THE ROLE OF ELITISM IN MULTIOBJECTIVE OPTIMIZATION WITH EVOLUTIONARY ALGORITHMS

Crina Groşan[1], Mihai Oltean[1], Mihaela Oltean[2]

Abstract. This paper studies the influence of elitism in Evolutionary Multiobjective Optimization. The true efficiency of inserting elitism in these algorithms was not yet sufficiently developed. Many algorithms for multiobjective optimization use different methods to keep the best individuals (the elite) founded on the duration of the search process. The problem is how this elitism can be preserved and, more than that, how this elitism can be efficiently incorporated in evolutionary algorithms. The paper presents some algorithms which use elitism and some algorithms which not use elitism and realizes a comparison of some algorithms for each category.

1. Introduction

Evolutionary algorithms have shown their efficiency for approximating the Pareto set of multiobjective optimization problems. Several surveys in multiobjective evolutionary algorithms can be founded in the literature. ([1], [4], [13]). While most of these algorithms were designed with regard to two common goals – fast and reliable convergence to the Pareto front and a good distribution of solutions along the front – each algorithm represents a unique combination of specific techniques to achieve these goals.

Laumans et al. ([9]) suggest that two things have remained open up to now:

- how a certain parameter or a certain operator affects the overall performance independent of the specific implementation and the other technique used;
- how the parameters and the operators influence each others performance.

Recently, several authors indicate that elitism could improve evolutionary multiobjective search significantly ([10], [11], [14]).

These assumptions are naturally based on a very individual notion of 'elitism'. In [8] Laumans et al. indicated that in the multiobjective case the followings are remained unclear up to now:

- how to define elite individuals or solutions;
- how to best incorporate the information from this elite effectively into the search ;
- which are the effects of elitism on evolutionary search for different algorithms and classes of multiobjective problems.

Zitzler ([17]) formulates two questions in what concern elitism:

- which individuals are kept for how long in the elite set;
- When and how are (and which) members of the elite set reinserted into the population.

2. An explanation of the elitism

The notion of elitism is strongly connected by the acceptance of newly generated solution. The algorithms use various modalities to assure an elitist character. Some algorithms use a secondary population (an archive) where the nondominated solutions founded are stored. The algorithms implement different modalities so that the elite take part to the production of offspring. According to the authors' notions elitism means that elite individuals cannot be exuded from the archive gene pool of the population in favor of worse individuals.

De Jong ([2]) suggested a policy to always include the best individual of the current population in the population of the next generation in order to prevent losing it due sampling effects of operator disruption. This strategy can be extended to copy the best *n* individuals. This is explanation of the elitism. In his experiments, De Jong found that elitism can improve the performance of a genetic algorithm on unimodal functions while, for multimodal functions it may cause premature convergence.

3. Elitist and non-elitist approaches

In what follows we short present some algorithms which don't use elitism and some algorithms which use elitism (in different forms).

3.1. Non-elitist Evolutionary Algorithms

The Vector Evaluated genetic Algorithm proposed in [12] uses a nonelitist strategy. At each generation the best individuals on each dimension are selected for reproduction. Crossover and mutation operators are applied over the selected individuals. The obtained individuals are reinserted into population of the next generation.

The Multiobjective Genetic Algorithm (MOGA) proposed in [4] sort population according to the rank. The rank of a solution represents the number of solutions from population which dominates that solution. Starting to the best solutions the mating pool is filled and the genetic operators (crossover and mutation) are applied.

The Niched Pareto Genetic Algorithm (NPGA) introduced by Horn and Nafpliotis in [5] combines tournament selection and the concept of Pareto dominance. A special mechanism of selection is used in order to fill the mating pool. Two individuals are randomly choose from the population and for each individual a set of solution for comparison are also choose from the population. The individual who is dominated by less individual from his set of comparison is preferred. In an equally case the agglomeration decides the winner.

