad |a_3| \leq \frac{4\alpha^2}{(\lambda+1)^2} + \frac{2\alpha}{2\lambda+1}.$$

**Definition 4.** *[7] A function $f(z) \in \Sigma$ given by (1) is said to be in the $\mathcal{B}_{\Sigma}(\beta, \lambda)$ $(0 \leq \beta < 1, \lambda \geq 1)$, if the following conditions are satisfied:*

$$Re((1-\lambda)\frac{f(z)}{z} + \lambda f'(z)) > \beta \ (z \in \mathbb{U}), \qquad Re((1-\lambda)\frac{g(w)}{w} + \lambda g'(w)) > \beta \ (w \in \mathbb{U}),$$

*where $g$ is the extension of $f^{-1}$ to $\mathbb{U}$.*

**Theorem 4.** *[7] Let the function $f(z)$ given by (1) be in the $\mathcal{B}_{\Sigma}(\beta, \lambda)$ $(0 \leq \beta < 1, \ \lambda \geq 1)$. Then*

$$|a_2| \leq \sqrt{\frac{2(1-\beta)}{2\lambda+1}}, \qquad |a_3| \leq \frac{4(1-\beta)^2}{(\lambda+1)^2} + \frac{2(1-\beta)}{2\lambda+1}.$$

The object of the present paper is to introduce a new subclass of the function class $\Sigma$ and obtain estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in this new subclass which generalize and improve those in related works of several earlier authors.

## 2. Coefficient bounds for the function class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$

In this section, we introduce the subclass $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$ and find the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in this subclass.

Let

$$h(z) = z + \sum_{n=2}^{\infty} h_n z^n, \qquad \text{where} \quad h_n > 0 \quad \text{for all} \;\; n \geq 2. \tag{2}$$

The Hadamard product $f(z),\ h(z)$ is defined as $(f * h)(z) = z + \sum_{n=2}^{\infty} a_n h_n z^n$, where $f(z) \in \mathcal{A}$ given by (1).

**Definition 5.** *Let the functions $p, q : \mathbb{U} \to \mathbb{C}$ be so constrained that*

$$\min\{Re(p(z)),\ Re(q(z))\} > 0 \quad (z \in \mathbb{U}) \quad \text{and} \quad p(0) = q(0) = 1.$$

*A function $f(z) \in \mathcal{A}$ given by (1) is said to be in the class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$, if the following conditions are satisfied:*

$$f \in \Sigma, \quad [(1-\lambda)\frac{(f*h)(z)}{z} + \lambda(f*h)'(z)] \in p(\mathbb{U}) \quad (z \in \mathbb{U};\ \lambda \geq 1) \tag{3}$$

*and*

$$[(1-\lambda)\frac{(f*h)^{-1}(w)}{w} + \lambda((f*h)^{-1})'(w)] \in q(\mathbb{U}) \quad (w \in \mathbb{U};\ \lambda \geq 1), \tag{4}$$

*where the function $h(z)$ is given by (2).*

**Remark 1.** *There are many choices of the functions $p(z)$ and $q(z)$ which would provide interesting subclasses of the analytic function class $\mathcal{A}$. For example, if we let*

$$p(z) = q(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \qquad (0 < \alpha \leq 1;\ z \in \mathbb{U}),$$

*it is easy to verify that the functions $p(z)$ and $q(z)$ satisfy the hypotheses of Definition 5. If $f(z) \in \mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$, then*

$$|arg\left((1-\lambda)\frac{(f*h)(z)}{z} + \lambda(f*h)'(z)\right)| < \frac{\alpha\pi}{2} \qquad (z \in \mathbb{U};\ \lambda \geq 1)$$

*and*

$$|arg\left((1-\lambda)\frac{(f*h)^{-1}(z)}{w} + \lambda((f*h)^{-1})'(w)\right)| < \frac{\alpha\pi}{2} \qquad (w \in \mathbb{U};\ \lambda \geq 1).$$

