A COMPARISON BETWEEN TWO INDICES OBTAINED BY MDF

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Abstract. A comparison between two indices obtained by FPM. It is well known that the price index obtained by FPM (factors path method) depends on the path. In this paper, we consider two particular paths namely the linear path and the exponential path. Then we make a comparison between the two indices from an axiomatically theory point of view.

A statistical index can be understood either by the algorithm used for the calculation of its values or by the related function proprieties.

For the index of integral variation of a variable $z = z(t)$ we have the formula $I_z^{k/j} = \frac{z(k)}{z(j)}$ where *j* and *k* are two values of *t*. Unlike of this, for the index of the factorial variation related to a variable $z(t) = f(x_1(t), x_2(t), ..., x_n(t))$ it have been propose more methods of calculus.

In this paper we focused on the factors path method (MDF).

Definition 1. The factorial index of a variable $z = z(t) = f(x_1(t), x_2(t), ... x_n(t))$ with respect to the x_i factor, measured in time interval (j, k) obtained by MDF method is given by the formula:

$$\mathrm{I}_{z/x_i}^{k/j} = \exp \int_{(P_j P_k)} \frac{f'_{x_i}(x_1, x_2, ... x_n)}{f(x_1, x_2, ... x_n)} dx \tag{1}$$

where $(P_{j,} P_k)$ represents the curve described by the factors evolution between the points $P_j(x_1(j)...x_n(j))$, $P_k(x_1(k)...x_n(k))$ from R^n that is parameterized by the equation $x_1 = x_1(t), ... x_n = x_n(t)$.

Remark 2. The factorial index MDF depends on the factors path.

Remark 3. If we want to measure the variation with t (time, space or socio-economical category) of a variable *z,* that is the value of a panel with *n* goods

$$z(t) = \sum_{i=1}^{n} p_i(t) q_i(t),$$

$p(t) = (p_1(t), p_2(t),..., p_n(t)), q(t) = (q_1(t), q_2(t),..., q_n(t))$ then the factorial index with respect to p is called the price index. Here p and q are the prices and the quantities vectors.

Definition 4. An index given by the formulas

$$\mathrm{I}_{z/p}^{k/j} = \mathrm{I}_{v/p}^{k/j} = \prod_{i=1}^{n} \mathrm{I}_{z/p_i}^{k/j} \qquad (2)$$

$$\mathrm{I}_{z/p_i}^{k/j} = \exp\int_{P_jP_k} \frac{v'_{p_i}}{v} dp_i = \exp\int_{P_jP_k} \frac{q_i}{\sum_{i=1}^{n} p_i q_i} dp_i = \exp\int_{t_0}^{t_1} \frac{q_1(t)p'_i(t)}{\sum_{i=1}^{n} q_i(t)p_i(t)} dt \qquad (2')$$

or

$$\mathrm{I}_{z/p}^{k/j} = \exp\int_{t_0}^{t_1} \frac{\sum_{i=1}^{n} q_i(t)p'_1(t)}{\sum_{i=1}^{n} q_i(t)p_i(t)} dt \qquad (2'')$$

is called the MDF price index.

Remark 5. The curve which links the two situations of price-quantity P_j, P_k is in general parametrised with respect to parameter $t \in [t_0, t_1]$, but it has an unknown form.

Case of a linear path-definition of the linear MDF price index

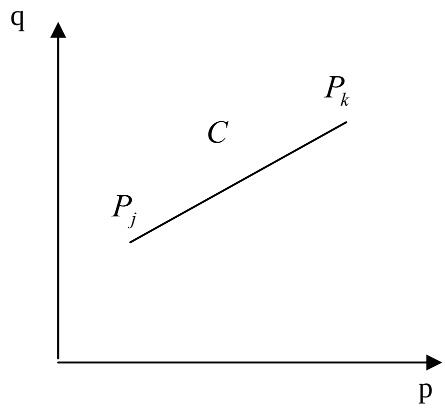
In the particullar case of a linear path given by the parametrised equation $x_i(t) = x_i(j) + t[x_i(k) - x_i(j)] = x_i(j) + t\Delta x_i$, $i = \overline{1,n}, t \in [0,1]$ the index from (1) become:

$$\mathrm{I}_{z/x_i}^{k/j} = \exp\int_0^1 \frac{f'_x(t)\Delta x_i}{f(t)} dt$$

For the calculation of a price index we use the parametrisation:

$$\begin{cases} p_i(t) = p_i(j) + t(p_i(k) - p_i(j)) = p_i(j) + t\Delta p_i \\ q_i(t) = q_i(j) + t(q_i(k) - q_i(j)) = q_i(j) + t\Delta q_i \end{cases} \qquad \text{with } i = \overline{1,n}, \; t \in [0,1]$$

