MIXED CONVECTION ON A VERTICAL FLAT PLATE EMBEDDED IN A FLUID SATURATED POROUS MEDIA

by
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Abstract: During the last five decades the convective flow through porous media has been the subject of many investigation many practical applications can be modeled as transport phenomena in porous media: geothermal energy extraction, storage of nuclear waste material, groundwater flow, industrial and agricultural water distribution, oil recovery processes, thermal insulation engineering, cooling of electronic components, packed-bed reactors, food processing. A large number of physical phenomena involve free convection driven by internal heat generation. The most important applications are in the field of nuclear energy and also to fire and combustion modeling, the development of metal waste form from spent nuclear fuel and for storage of spent nuclear fuel. Literature concerning these processes can be found in recent books by Ingham and Pop (1998), Nield and Bejan (1999) and Vafai (2000), Pop and Ingham (2001).

The boundary layer theory developed first by Prandl (1904) has a great importance in Newtonian fluids because we can simplify the Navier-Stokes equations. In 1960's Wooding(1963) used boundary layer assumptions to solve the equations which govern flow and heat transfer in porous media. Several models were proposed to explain mathematical and physical aspects associated with boundary layer flow and convective heat transfer in porous media. Among this, the Darcy law gained many acceptances. Boundary layer assumptions were successfully applied to these models and much work has been done on them for different body geometries in the last three decades.

Basic Equations

We consider an uniform flow with constant velocity U vertically past a semi infinite vertical flat plate at the temperature T_w which is embedded in a fluid saturated porous medium at the temperature T_∞ (see Fig. 1). We also consider that the physical phenomenon of internal heat generation is present. Using the boundary layer and the Boussinesq approximation

$$\rho = \rho_\infty\left[1 - \beta\left(T - T_\infty\right)\right]$$

the governing equations are given by the continuity equation, the Darcy's law and the energy equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=U+\frac{g\beta K}{\nu}(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{q'''}{\rho c_p} \tag{3}$$

The boundary conditions are:

$$\begin{aligned} &v=0 \quad T=T_w \qquad \text{at } y=0,\ x\geq 0\\ &u\to U \quad T\to T_\infty \qquad \text{as } y\to\infty,\ x\geq 0 \end{aligned} \tag{4}$$

Using the transformation

$$\psi=(2\alpha Ux)^{1/2}f(\eta),\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\eta=y\left[\frac{U}{2\alpha x}\right]^{1/2}$$

where ψ is the stream function defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ and Eqs.(1) –(4) the governing equations become

$$f'=1+\bar{\lambda}\,\theta \tag{5}$$

$$\theta''+f\theta'+e^{-\eta}=0 \tag{6}$$

subject to the boundary conditions

$$\begin{aligned} &f(0)=0, f'(0)=1+\bar{\lambda}, f'(\infty)=1.\\ &\theta(0)=1,\ \theta(\infty)=0 \end{aligned} \tag{7}$$

The parameter $\bar{\lambda}$ is called the mixed convection parameter or the buoyancy parameter defined and it is defined as:

$$\bar{\lambda}=\frac{g\beta K(T_w-T_\infty)}{U\nu}=\frac{Ra}{Pe}=\frac{Gr}{\mathrm{Re}}$$

where Ra is the Rayleigh number, Pe is the Peclet number, Gr is the Grashof number and Re is the Reynolds number

$$Ra=\frac{gK\beta(T_w-T_\infty)l}{\alpha_f\nu},\quad Pe=\frac{Ul}{\alpha},\quad Gr=\frac{g\beta K(T_w-T_\infty)l}{\nu^2},\quad \mathrm{Re}=\frac{Ul}{\nu}.$$

We notice that for $\bar{\lambda}>0$ $T_w>T_\infty$ (the plate is heated) and for $\bar{\lambda}<0$ $T_w<T_\infty$ (the plate is cooled). In order to obtain similarity solutions we use for the term of internal heat generation the form proposed by Crepeau şi Clarksean (1997):

$$q'''=\frac{\rho c_p(T_w-T_\infty)U_\infty}{2x}e^{-\eta} \tag{8}$$

Case $\bar{\lambda}=0$.