3.2. Elitist Evolutionary Algorithms

The Strength Pareto Evolutionary Algorithm (SPEA) ([15], [17]) and its recent variant SPEA 2 ([16]) use an external population (an archive) where all nondominated solutions founded are stored. The archive is update at each generation and all solutions from archive are nondominated (respecting the archive). The solutions from united current and external population take part to selection, recombination and mutation.

Parks and Miller also use in their algorithm proposed in [10] an archive of nondominated solutions. A random subset of this archive is reinserted into the population at each generation.

The Pareto Archive Evolution Strategy (PAES) proposed in [6] (see also, [7]) is a multiobjective (1 + 1) Evolution Strategy. The domination relation is the selection criterion. This algorithm uses also an external population (an archive). The archive is use only as a comparison set for incomparable individuals (the density of solution in this archive decides which solution will be kept). In these conditions the elitism is guaranteed.

The Nondominated Sorting Genetic Algorithm (NSGA II) ([3]) is another algorithm which uses elitism. All nondominated levels of solutions are detected from each population. After genetic algorithms application from resulted population of parents and offspring are introduced solutions in the population of the next generation starting with the first level of nondominance and the followings until a prescribed number of solutions are introduced.

4. Numerical experiments

We will compare in this section two evolutionary algorithms: SPEA which uses elitism and VEGA which don’t uses elitism. For this comparison three test functions are used. These functions have been introduced in [13] and are presented below.
Each test function considered is built by using three functions f_1, g, h.
Let us define T(x) = ($f_1(x)$, $f_2(x)$). The optimization problem is:
Minimize T(x),
subject to $f_2(x) = g(x_2,\ldots,x_m)h(f_1(x_1),g(x_2,\ldots,x_m))$,
where x = $(x_1,..,x_m)$.
The parameters used by each of these algorithms are presented in Table 1.

Parameter	Value
Population size	100
Number of generations	250
Crossover rate	0.8
Mutation rate	0.01

4.1 Experiment 1

The test function T_1 is defined using the functions:
where m = 30 and $x_i \in [0,1]$.

$$f_1(x_1) = x_1,$$

$$g(x_2,\ldots.x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i / (m-1),$$

$$h(f_1, g) = 1 - \sqrt{f_1/g},$$

Pareto optimal front for the problem T_1 is convex and it is formed with $g(x) = 1$.

For the test function T1 the result obtained by these two algorithms are depicted in Figure 1.