In *2n* dimensional price-quantity space the situation can be represented as:



Then the index from the definition 4 become:

$$\mathrm{I}_{z/p_i}^{k/j} = \exp\int_0^1 \frac{\left[q_i(j)+t\Delta q_i\right]\Delta p_i}{\sum_{i=1}^{n}\left[q_i(j)+t\Delta q_i\right]\left(p_i(j)+t\Delta p_i\right)}dt \qquad (3)$$

or

$$\mathrm{I}_{z/p}^{k/j} = \exp\int_0^1 \frac{\sum_{i=1}^{n}\left[q_i(j)+t\Delta q_i(t)\right]\Delta p_i}{\sum_{i=1}^{n}\left[q_i(j)+t\Delta q_i\right]\left(p_i(j)+t\Delta p_i\right)}dt \qquad (3')$$

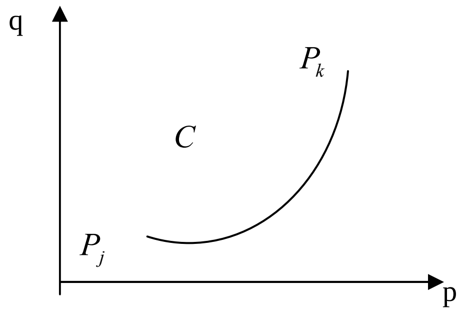
since $dp_i = \Delta p_i dt$.

Case of an exponential path-definition of the exponential MDF price index

After we use the parametrisation $x_i(t) = x_i(j)\left(\frac{x_i(k)}{x_i(j)}\right)^t$, $t \in [0,1]$, $i = \overline{1,n}$, the index from definition 4 becomes

$$I_{z/x_i}^{k/j} = \exp\int_0^1 \frac{f_x^{'}(t)x_i(j)\ln\frac{x_i(k)}{x_i(j)}x_i(k)}{f(t)}dt = \exp\int_0^1 \frac{f_x^{'}(t)x_i(j)x_i(k)\ln I_{x_i}^{k/j}}{f(t)}dt \qquad (4)$$

In case of the value of a panel $v = \sum_{i=1}^{n} p_i q_i$, we have the following representation in *2n* dimensional price-quantity space:



With the equations

$$\begin{cases} q_i(t) = q_i(j)\cdot\left(\dfrac{q_i(k)}{q_i(j)}\right)^t = q_i(j)\cdot\left(\mathrm{I}_{q_i}^{k/j}\right)^t \\ p_i(t) = p_i(j)\cdot\left(\dfrac{p_i(k)}{p_i(j)}\right)^t = p_i(j)\cdot\left(\mathrm{I}_{p_i}^{k/j}\right)^t \end{cases} \quad \text{with } i=\overline{1,n}\ , t\in[0,1]$$

the MDF price index becomes :

$$\mathrm{I}_{v/p}^{k/j} = \exp\int_0^1 \frac{\sum_{i=1}^{n} q_i(j)\cdot\left(\dfrac{q_i(k)}{q_i(j)}\right)^t \cdot p_i(j)\cdot\left(\dfrac{p_i(k)}{p_i(j)}\right)^t \ln\left(\dfrac{p_i(k)}{p_i(j)}\right)}{\sum_{i=1}^{n} q_i(j)\cdot p_i(j)\cdot\left(\dfrac{q_i(k)\cdot p_i(k)}{q_i(j)\cdot p_i(j)}\right)^t} dt =$$

$$= \exp\int_0^1 \frac{\sum_{i=1}^{n} q_i(j)p_i(j)\left(\dfrac{q_i(k)\cdot p_i(k)}{q_i(j)\cdot p_i(j)}\right)^t \ln\left(\dfrac{p_i(k)}{p_i(j)}\right)}{\sum_{i=1}^{n} q_i(j)\cdot p_i(j)\cdot\left(\dfrac{q_i(k)\cdot p_i(k)}{q_i(j)\cdot p_i(j)}\right)^t} dt \qquad (5)$$

