

## APPLICATIONS OF THE CONSTANT ELASTICITY OF SUBSTITUTIONS FUNCTIONS

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**ABSTRACT.** The paper presents an overview of CES (Constant Elasticity of Substitutions Functions) which are frequently used in international commerce for analyzing the effect of new open markets or the effects of commercial politics in the presence of different products.

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The representative consumer's theory and CES functions (Constant Elasticity of Substitutions Functions) have been frequently used in international commerce in order to analyze the effect of new open markets or the effect of commercial politics on wellbeing when different products are present. This frequent use is due, mainly, to the simplicity of these functions. In fact, it represents an extension of the traditional theory of demand, where all the individuals considered (and not each individual) have similar preferences. Thus, the equilibrium is stable and unique. There is only one point where the utility is at its utmost, taking into account the budget restraints, this point representing a combination of all the products varieties.

The initial application in international commerce from A.DIXIT and J.E. STIGLITZ's article gave rise to international commerce models as part of the monopoly competition. The authors that followed have developed other applications in the same field of activity.

The main objective of these theoretical models is to establish the existence of the trade profit in monopoly competition, on condition that:

- If the branches are identical with each other in autarchy, the opening of the markets is achieved through scale economies (according to the fixed costs  $f$ ), because the quality of the market is getting double and each variety of the product is made in the same country by the same company;
- If the branches provide different varieties of products before the trade has been made, the opening in declined and every company sells less on the internal market due to the competition of different products and especially due to the international commerce. Thus, the profit is generated by a wide range of available products for the consumers of the two countries, while the production costs of the companies are the same.

From this point of view, the international commerce increases the wellbeing, but the direction of the trade and the nature of the varieties of products are not taken into consideration. As an observation, the monopoly competition it's the only market structure that brings obvious results for free trade's benefit.

The next section presents an interpretation of CES functions specific to international commerce and some conclusions regarding these applications.

When developing the theory of the representative consumer, C.BISMUT and J.OLIVEIRA-MARTINS (1989), have included parts from the new theory of the international commerce and the horizontal differentiation of products, in order to emphasize the connection between competition-price, the differentiation of products and partition of the markets between producers of different nationalities. A two level utility CES function describes the choice of the aggregated agent as a two stages process. Firstly the consumer proceeds to the distribution of his budget between two different categories of products, as imported products and national products. Secondly, he chooses different varieties from each category. The products of different origin form aggregates of different products made by multiproduct monopolies. The horizontal differentiation is included as to Dixit-Stiglitz's theory inside each aggregate. Thus, the national and international producers are supposed to offer two aggregates of different products, which constitutes imperfect substitutes. The partition of the internal market is therefore determined in accordance with the relative prices, the substitution elasticity between the two aggregates and the number of varieties of each aggregate.

The starting point of the model, where the supply is supposed to be hexogen and the conduct of the demand is modeled, consists in the characteristic of the utility function that is being used. Thus, an utility function in two stages is used:

$$U = [aU_a^\theta + bU_b^\theta]^{1/\theta},$$

where  $a$  indexes the imported goods  $[A]$ ;  $b$  indexes the national goods  $[B]$ ;  $\theta \leq 1$  in order to assure the concavity of the utility function, and  $a$  and  $b$  are of positive value. The functions of sub-utility  $U_a$  and  $U_b$  are also CES functions as against to the quantities of the  $n_a$  possible varieties of the product  $A$  and  $n_b$  possible varieties of the product  $B$ :

$$U_a = \left[ \sum_{i=1}^{n_a} x_{ai} \right]^{1/\theta_a}$$

$$U_b = \left[ \sum_{i=1}^{n_b} x_{bi} \right]^{1/\theta_b},$$

where  $\theta_a < 1$  and  $\theta_b < 1$  in order to provide the concavity of the sub-utility functions and  $\theta_a > 0$  and  $\theta_b > 0$  as to consider that some varieties could not be available on the market, but  $m_a$  with  $0 < m_a < n_a$  varieties from  $A$  category and  $m_b$  with  $0 < m_b < n_b$  varieties from  $B$  category can be found on the market. Thus, if the varieties of an aggregate have the same price, the market share of a certain variety will be a descending function of the total number of available varieties for the consumer. The market share of variety 1, for instance, is rendered by:

$$\frac{p_1 x_1}{\sum_{i=1}^n p_i x_i} = \frac{1}{n},$$

This result, regarding the varieties of a category of goods, reveals the fact that the consumers are supposed to prefer a large number of varieties rather than a small one.