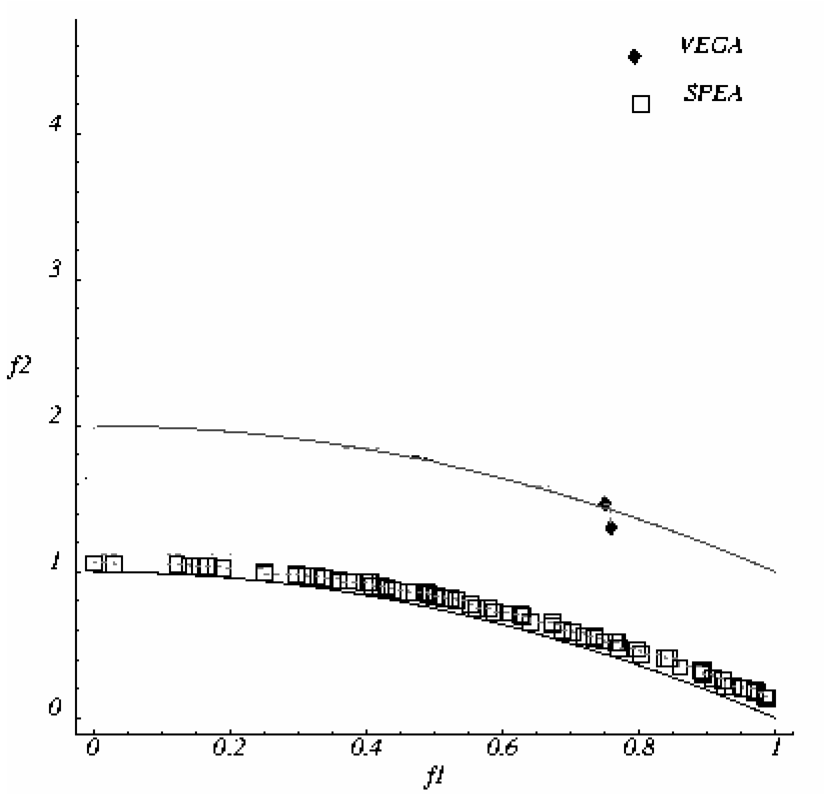

Figure 1. The results obtained for test function T1 by SPEA and VEGA.

From Figure 1 we can see that the result obtained by SPEA is better than the result obtained by VEGA. SPEA converge to the true Pareto front while VEGA stops in a local Pareto front.

4.2 Experiment 2

The test function T_2 is defined by considering the following functions:

$$f_1(x_1) = x_1$$
$$g(x_2,\ldots.x_m) = 1 + 9 \cdot \sum_{i=2}^{m} x_i / (m-1)$$
$$h(f_1, g) = 1 - (f_1/g)^2$$

where $m = 30$ and $x_i \in [0,1]$.

The Pareto optimal front is formed with $g(x) = 1$. T_2 is the nonconvex counterpart to T_1.

The results obtained by SPEA and VEGA for this test function are depicted in Figure 2.

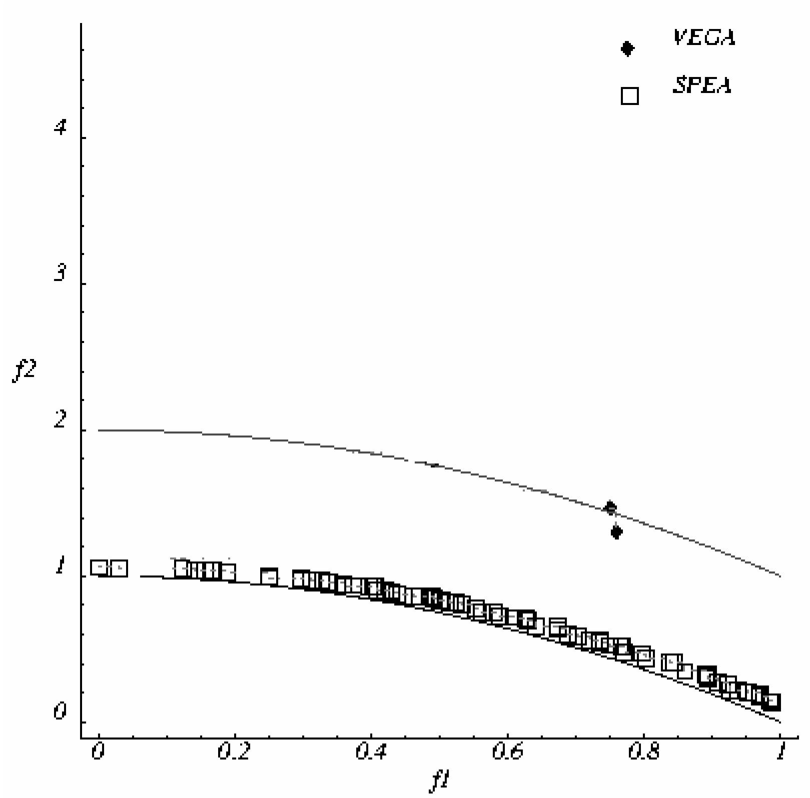

Figure 2. The results obtained by SPEA and VEGA for the test function T2.

As you can see, SPEA converge to the Pareto front. VEGA converge only to a local Pareto front. SPEA assure a good distribution of the solutions on the front.