*Therefore for $p(z) = q(z) = \left(\frac{1+z}{1-z}\right)^{\alpha}$ and $h(z) = \frac{z}{1-z}$, the class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$ reduce to Definition 3 and in special case $\lambda = 1$ it reduce to Definition 1.*

*If we take*

$$p(z) = q(z) = \frac{1 + (1 - 2\beta)z}{1 - z} \qquad (0 \le \beta < 1;\ z \in \mathbb{U}),$$

*then the functions $p(z)$ and $q(z)$ satisfy the hypotheses of Definition 5. If $f(z) \in \mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$, then*

$$Re\left((1 - \lambda)\frac{(f * h)(z)}{z} + \lambda(f * h)'(z)\right) > \beta \qquad (z \in \mathbb{U};\ \lambda \ge 1)$$

*and*

$$Re\left((1 - \lambda)\frac{(f * h)^{-1}(z)}{w} + \lambda((f * h)^{-1})'(w)\right) > \beta \qquad (w \in \mathbb{U};\ \lambda \ge 1).$$

*Therefore for $p(z) = q(z) = \frac{1+(1-2\beta)z}{1-z}$ and $h(z) = \frac{z}{1-z}$, the class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$ reduce to Definition 4 and in special case $\lambda = 1$ it reduce to Definition 2.*

### 2.1. Coefficients estimates

Now, we derive the estimates of the coefficients $|a_2|$ and $|a_3|$ for class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$.

**Theorem 5.** *Let a function $f(z)$ given by (1) be in the class $\mathcal{B}_{\Sigma}^{p,q}(h, \lambda)$ $(\lambda \ge 1)$. Then*

$$|a_2| \le min\left\{\frac{1}{h_2(\lambda + 1)}\sqrt{\frac{|p'(0)|^2 + |q'(0)|^2}{2}}, \frac{1}{2h_2}\sqrt{\frac{|p''(0)| + |q''(0)|}{2\lambda + 1}}\right\}$$

*and*

$$|a_3| \le min\left\{\frac{|p'(0)|^2 + |q'(0)|^2}{2h_3(\lambda + 1)^2} + \frac{|p''(0)| + |q''(0)|}{4h_3(2\lambda + 1)}, \frac{|p''(0)|}{2h_3(2\lambda + 1)}\right\}.$$

*Proof.* First of all, we write the argument inequalities in (3) and (4) in their equivalent forms as follows:

$$(1 - \lambda)\frac{(f * h)(z)}{z} + \lambda(f * h)'(z) = p(z) \qquad (z \in \mathbb{U}), \tag{5}$$

$$(1 - \lambda)\frac{(f * h)^{-1}(w)}{w} + \lambda((f * h)^{-1})'(w) = q(w) \quad (w \in \mathbb{U}), \tag{6}$$

respectively, where functions $p(z)$ and $q(w)$ satisfy the conditions of Definition 5. Furthermore, the functions $p(z)$ and $q(w)$ have the following Taylor-Maclaurin series expansions:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 ... \tag{7}$$

and

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 ... \quad , \tag{8}$$

respectively. Now, upon substituting from (7) and (8) into (5) and (6), respectively, and equating the coefficients, we get

$$(\lambda + 1)a_2 h_2 = p_1, \tag{9}$$

$$(2\lambda + 1)a_3 h_3 = p_2, \tag{10}$$

$$-(\lambda + 1)a_2 h_2 = q_1 \tag{11}$$

and

$$2(2\lambda + 1)a_2^2 h_2^2 - (2\lambda + 1)a_3 h_3 = q_2. \tag{12}$$

From (9) and (11), we obtain

$$p_1 = -q_1, \tag{13}$$

$$a_2^2 = \frac{p_1^2 + q_1^2}{2(\lambda + 1)^2 h_2^2}. \tag{14}$$

By adding (10) and (12), we get

$$a_2^2 = \frac{p_2 + q_2}{2(2\lambda + 1)h_2^2}. \tag{15}$$

Therefore, we find from the equations (14) and (15) that

$$|a_2| \leq \frac{1}{h_2(\lambda + 1)} \sqrt{\frac{|p'(0)|^2 + |q'(0)|^2}{2}}$$

and

$$|a_2| \leq \frac{1}{2h_2} \sqrt{\frac{|p''(0)| + |q''(0)|}{2\lambda + 1}},$$

respectively. So we get the desired estimate on the coefficient $|a_2|$ asserted. Next, in order to find the bound on the coefficient $|a_3|$, we subtract (12) from (10). We thus get