For $\overline{\lambda} = 0$ using equation (4) and the boundary conditions (7) we obtain $f(\eta) = \eta$, and equation (6) becomes:

$$\theta'' + \eta\theta' + e^{-\eta} = 0 \tag{9}$$

Equation (9) have the following analytic solution:

$$\theta(\eta) = \left[\int_0^\infty \left(\int_0^\eta e^{-\eta+\frac{\eta^2}{2}} d\eta\right) e^{-\frac{\eta^2}{2}} d\eta - 1\right] erf\left(\frac{\eta}{\sqrt{2}}\right) - \int_0^\eta \left(\int_0^\eta e^{-\eta+\frac{\eta^2}{2}} d\eta\right) e^{-\frac{\eta^2}{2}} d\eta + 1 \tag{10}$$

where $$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

Case $\overline{\lambda} \neq 0$.

In the case of $\overline{\lambda} \neq 0$ we can reduce equation (5) and (6) at only one equation

$$f''' + ff'' + \overline{\lambda} e^{-\eta} = 0 \tag{11}$$

which have to be solved subject to the boundary conditions

$$f(0) = 0, f'(0) = 1+\overline{\lambda}, f'(\infty) = 1. \tag{12}$$

We can see that if the internal heat generation is absent equation (11) is similar with that obtained by Merkin (1980) and Harris et all.(1999).

Results and Discussions

When the internal heat generation is absent for $\overline{\lambda} > 0$ the solution of equation (12) can be obtain for all $\overline{\lambda}$ (see, Merkin 1980) and for $\overline{\lambda} < 0$ we have solution only for -1.354 ≤ $\overline{\lambda}$ ≤ 0. If -1.354 < $\overline{\lambda}$ < -1 we have a dual solution. This can be seen from Fig 2. where $f''(0)$ is plotted against $\overline{\lambda}$. In Table 1 we present some values for $f''_1(0)$ şi $f''_2(0)$ where subscripts 1 and 2 denote dual solutions. We also can see that for $\overline{\lambda} = 0$, $f(\eta) = \eta$ and for $\overline{\lambda} = -1$ we have the Blasius solution.

$\overline{\lambda}$	$f''_1(0)$	$f''_2(0)$
-1.00	0.46960	
-1.05	0.46758	0.00004
-1.10	0.46105	0.00194
-1.15	0.44907	0.00866
-1.20	0.43015	0.02219
-1.25	0.40152	0.04539
-1.30	0.35664	0.08497
-1.35	0.25758	0.17856
-1.354	0.22428	

Table 6. The values of skin frictions coefficients in the case of dual solutions

In Fig. 3 we can see the velocity profiles for different values of $\overline{\lambda}$, namely $\overline{\lambda}$ = 1,0.5, 0, -0.5, -1. In the case of dual solution, for -1.354 < $\overline{\lambda}$ < -1, the upper branch solution

is the valid solution, while the lower branch solution are unstable. In the Fig. 4 the valid solution is plotted with full line and the unstable solution is plotted with broken line.If the internal heat generation is present we notice that we have solution only for $0.94 \le \bar{\lambda} \le \infty$ (see Fig. 5). In Fig. 6 we present the velocity profiles for some values of $\bar{\lambda}$, $\bar{\lambda}$=-0.94, -0.8, -0.7, -0.5, -0.3, -0.1, 0.1, 0.3, 0.5 and 1.We can see that the effect of internal heat generation is greater for negative values of the mixed convection parameter $\bar{\lambda}$.
If $T_w > T_\infty$ ($\lambda > 0$) the free stream and the buoyancy forces are in the same direction and for $T_w < T_\infty$ ($\lambda <0$) the free stream and the buoyancy forces are in the opposite direction. These situations are very well illustrated in Fig. 3 and 6.