or $\quad \mathrm{I}_{v/p}^{k/j} = \prod_{i=1}^{n} \mathrm{I}_{v/p}^{k/j}$

where $$\mathrm{I}^{k/j}_{v/p_i} = \exp\int_0^1 \frac{q_i(j)p_i(j)\left(\frac{q_i(k)\cdot p_i(k)}{q_i(j)\cdot p_i(j)}\right)^t \ln\left(\frac{p_i(k)}{p_i(j)}\right)}{\sum_{i=1}^n q_i(j)\cdot p_i(j)\cdot\left(\frac{q_i(k)\cdot p_i(k)}{q_i(j)\cdot p_i(j)}\right)^t} dt \qquad (5')$$

Remark 6. Is obviously that the factors path is neither a linear nor an exponential one except the particular cases. The real path is unknown. However, on small time intervals we can suppose that the path is one of a linear or an exponential path. The problem which arising here is which of these two path are more appropiate for the calculus of a price index. We can partialy solve such a problem if we use the axiomatical approach for an index. In this approach, the index has the following definition:

Definition 7. Is called the statistical index, a function $f : D \to R$ which maps an economical object set *D* into the real number set *R* and which satisfies a system of economically relevant conditions. The form of these conditions depends on the information which one wants to obtain from the particular index. In 1911, I. Fisher gave an axioms list suitable for a price index. Later, these axioms are grouped in systems that satisfy natural requirements of independence and consistency. Thus, these systems can be used as definitions for different types of price index. Here we work with three systems of axioms that are used as comparison tool for the linear MDF price index and the exponential MDF price index. Since the axioms (properties) are desirable for a price index we call a better index that index which satisfies more axioms.

Definition 8.A function $P : R^{4n}_{++} \to R_{++}, (q^j, p^j, q^k, p^k) \to P(q^j, p^j, q^k, p^k)$ is called a price index if *P* satisfies the following four properties (axioms): monotonicity, proportionality, dimensionality and comensurability.

Then $P(q^j, p^j, q^k, p^k)$ represents the value of price index on price-quantity situation (q^j, p^j, q^k, p^k).

Definition 9.(Eichhorn-Voeller's axioms system)(1978).

A function $P : R^{4n}_{++} \to R_{++}, (q^j, p^j, q^k, p^k) \to P(q^j, p^j, q^k, p^k)$ is called a price index if *P* satisfies the next five properties: monotonicity, dimensionality, comensurability, identity and linear homogeneity.

Remark 10. In 1979 the definition given by Eichhorn and Voeller is modified in sense that the proportionality substitutes the identity and is required a weak linear homogeneity: $P(q^j, p^j, q^j, \lambda p^k) = \lambda P(q^j, p^j, q^j, p^k), \lambda \in R_{++}$

Definition 11. (Olt's axioms system)(1995). A function $P : R^{4n}_{++} \to R_{++}, (q^j, p^j, q^k, p^k) \to P(q^j, p^j, q^k, p^k)$ is called a price index if *P* satisfies the following four properties (axioms):dimensionality, comensurability, simetry and the mean value test. Next we state these mentioned properties and other that are necesary for our goal.

Proportionality axiom. The index equals the individual price relatives when they agree with each other. $P(q^j, p^j, q^k, \lambda p^j) = \lambda$.

Comensurability axiom. A change in the units of measurement of comodities (goods) does not change the value of the function *P*.

$P(\frac{q_1^j}{\lambda_1},...,\frac{q_n^j}{\lambda_n},\lambda_1 p_1^j,...,\lambda_n p_n^j,\frac{q_1^k}{\lambda_1},...,\frac{q_n^k}{\lambda_n},\lambda_1 p_1^k,...,\lambda_n p_n^k) = P(q^j, p^j, q^k, p^k)$ where $\lambda_i \in R_{++}$

Time reversal test. When the two situations are interchanged, the price index yields the reciprocal value.

$$P(q^j, p^j, q^k, p^k) = \frac{1}{P(q^k, p^k, q^j, p^j)} .$$

Linear homogeneity. The value of the function *P* changes by the factors λ if all prices of the observed situation change λ-fold.

$$P(q^j, p^j, q^k, \lambda p^k) = \lambda P(q^j, p^j, q^k, p^k), \lambda \in R_{++}$$

Monotonicity. The function P is strictly increasing with respect to p^k and strictly decreasing with respect to p^j.