$$2(2\lambda+1)a_3h_3 - 2(2\lambda+1)a_2^2h_2^2 = p_2 - q_2. \tag{16}$$

Upon substituting the value of $a_2^2$ from (14) into (16), it follows that

$$a_3 = \frac{p_1^2+q_1^2}{2h_3(\lambda+1)^2} + \frac{p_2-q_2}{2h_3(2\lambda+1)}. \tag{17}$$

We thus find that

$$|a_3| \leq \frac{|p'(0)|^2+|q'(0)|^2}{2h_3(\lambda+1)^2} + \frac{|p''(0)|+|q''(0)|}{4h_3(2\lambda+1)}.$$

On the other hand, upon substituting the value of $a_2^2$ from (15) into (16), it follows that

$$a_3 = \frac{p_2+q_2}{2h_3(2\lambda+1)} + \frac{p_2-q_2}{2h_3(2\lambda+1)}. \tag{18}$$

Consequently, we have

$$|a_3| \leq \frac{|p''(0)|}{2h_3(2\lambda+1)}.$$

## 3. Corollaries and Consequences

By setting

$$h(z) = p(z) = (\frac{1+z}{1-z})^{\alpha} \qquad (0<\alpha\leq 1,\ z\in\mathbb{U}),$$

in Theorem 5, we obtain the following result.

**Corollary 6.** *Let the function $f(z)$ given by* (1) *be in the bi-univalent function class* $\mathcal{B}_\Sigma(h,\alpha,\lambda)$ $(0<\alpha\leq 1;\ \lambda\geq 1)$. *Then*

$$|a_2| \leq min\left\{\frac{2\alpha}{h_2(\lambda+1)}, \frac{\alpha}{h_2}\sqrt{\frac{2}{2\lambda+1}}\right\}$$

*and*

$$|a_3| \leq \frac{2\alpha^2}{h_3(2\lambda+1)}.$$

**Remark 2.** *The bounds on $|a_2|$, $|a_3|$ given in Corollary 6 are better than those given by El-Ashwah[6, Theorem1].*

By setting $h(z) = \frac{z}{1-z}$ and $\lambda = 1$ in Corollary 6, we conclude the following corollary.

**Corollary 7.** *Let the function $f(z)$ given by (1) be in the bi-univalent function class $\mathcal{H}_{\Sigma}^{\alpha}$ $(0 < \alpha \leq 1)$. Then*

$$|a_2| \leq min\{\alpha, \sqrt{\frac{2}{3}}\alpha\} = \sqrt{\frac{2}{3}}\alpha$$

*and*

$$|a_3| \leq \frac{2}{3}\alpha^2.$$

**Remark 3.** *The bounds on $|a_2|$, $|a_3|$ given in Corollary 7 are better than those given in Theorem 1. Because*

$$\sqrt{\frac{2}{3}}\alpha \leq \alpha\sqrt{\frac{2}{\alpha+2}}$$

*and*

$$\frac{2}{3}\alpha^2 \leq \alpha^2 + \frac{2}{3}\alpha.$$

By setting $h(z) = \frac{z}{1-z}$ in Corollary 6, we conclude the following corollary.