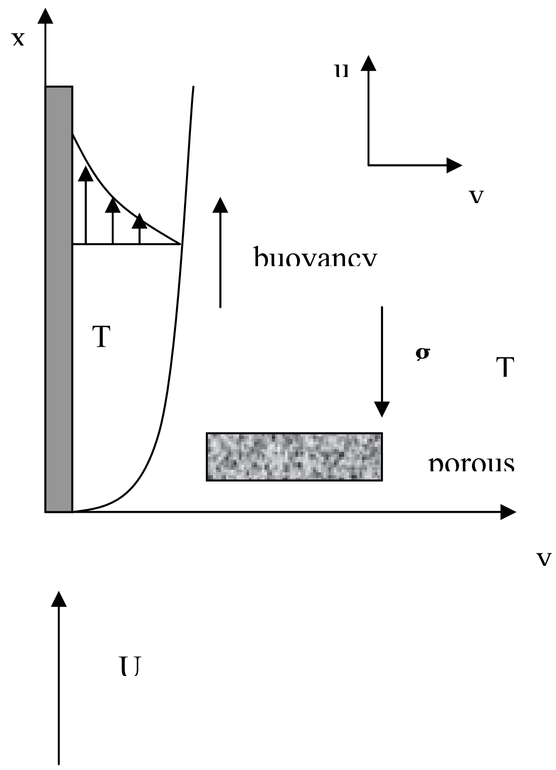

Fig. 1. Definition sketch for mixed convection over a vertical flat plate

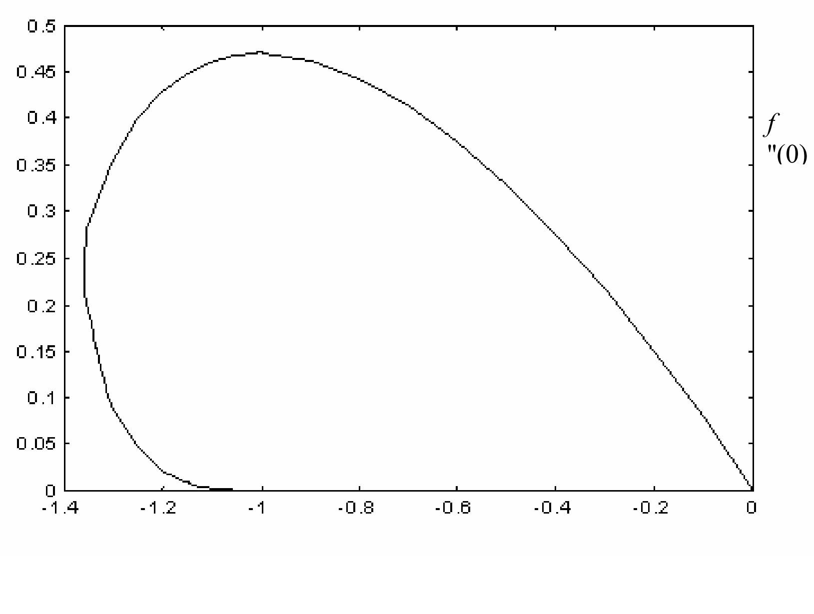

Fig. 2. Variation of $f''(0)$ with $\overline{\lambda}$ when internal heat generation is absent

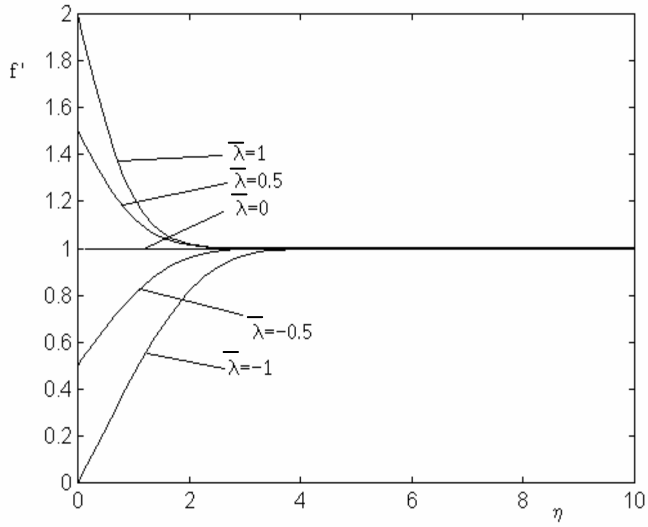

Fig. 3. Velocity profiles for $\overline{\lambda}$ = 1, 0.5, 0, -0.5, -1 when internal heat generation is absent

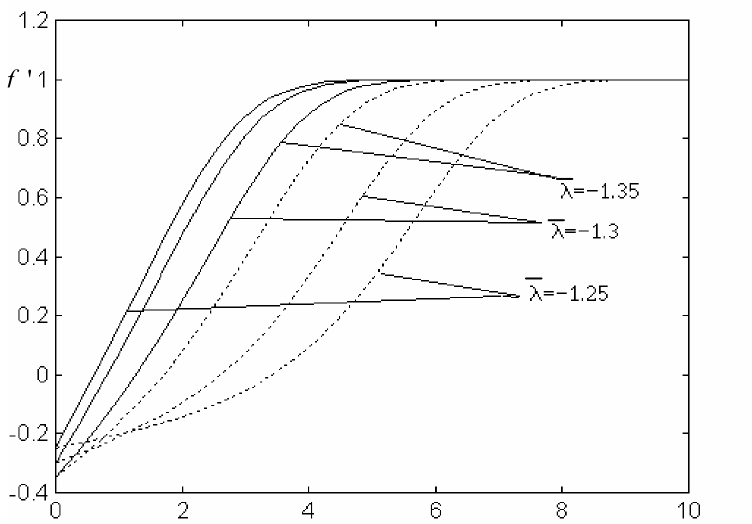

Fig. 4. Dual solution for different values of $\bar{\lambda}$. (– valid solution, ... instable solution) when internal heat generation is absent