$P(q^j, p^j, q^k, p^k) > P(q^j, p^j, q^k, \tilde{p}^k), \text{if}, p^k > \tilde{p}^k$

$P(q^j, p^j, q^k, p^k) < P(q^j, \tilde{p}^j, q^k, p^k), \text{if}, p^j > \tilde{p}^j$

Dimensionality. A dimensional change in the unit of currency in which all prices are measured does not change the value of the function *P*.

$$P(q^j, \lambda p^j, q^k, \lambda pk) = P(q^j, p^j, q^k, p^k), \lambda \in R_{++}$$

Identity. The value of the function P equals one if all prices remain constant.

$$P(q^j, p^j, q^k, p^j) = 1 .$$

Mean value test. The value of the price index can be represented as a convex combination of the smallest and the biggest price relative.

$\min\left\{\frac{p_1^k}{p_1^j},...,\frac{p_n^{k1}}{p_n^j}\right\} \le P(q^j, p^j, q^k, p^k) \le \max\left\{\frac{p_1^k}{p_1^j},...,\frac{p_n^k}{p_n^j}\right\}$

or

$P(q^j, p^j, q^k, p^k) = \lambda \min_i \left\{\frac{p_i^k}{p_i^j}\right\} + (1-\lambda)\max\left\{\frac{p_i^k}{p_i^j}\right\}$

Simetry. The same permutation of the components of the four vectors does not change the value of the index.

$$P(q^j, p^j, q^k, p^k) = P(\tilde{q}^j, \tilde{p}^j, \tilde{q}^k, \tilde{p}^k).$$

Theorem 12. The MDF price index $I^{k/j}_{z/p} = \exp\int_{P_jP_k} \frac{z'_p}{z} dp$ on the linear path is a price index in sense of definition (8).

Proof:

We will proove that the index satisfies the monotonicity, the proportionality, the dimensionality and the comensurability.

Let $z = \sum_{i=1}^{n} p_i q_i, z = z(t), t \in [j,k],$ and $\begin{cases} q_i = q_i(j) + t(q_i(k) - q_i(j)) \\ p_i = p_i(j) + t(p_i(k) - p_i(j)) \end{cases}, t \in [0,1]$ the parametrisation of the linear path between the points $P_j(q(j), p(j)), P_k(q(k), p(k))$.

We get $I^{k/j}_{z/p_i} = \exp\int_0^1 \frac{[q_i(j) + t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}(q_i(j) + t\Delta q_i)(p_i(j) + t\Delta p_i)} dt$

and $I^{k/j}_{z/p} = \prod_{i=1}^{n} I^{k/j}_{z/p_i} = \exp\int_0^1 \frac{\sum_{i=1}^{n}[q_i(j) + t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}(q_i(j) + t\Delta q_i)(p_i(j) + t\Delta p_i)} dt$

-dimensionality

$$I^{k/j}_{z/p}(q(j), \lambda p(j), q(k), \lambda p(k)) = I^{k/j}_{z/p}(q(j), p(j), q(k), p(k))$$

$$I^{k/j}_{z/p}(q(j), \lambda p(j), q(k), \lambda p(k)) = \exp\int_0^1 \frac{\sum_{i=1}^{n}[q_i(j) + t\Delta q_i]\lambda\Delta p_i}{\sum_{i=1}^{n}(q_i(j) + t\Delta q_i)(\lambda p_i(j) + t\lambda\Delta p_i)} dt$$

$$= \exp\int_0^1 \frac{\sum_{i=1}^{n}[q_i(j) + t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}[q_i(j) + t\Delta q_i][p_i(j) + t\Delta p_i]} dt = I^{k/j}_{z/p}(q(j), p(j), q(k), p(k))$$

-proportionality

$$I_{z/p}^{k/j}(q(j),p(j),q(k),\lambda p(j))=\lambda$$

$$I_{z/p}^{k/j}(q(j),p(j),q(k),\lambda p(j))=\exp\int_0^1\frac{\sum_{i=1}^{n}[q_i(j)+t\Delta q_i]p_i(j)(\lambda-1)}{\sum_{i=1}^{n}[q_i(j)+t\Delta q_i][p_i(j)+tp_i(j)(\lambda-1)]}dt=$$

$$=\exp\int_0^1\frac{\lambda-1}{1+t(\lambda-1)}dt=\exp\ln[1+t(\lambda-1)]/_0^1=\exp\ln\lambda=\lambda$$