4.3 Experiment 3

The test function T_4 contains 21^9 local Pareto optimal fronts and, therefore, it tests the EA ability to deal with multimodality. The involved functions are defined by:

$$f_1(x_1) = x_1$$
$$g(x_2,\ldots x_m) = 1 + 10(m-1) + \sum_{i=2}^{m}(x_i^2 - 10\cos(4\pi x_i))$$
$$h(f_1, g) = 1 - \sqrt{f_1/g}$$

where $m = 10$, $x_1 \in [0,1]$ and $x_2,\ldots,x_m \in [-5,5]$.

Global Pareto optimal front is formed with $g(x) = 1$. The best local Pareto optimal front with $g(x) = 1.25$.

Note that not all local Pareto optimal sets are distinguishable in the objective space.

The results Obtained by SPEA and VEGA are depicted in Figure 3.

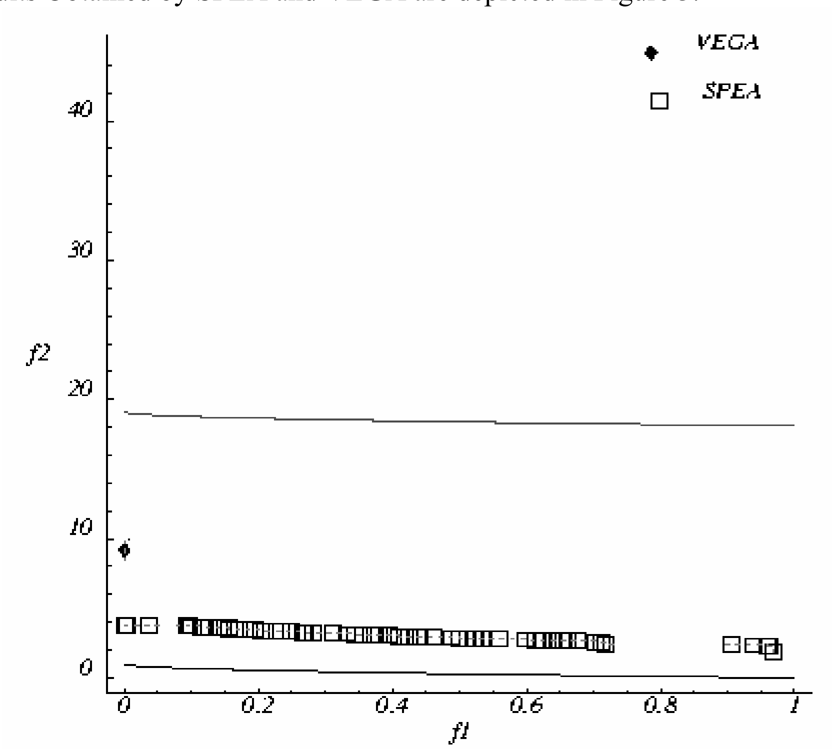

Figure 3. The results obtained by SPEA and VEGA for the test function T4.

Both SPEA and VEGA converge to a local Pareto front only. The result obtained by SPEA is better than the result obtained by VEGA regarding the convergence to the Pareto front and the diversity of solutions on the Pareto front.

5. Conclusions

From the experiments considered in the previous section we can see that the result obtained by an algorithm which uses elitism is better than the result obtained by an algorithm which doesn't use elitism. Keeping in the population of the next generations the best individuals of the current populations seems to be a very good idea. Note that the performance of SPEA is not the best from the existing techniques (SPEA 2, for instance is better than SPEA) and VEGA is not a worst algorithm which don't use elitism (is better, for instance like MOGA).

All recent algorithms use different forms of elitism.