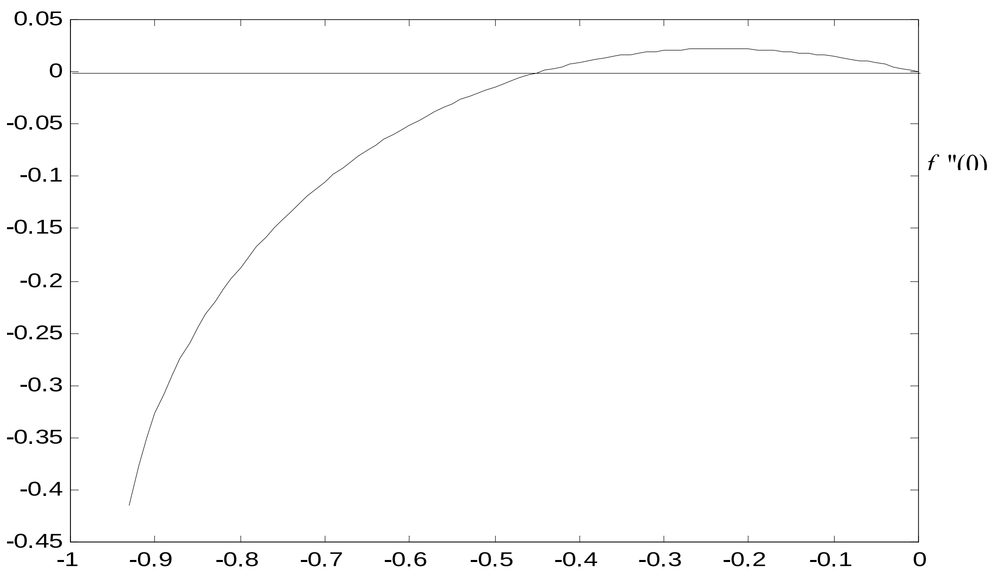

Fig. 5. Variation of $f''(0)$ with $\bar{\lambda}$ when internal heat generation is present

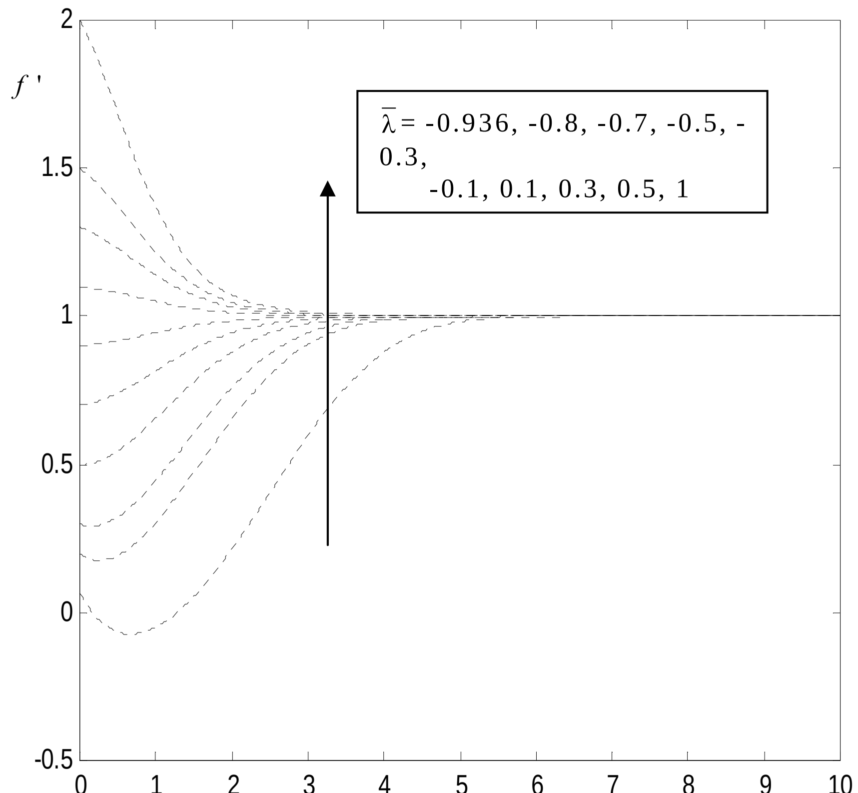

Fig. 6. Velocity profiles for different values of $\overline{\lambda}$ when internal heat generation is present

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