**Corollary 8.** *Let the function $f(z)$ given by (1) be in the bi-univalent function class $\mathcal{B}_{\Sigma}(\alpha, \lambda)$ $(0 < \alpha \leq 1, \lambda \geq 1)$. Then*

$$|a_2| \leq min\{\frac{2\alpha}{\lambda+1}, \alpha\sqrt{\frac{2}{2\lambda+1}}\ \}$$

*and*

$$|a_3| \leq \frac{2\alpha^2}{2\lambda+1}.$$

**Remark 4.** *The bounds on $|a_2|$, $|a_3|$ given in Corollary 8 are better than those given in Theorem 3. Because*

$$\frac{2\alpha}{\lambda+1} \leq \frac{2\alpha}{\sqrt{(\lambda+1)^2 + \alpha(1+2\lambda-\lambda^2)}} \qquad (\lambda \geq 1+\sqrt{2})$$

*and*

$$\frac{2\alpha^2}{2\lambda+1} \leq \frac{4\alpha^2}{(\lambda+1)^2} + \frac{2\alpha}{2\lambda+1}.$$

By setting

$$h(z) = p(z) = \frac{1+(1-2\beta)z}{1-z} \qquad (0 \leq \beta < 1,\ z \in \mathbb{U}),$$

in Theorem 5, we obtain the following result.

**Corollary 9.** *Let the function $f(z)$ given by (1) be in the bi-univalent function class $\mathcal{B}_\Sigma(h,\beta,\lambda)$ $(0 \leq \beta < 1, \lambda \geq 1)$. Then*

$$|a_2| \leq min\{\frac{2(1-\beta)}{h_2(\lambda+1)}, \frac{1}{h_2}\sqrt{\frac{2(1-\beta)}{2\lambda+1}}\}$$

*and*

$$|a_3| \leq \frac{2(1-\beta)}{h_3(2\lambda+1)}.$$

**Remark 5.** *The bounds on $|a_2|$, $|a_3|$ given in Corollary 9 are better than those given by El-Ashwah[6, Theorem 2].*

By setting $h(z) = \frac{z}{1-z}$ and $\lambda = 1$ in Corollary 9, we conclude the following corollary.

**Corollary 10.** *Let the function $f(z)$ given by (1) be in the bi-univalent function class $\mathcal{H}_\Sigma(\beta)$ $(0 \leq \beta < 1)$. Then*

$$|a_2| \leq \begin{cases} \sqrt{\frac{2}{3}(1-\beta)} & ;\ 0 \leq \beta \leq \frac{1}{3} \\ (1-\beta) & ;\ \frac{1}{3} \leq \beta < 1 \end{cases}$$

*and*

$$|a_3| \leq \frac{2}{3}(1-\beta).$$

**Remark 6.** *The bound on $|a_2|$, $|a_3|$ given in Corollary 10 are better than those given in Theorem 2.*

By setting $h(z) = \frac{z}{1-z}$ in Corollary 9, we conclude the following corollary.

**Corollary 11.** *Let the function $f(z)$ given by (1) be in the bi-univalent function class $\mathcal{B}_\Sigma(\beta,\lambda)$ $(0 \leq \beta < 1, \lambda \geq 1)$. Then*

$$|a_2| \leq min\{\frac{2(1-\beta)}{\lambda+1}, \sqrt{\frac{2(1-\beta)}{2\lambda+1}}\}$$

*and*

$$|a_3| \le \frac{2(1-\beta)}{2\lambda+1}.$$

**Remark 7.** *The bounds on $|a_2|$, $|a_3|$ given in Corollary 11 are better than those given in Theorem 4. Because*

$$\frac{2(1-\beta)}{(\lambda+1)} \le \sqrt{\frac{2(1-\beta)}{2\lambda+1}} \qquad (\lambda \ge 1-2\beta+\sqrt{4\beta^2-6\beta+2};\ 0 \le \beta \le \frac{1}{3})$$

*and*

$$\frac{2(1-\beta)}{(2\lambda+1)} \le \frac{4(1-\beta)^2}{(\lambda+1)^2} + \frac{2(1-\beta)}{2\lambda+1}.$$

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