-commensurability

$$I_{z/p}^{k/j}(\frac{q_1(j)}{\lambda_1},...,\frac{q_n(j)}{\lambda_n},\lambda_1 p_1(j),....,\lambda_n p_n(j),\frac{q_1(k)}{\lambda_1},....,\frac{q_n(k)}{\lambda_n},\lambda_1 p_1(k),....,\lambda_n p_n(k))=$$
$$=I_{z/p}^{k/j}(q(j),p(j),q(k),p(k)).$$

$$I_{z/p}^{k/j}(....)=\exp\int_0^1\frac{\sum_{i=1}^{n}\frac{1}{\lambda_i}[q_i(j)+t\Delta q_i]\lambda_i\Delta p_i}{\sum_{i=1}^{n}\lambda_i[p_i(j)+t\Delta p_i]\cdot\frac{1}{\lambda_i}[q_i(j)+t\Delta q_i]}dt=$$

$$=\exp\int_0^1\frac{\sum_{i=1}^{n}[q_i(j)+t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}[p_i(j)+t\Delta p_i][q_i(j)+t\Delta q_i]}dt=I_{z/p}^{k/j}(q(j),p(j),q(k),p(k))$$

-monotonicity

a) $I_{z/p}^{k/j}(q(j),p(j),q(k),p(k))>I_{z/p}^{k'/j'}(q(j'),p(j'),q(k'),p(k'))$
if $q(j)=q(j'),p(j)=p(j'),q(k)=q(k')$ and $p(k)\geq p(k')$

b) $I_{z/p}^{k/j}(q(j),p(j),q(k),p(k))<I_{z/p}^{k'/j'}(q(j'),p(j'),q(k'),p(k'))$
if $q(j)=q(j'),p(j)\geq p(j'),q(k)=q(k')$ and $p(k)=p(k')$

First we prove the part b) of the property

$$I_{z/p}^{k\ /j} = \exp\int_0^1 \frac{\sum_{i=1}^{n}[q_i(j)+t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}(p_i(j\ \ )+t\Delta p_i\ )(q_i(j)+t\Delta q_i)}dt$$

Denoting

$$F(p_1(j), p_2(j),..., p_n(j)) = \frac{\sum_{i=1}^{n}[q_i(j)+t\Delta q_i]\Delta p_i}{\sum_{i=1}^{n}[p_i(j)+t\Delta p_i][q_i(j)+t\Delta q_i]}$$

we obtain

$$\frac{\partial F}{\partial p_i(j)} < 0, \forall \mathrm{i} = \overline{1,\mathrm{n}}\ ,$$

for any $t \in [0,1]$.

From $(p_1(j), p_2(j),..., p_n(j)) \geq (p_1(j^{'}), p_2(j^{'}),..., p_n(j^{'}))$ follows

$$F((p_1(j),p_2(j),...,p_n(j)))\leq F((p_1(j^{'}),p_2(j\ \ ),...,p_n(j)))\leq F((p_1(j^{'}),p_2(j^{'}),...,p_n(j)))\leq$$
$$\leq ... \leq F((p_1(j^{'}), p_2(j^{'}),..., p_n(j^{'})))$$

and now we apply the integrand with respect to *t* and the exponential function and we have $I_{z/p}^{k/j} < I_{z/p}^{k'/j'}$.

a) According [1], the MDF index satisfies the time reversal test independently of the path, so we have:

$$I_{z/p}^{k/j}\big(q(j),p(j),q(k),p(k)\big) = \frac{1}{I_{z/p}^{j/k}\big(q(k),p(k),q(j),p(j)\big)} > \frac{1}{I_{z/p}^{k'/j'}(q(k),p(k'),q(j)p(j)} =$$
$$= I_{z/p}^{k'/j'}(q(j),p(j),q(k),p(k'))$$

where we use the hypothesis $p(k) \geq p(k')$ and the part b) of the monotonicity.

Theorem 13. The exponential MDF price index is a price index in sense of the definition (8).

Proof:

The most part of these axioms are verified in [3]. Here we prove the monotonicity.