References

1. Carlos A. Coello Coello. A comprehensive survey on evolutionary based multiobjective optimization. Knowledge and Information Systems, 1(3): 269-308, 1999.
2. K. A. De Jong. An analysis of the behavior of a class of genetic adaptive systems. PhD thesis, University of Michigan.
3. Deb, K., Agrawal, S., Pratap, A., Meyarivan, T. A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multiobjective Optimization: NSGA II, Kanpur Genetic Algorithms Laboratory (KanGal), Indian Institute of Technology Kanpur, Kanpur, PIN 208 016, India, KanGal Report No. 200001, 2001.
4. C. M. Fonseca and P.J. Fleming. An overview of evolutionary algorithms in multiobjective optimization. Evolutionary Computation, 3(1): 1-16, 1995.
5. J. Horn, N. Nafpliotis and D. Goldberg. A niched Pareto genetic algorithm for multiobjective optimization. In proceeding of the first IEEE Conference on Evolutionary Computation, IEEE World Congress on Computational Computation, 1: 82-87, Piscataway, NJ, 1994.
6. Knowles, J.D., Corne, D., The Pareto Archived Evolution Strategy: A New Baseline Algorithm for Multiobjective Optimization, In Congress on Evolutionary Computation, p. 98-105, Washington D.C., July 1999;
7. Knowles, J.D., Local-Search and Hybrid Evolutionary Algorithms for Pareto Optimization, PhD Thesis, The University of Reading, Reading, UK, 2002;
8. M. Laumans, E. Zitzler, and L. Thiele. A unified model for multiobjective evolutionary algorithms with elitism. In Congress on Evolutionary computation (CEC 2000), 1: 46- 53, Piscataway, NJ, 2000.
9. M. Laumans, E. Zitzler, and L. Thiele. On the effects of archiving, elitism and density based selection in evolutionary multiobjective optimization.
10. G. T. Parks and I. Miller. Selective breeding in a multiobjective genetic algorithm. In A. E. Eiben, T. Bäck, M. Schoenauer and H. p. Schwefel (Eds.), Fifth International Conference on Parallel Problem Solving from Nature (PPSN-V), 250-259, Berlin, Germany, 1998.

11. S. Obayashi, S. Takahashi, and Y. Takeguchi. Niching and elitist models for mogas. In A. E. Eiben, T. Bäck, M. Schoenauer and H. p. Schwefel (Eds.), Fifth International Conference on Parallel Problem Solving from Nature (PPSN-V), 260-269, Berlin, Germany, 1998.
12. J. D. Schaffer. Multiple objective optimization with vector evaluated genetic algorithms. In J. J. Grefenstette (Ed), Proceeding of an International Conference on Genetic Algorithms and their Applications, Pittsburg, PA, 93-100, 1985.
13. D. A. van Veldhuizen. Multiobjective Evolutionary Algorithms: Classifications, Analyses and New Inovations. PhD thesis, Graduate School of Engineering of the Air Force Institute of Technology, Air University, 1999.
14. E. Zitzler, K. Deb and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. Evolutionary Computation 8(2): 173-195, 2000.
15. Zitzler, E., and Thiele, L., Multiobjective evolutionary algorithms: A comparative case-study and the Strength Pareto Approach, IEEE Transactions on Evolutionary computation 3(4), p. 257-271, 1999.
16. Zitzler, E., Marco Laumanns and Thiele, L., SPEA 2: Improving the Strength Pareto Evolutionary Algorithm, TIK Report 103, Computer Engineering and Networks Laboratory (TIK), Departament of Electrical Engineering Swiss federal Institute of Technology (ETH) Zurich, 2001.
17. Zitzler, E., *Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications*, Ph. D. thesis, Swiss Federal Institute of Technology (ETH) Zurich, Switzerland, 1999.

[1] Department of Computer Science, Faculty of Mathematics and Computer Science, Babeş- Bolyai University of Cluj-Napoca,Kogălniceanu 1, Cluj-Napoca 3400
e-mail: {cgrosan, moltean}@cs.ubbcluj.ro
[2] Faculty of Sciences, Lucian Blaga University, Sibiu, e-mail: mihaelaoltean@personal.ro