The conditions applied to  $\theta, \theta_a, \theta_b$  such as  $\theta < 1, 0 < \theta_a < 1, 0 < \theta_b < 1$ , generate:

$$\sigma > 0$$

$$\sigma_a = \frac{1}{1 - \theta_a} > 1$$

$$\sigma_b = \frac{1}{1 - \theta_b} > 1$$

These assumptions allow that the substitution between the aggregates [A] and [B], expressed by  $\sigma$  to be weaker than the substitutions between the varieties from every aggregate, expressed by  $\sigma_a$  and  $\sigma_b$ . This is in accordance with the perspectives of the model where the varieties of a product (horizontally differentiated) are easy to substitute, while between varieties of products made by different nationalities the substitution is less important. The consumer chooses the varieties  $x_{ai}$  and  $x_{bi}$  that are available at the prices  $p_{ai}$  and  $p_{bi}$  so that it maximizes the utility function  $U$  because of the budget restraint:

$$R = \sum_{i=1}^{m_a} p_{ai}x_{ai} + \sum_{i=1}^{m_b} p_{bi}x_{bi},$$

where  $R$  is hexogen. Due to the assumption of concavity, the first conditions are enough as to obtain a maximum. Taking into account these conditions, the expression of the partition of the market between the imported goods and the national ones derives as a function of the ratio of the price indexes CES, of the products aggregates [A] and [B], and of the number of varieties  $m_a$  and  $m_b$ .

Without any details, it's worth mentioning that a price index CES is defined as:

$$\Pi_a = \left[ \sum_{i=1}^{m_a} \frac{1}{m_a} p_{ai}^{1-\sigma_a} \right]^{1/(1-\sigma_a)}$$

$$\Pi_b = \left[ \sum_{i=1}^{m_b} \frac{1}{m_b} p_{bi}^{1-\sigma_b} \right]^{1/(1-\sigma_b)}$$

Thus, considering the relative prices:

$$\Pi = \frac{\Pi_a}{\Pi_b}$$

and defining the partition of the internal market as the ratio between the value of imported goods and the value of the national goods consumed by the internal market, meaning:

$$V = \frac{\sum_{i=1}^{m_a} p_{ai} x_{ai}}{\sum_{i=1}^{m_b} p_{bi} x_{bi}}$$

the final expression becomes:

$$V = \left[ \frac{b}{a} \right]^{-\sigma} \cdot \left[ \frac{m_a^{1/(1-\sigma_a)}}{m_b^{1/(1-\sigma_b)}} \right]^{1-\sigma} \cdot \Pi^{1-\sigma}$$

This expression is different from the usual forms of the market partition functions due to the intervention of the great number of varieties.

Nevertheless, the main disadvantage of the model C. BISMUT and J. OLIVEIRA-MARTINS [1989], is the fact that the competition sources that can modify  $\sigma$  are not explicit. These sources are not to be found but in the vertical differentiation or in the elements of the localized competition.

In order to render a horizontal differentiation strategy, the following elements will be taken into account. In the formula of the partition of the market, both  $\sigma_a$  and  $\sigma_b$  being greater than "0" in accordance with the conditions considered, the sign of the effect of the number of the varieties between [A] and [B] depends only on the value of the substitution elasticity between the two aggregates of different products, respectively  $\sigma$ . If  $\sigma > 1$ , more competitive would be achieved by means of horizontal differentiation, that is by rising the number of varieties offered by the producers, because the partition of the market between importers and national producers is an ascending function reported to  $m_a$ , the number of international varieties and descending reported to  $m_b$ , the number of local varieties. This being the case, every producer should try to grow his market share by offering a wider range of products. On the other hand, if  $\sigma < 1$ , meaning that the substitution level is weak enough, V becomes a descending function reported to  $m_a$ , and a descending function reported to  $m_b$ .

The amplitude of the effect of the number of varieties depends on the values of the substitutions elasticities of the aggregate, namely  $\sigma_a$  and  $\sigma_b$ . For instance, the lower the substitution of the aggregate [A] is, the stronger the effect that follows after a variation of  $m_a$  will be. This effect gets lower when the substitution level grows, until it is null in case of a perfect substitution. Indeed, a supplementary variety generates a redistribution of the market between the varieties of the aggregate [A], but it does not modify the separation

between the two categories of goods.

As a whole, the horizontal differentiation, considered as a growing of the number of varieties among a group of products, turns out to be the more efficient strategy as to achieve market shares, the stronger capacity of substitution the national and international products have, but the varieties being less capable of substitution.

What should be remembered from the approach of the representative consumer? It is a vague notion, coming from the idea of the medium consumer, which won't specify anything related to the wellbeing if the form of the utility function is not mentioned.

A specification implies a simple functional form, such as CES, which reduces the limits of the economic analysis.

To conclude with, the representative consumer, with a utility function CES can not reflect situations of oligopolistic competition.

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