Let the function

$$F\left(p_1(j), p_2(j), ..., p_n(j)\right) = \frac{\sum_{i=1}^{n} \left[q_i(j)p_i(j)\right]^{1-t} \left[q_i(k)p_i(k)\right]^t \ln \frac{p_i(k)}{p_i(j)}}{\sum_{i=1}^{n} \left[q_i(j)p_i(j)\right]^{1-t} \left[q_i(k)p_i(k)\right]^t}.$$

We consider $p_i, i = \overline{1,n}$,

$$\frac{p_1(k)}{p_1(j)} \le \frac{p_2(k)}{p_2(j)} \le ... \le \frac{p_n(k)}{p_n(j)} \quad (6)$$

without loss the generality. Moreover we make the assumption that

$$\frac{p_n(k)}{p_n(j)} \le \min\left\{\frac{p_1(k)}{p_1(j')}, \frac{p_2(k)}{p_2(j')}, ..., \frac{p_n(k)}{p_n(j')}\right\} \qquad (7)$$

For fixed $p_2(j), p_3(j), ..., p_n(j)$, the function becomes a function of single variable $p_1(j)$.

$$F'_{p_1(j)} = \frac{\sum_{i=1}^{n} [q_i(j)p_i(j)]^{-t} [q_i(k)p_i(k)]^t q_1(j) [q_i(j)p_i(j)]^{1-t} [q_i(k)p_i(k)]^t \left[(1-t)\ln\frac{p_1(k)}{p_1(j)} - 1\right]}{\left[\sum_{i=1}^{n} [q_i(j)p_i(j)]^{1-t} [q_i(k)p_i(k)]^t\right]^2}$$

$$- \frac{\sum_{i=1}^{n} [q_i(j)p_i(j)]^{-t} [q_i(k)p_i(k)]^t q_1(j) [q_i(j)p_i(j)]^{1-t} [q_i(k)p_i(k)]^t \left[(1-t)\ln\frac{p_i(k)}{p_i(j)}\right]}{\left[\sum_{i=1}^{n} [q_i(j)p_i(j)]^{1-t} [q_i(k)p_i(k)]^t\right]^2}$$

Since the prices and the quantities at different moments are positive real numbers if

$$\left[(1-t)\ln\frac{p_1(k)}{p_1(j)} - 1 - (1-t)\ln\frac{p_i(k)}{p_i(j)}\right] \le 0$$

for any $i = \overline{1,n}$ then we have $F'_{p_1(j)} \le 0$.

From (6) the above inequality holds because

$$\frac{p_1(k)}{p_1(j)} \le \frac{p_2(k)}{p_2(j)},\ \frac{p_1(k)}{p_1(j)} \le \frac{p_3(k)}{p_3(j)},\ldots,\frac{p_1(k)}{p_1(j)} \le \frac{p_n(k)}{p_n(j)}.$$

So the function $F_{p_1(j)}$ is monoton decreasing.

From the hypothesis we have $p(j) \ge p(j^{'})$, and further $p_1(j) \ge p_1(j^{'})$ so it results $F\big(p_1(j), p_2(j),..., p_n(j)\big) \le F\big(p_1(j^{'}), p_2(j),..., p_n(j)\big)$.

Now we fix $p_1(j^{'}), p_3(j),..., p_n(j)$ and prove that F is decreasing with respect to $p_2(j)$.

Similarly if

$$\frac{p_2(k)}{p_2(j)} \le \frac{p_1(k)}{p_1(j^{'})},\ \frac{p_2(k)}{p_2(j)} \le \frac{p_i(k)}{p_i(j)}\ ,i = \overline{2,n},$$

then we have

$F^{'}_{p_2(j)} \le 0$.

But according to (6) and (7) the above relations hold.

So if $p_2(j) \ge p_2(j^{'})$ we can write

$$F\big(p_1(j), p_2(j),..., p_n(j)\big) \le F\big(p_1(j^{'}), p_2(j),..., p_n(j)\big) \le F\big(p_1(j^{'}), p_2(j^{'}), p_3(j),..., p_n(j)\big)$$
.

We repeat this method and if $p(j) \ge p(j^{'})$ then results

$$F\big(p_1(j), p_2(j),..., p_n(j)\big) \le F\big(p_1(j^{'}), p_2(j^{'}),..., p_n(j^{'})\big).$$

According to the properties of integrand and exponential function the second part of monotonicity is proved. For the first part one can be repeat the proof for the linear case.

Proposition 14. Any function $P : R^{4n}_{++} \to R_{++}$, that satisfies the proportionality and the monotonicity satisfies the mean value test, too.

