ON SOME SECOND-ORDER NONLIMBAR BOUNDARY VALUE PROBLEMS

by
Dorina Trifon

Abstract: In this article, a problem of limit for differential equations of second order is studied. In the conditions imposed the problem (1) has a unique solution, solution obtained by using the method of successive approximations.The problem (1) could be written as an integral equation and for the computation of the integrals we applied more quadrature formulae among which the trapeze formula, too.When comparing the numerical results displayed in the tables, we infer that the trapaye formula gives a more exact approximation.

In this paper we give a method for calculating the solution of the following problems

(1) $$\begin{cases} y^n = f(x, y) \\ y(a) = A, y(b) = B \end{cases}$$

derived from the Euler-Maclauren formula and a Fadeeva's numerical derivation formula.

The solution y of the boundary value problem (1) satisfies the identity:

(2) $$y(x) = H(y) + \int_a^b G(x,s)\, f(s, y(s))ds$$

where

(3) $$H(x) = \frac{A(b-x) + B(x-a)}{b-a}$$

and G is the Green's function:

(4) $$G(x,s) = \begin{cases} -\dfrac{(b-x)(s-a)}{b-a} & a \le s \le x \\ -\dfrac{(x-a)(b-s)}{b-a} & x \le s \le b \end{cases}$$

The Picard's iterations are:

(5) $$y_{m+1}(x) = H(x) + \int_a^b G(x,s)\, f(s, y_m(s))ds\,; \qquad m = 0,1,2,...$$

where H and G are defined in (3) and (4) and do not change with each iterations.

When calculating an iteration of the Picard method we use the following formula:

$$(6)\int_a^b f(x)dx = \frac{b-a}{n}\left[\frac{f(a)+f(b)}{2}+\sum_{i=1}^{n-1} f(x_i)\right]+\frac{(b-a)^2}{10n^2}\left[f'(a)-f'(b)\right]+$$
$$+\frac{(b-a)^3}{s!n^3}\left[f''(a)+f''(b)+2\sum_{i=1}^{n-1} f''(x_i)\right]+R$$

where

$$(7) \qquad |R| \le M \cdot \frac{(b-a)^7}{20\cdot 7!\ n^6}$$

and M is the upper limit of $\left|f^{VI}(x)\right|$ in the interval $[a,b]$. This formula is derived from the $0(h^6)$ numerical quadrature formula:

$$(8) \qquad \int_a^b f(x)dx = \frac{b-a}{2}\left[f(a)+f(b)\right]+\frac{(b-a)^2}{10}\left[f'(a)-f'(b)\right]+$$
$$+\frac{(b-a)^3}{5!}\left[f''(a)+f''(b)\right]+R$$

which was applied on each subinterval $[x_i, x_{i+1}]$ where

$x_i = a+ih$, $i = 0,1,...,n$ and $h = \frac{b-a}{n}$.

Using the "φ" function method by D.V. Ionescu [1], we found the numerical quadrature formula (8).

We have

Theorem 1. The rest of (8) is:

$$(9) \qquad R = \int_a^b \varphi(x) f^{VI}(x)dx$$

and φ-function is negative on the interval $[a,b]$.

Also we use the nimerical derivation formula

$$\Delta^4 f(x_1) = -\frac{h}{2} f'(x_1) + h\ f'(x_2) - hf'(x_4) + \frac{h}{2} f'(x_5) + R$$

with

$$\Delta^4 f(x_1) = f(x_5) - C_4^1 f(x_4) + C_4^2 f(x_3) - C_4^3 f(x_2) + C_4^4 f(x_1)$$

and

$$R = \int_{x_1}^{x_5} \varphi(x) \quad f^{VI} \quad (x)dx$$

where

$x_{i+1} = x_{i+h}; i = 1,2,3,4.$

This formula is a particualry case of Fadeeva's numerical derivation formula which was studied by D.V. Ionescu [2].

We consider the following problem to illustrate the new method:

$$\begin{cases} y^{n} = 0,5(1+x+y)^3 \\ y(0)=0, \quad y(1)=0 \end{cases}$$

The exact solution is :

$$y(x) = \frac{2}{2-x} - x - 1.$$

We use this new method for solving this problem and the other one based on the trapesoid rule[3].

We found the following results:

m	$\max_i \lvert y_m(x_1) - y_{m-1}(x_1) \rvert$
1	0.636458 D 00
5	0.219707 D-01
10	0.219707 D-03
15	0.219707 D-05

Rate of convergence with $h = \frac{1}{20}$ and using the trapezoid rule

m	$\max_i \lvert y_m(x_1) - y_{m-1}(x_1) \rvert$
1	0.212636 D 00
5	0.282373 D-01
10	0.707769 D 00
14	0.351635 D 05
15	0.148760 D 14

Rate of convergence with $h = \frac{1}{20}$ and using the $0(h^6)$ numerical quadrature formula.

The next table shows the accuracy of the limizing solution vor various integration methods:

$h = \frac{1}{20}$	1.3519 D - 4	0.1487 D 14
$h = \frac{1}{20}$	4.6395 D - 5	0.7338 D 13

$\max_i \lvert y(x_1) - y_m(x_1) \rvert$

REFERENCES

[1] D.V. Ionescu, *Cuadraturi numerice*, Bucuresti, 1957.
[2] D.V. Ionescu, *Généralisation d'une formule de derivation numerique de V.M. Fadeeva*, Annales Polonici Matematici, M(1964), 169-174. 1957
[3] D.S. MEEK and R.A. USMANT, *Some experiments with Picard's iteration for second-order nonlinear boundary value problems*, Congressus Numeratium 46(1985)University of Cluj Napoca,Kogălniceanu Nr.1,3400 CLUJ NAPOCA ROMANIA

Author:

Dorina Trifon - Military College "Mihai Viteazul" Alba Iulia, e-mail: trifon_dorina@personal.ro