Proof:

Let

$$a = \min\left\{\frac{p_1(k)}{p_1(j)},...,\frac{p_n(k)}{p_n(j)}\right\}$$

and

$$b = \max\left\{\frac{p_1(k)}{p_1(j)},\dots,\frac{p_n(k)}{p_n(j)}\right\}.$$

If the price index satisfies the proportionality then results $a = P\big(q(j), p(j), q(k), ap(j)\big)$,and according to the monotonicity we have $a = P\big(q(j), p(j), q(k), ap(j)\big) \le P\big(q(j), p(j), q(k), p(k)\big)$ since $ap(j) \le p(k)$.The last inequality means that $ap_1(j) \le p_1(k), ap_2(j) \le p_2(k), \dots, ap_n(j) \le p_n(k)$,which is true since

$$a \le \frac{p_i(k)}{p_i(j)} \quad \forall\, i = \overline{1,n}\,.$$

Similarly, we have $b = P\big(q(j), p(j), q(k), bp(j)\big) \le P\big(q(j), p(j), q(k), p(k)\big)$, so the mean value test is verified.

Corollary 15. The MDF price index both on the linear and the exponential path satisfies the mean value test.

Remark 16. It can be proved that the MDF linear factorial index does not satisfy the linear homogeneity.

Proposition 17. The MDF exponential price index satisfies the linear homogeneity.

Proof: $P\big(q(j), p(j), q(k), \lambda p(k)\big) = \lambda P\big(q(j), p(j), q(k), p(k)\big), \lambda \in R_{++}$.

$$P\big(q(j), p(j), q(k), \lambda p(k)\big) = \exp\int_0^1 \frac{\sum_{i=1}^{n} q_i(j)p_i(j)\left(\frac{q_i(k)\lambda p_i(k)}{q_i(j)p_i(j)}\right)^t \ln\left(\frac{\lambda p_i(k)}{p_i(j)}\right)}{\sum_{i=1}^{n} q_i(j)p_i(j)\left(\frac{q_i(k)p_i(k)\lambda}{q_i(j)p_i(j)}\right)^t} dt =$$

$$\exp\int_0^1 \frac{\lambda^t \sum_{i=1}^{n} q_i(j)p_i(j)\left(\frac{q_i(k)p_i(k)}{q_i(j)p_i(j)}\right)^t \left(\ln\lambda + \ln\frac{p_i(k)}{p_i(j)}\right)}{\lambda^t \sum_{i=1}^{n} q_i(j)p_i(j)\left(\frac{q_i(k)p_i(k)}{q_i(j)p_i(j)}\right)^t} dt =$$

$$\exp\int_0^1\left(\ln\lambda+\frac{\sum_{i=1}^{n}q_i(j)p_i(j)\left(\frac{q_i(k)p_i(k)}{q_i(j)p_i(j)}\right)^t\ln\left(\frac{p_i(k)}{p_i(j)}\right)}{\sum_{i=1}^{n}q_i(j)p_i(j)\left(\frac{q_i(k)p_i(k)}{q_i(j)p_i(j)}\right)^t}\right)dt=\exp\int_0^1\ln\lambda dt\cdot P(q(j),p(j),q(k),p(k))=$$
$$\lambda P(q(j),p(j),q(k),p(k)).$$

Proposition 18. Any function $f:R_{++}^{4n}\to R_{++}$, that satisfies the proportionality satisfies also the identity.

Proof:
We have $P(q(j),p(j),q(k),\lambda p(j))=\lambda$.

$$P(q(j),p(j),q(k),p(j))=P(q(j),p(j),q(k),1\cdot p(j))=1.$$

Theorem 19. The MDF exponential price index is a price index in sense of definition (9).
Proof: The theorem is a consequence of (17) and (18) and the theorem (13).

Remark 20. The MDF linear price index is not a price index by the definition (9) since does not satisfy the linear homogeneity.

Proposition 21. The MDF price index both on linear and exponential index satisfy the symmetry.
Proof: This will results from the form of the two indices inside we have $\sum_{i=1}^{n}$.

Theorem 22. The MDF price index (linear or exponential) is a price index in sense of the definition (11).

Proof: The theorem results since we have (12), (13), (15) and (21).

In conclusion, the MDF linear price index satisfies less axioms than the MDF exponential price index. The first index satisfies just two of the three systems given here. From the three filter definitions (axioms systems) point of view, the MDF exponential price index is more appropiate than MDF linear price index.

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