ANALYTICAL TREATMENT OF THE COUPLED HIGGS EQUATION AND THE MACCARI SYSTEM VIA EXP-FUNCTION METHOD

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ABSTRACT. In this article, He's Exp-function method (EFM) is used to construct solitary and soliton solutions of the nonlinear evolution equation. This technique is straightforward and simple to use and is a powerful method to overcome some difficulties in the nonlinear problems. This method is developed for searching exact traveling wave solutions of the nonlinear partial differential equations. The EFM presents a wider applicability for handling nonlinear wave equations. Also, it is shown that EFM, with the help of symbolic computation, provides a straightforward and powerful mathematical tool for solving nonlinear evolution equations. Application of Exp function method to coupled Higgs equation and the Maccari system illustrates its effectiveness.

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1. INTRODUCTION

The investigation of exact travelling wave solutions to nonlinear wave equations plays an important role in the study of nonlinear physical phenomena. In the recent decade, the study of nonlinear partial differential equations in modelling physical phenomena, has become an important tool. Here, we use of an effective method for constructing a range of exact solutions for following nonlinear partial differential equations that proposed by J. H. He [1]. In this article an application of the proposed method to two complex coupled equations is illustrated. We consider the coupled Higgs field equation [2,3] in the form

$$u_{tt} - u_{xx} - au + b|u|^2 u - 2uv = 0, \tag{1}$$

$$v_{tt} + v_{xx} - b(|u|^2)_{xx} = 0,$$

where Eq. (1) is the coupled Higgs field equation for $a \geq 0$, $b > 0$ [2, 3]. Here, we choose $a = 0$ and $b = 1$. N-soliton solutions to the system Eq. (1) are obtained in [2].

More general travelling wave solutions constructed by Bekir and Zhao of Eq. (1) in [4, 5]. We next consider the following Maccari new integrable $(2 + 1)$-dimensional nonlinear system [6]

$$iu_t + u_{xx} + uv = 0, \tag{2}$$

$$v_t + v_y + (|u|^2)_x = 0.$$

Also, more general travelling wave solutions constructed by Bekir and Zhao of Eq. (2) in [4, 5]. Several powerful methods have been proposed to obtain exact solutions of nonlinear partial differential equations, such as inverse scattering method [7], the tanh method [8, 9], tanh-coth method [10], the homotopy perturbation method [11, 12], the homotopy analysis method [13, 14] and variational iteration method [15]. In recent years, the direct search for exact solutions of PDEs has become more and more attractive partly due to the availability of computer symbolic systems like Maple or Mathematica, which allows us to perform the complicated and tedious algebraic calculations on computer. In particular, one of the most effective direct methods to construct exact solutions of PDEs is the EFM, which was first proposed by He in [1]. The EFM can be used to seek solitary solutions, periodic solutions and compacton-like solutions of nonlinear differential equations. Wu and He [16] have used the Exp-function method to give new periodic solutions for nonlinear evolution equations. Dehghan and et. al [17] have applied the EFM and its application for solving a partial differential equation arising in biology and population genetics. The EFM has recently been solved by Zhang [18] to high-dimensional nonlinear evolution equation. The new exact solutions of modified KdV and the generalized KdV equations with Exp-function method have been obtained by Manafian and Bagheri [19]. The proposed method not only gives a unified formulation to construct various travelling wave solutions, but also provides a rule to classify the types of solutions according to the given parameters. Furthermore, the proposed method is readily computerizable in solving equation by using symbolic software like Mathematica or Maple. Our aim of this paper is to obtain analytical solutions of the coupled Higgs equation and the Maccari system, and to determine the accuracy of the Exp–function method in solving these kinds of problems. The article is organized as follows: in Section 2, first we briefly give the steps of the EFM and apply the method to solve the nonlinear partial differential equations. In Section 3, the application of the EFM to the couple Higgs equation will be introduced briefly. Also, Section 4 by using the results obtained in Section 2, the corresponding solutions of the Maccari system can be obtained. Finally some references are given at the end of this paper.

2. Basic idea of the Exp–function method

We first consider the nonlinear equation of the form

$$\mathcal{N}(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{tt}, u_{tx}, ...) = 0, \tag{3}$$

and introduce a transformation

$$u(x, y, t) = u(\eta), \quad \eta = x + y + ct, \tag{4}$$

where c is constant to be determined later. Therefore Eq. (3) is reduced to an ODE as follows

$$\mathcal{M}(u, cu', u', u', u'', ...) = 0. \tag{5}$$

The EFM is based on the assumption that travelling wave solutions as in ([?]) can be expressed in the form

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)}, \tag{6}$$

where c, d, p and q are positive integers which could be freely chosen and a_n' and b_m' are unknown constants to be determined. To determine the values of c and p, we balance the linear term of highest order in Eq. (5) with the highest order nonlinear term. Also to determine the values of d and q, we balance the linear term of lowest order in Eq. (5) with the lowest order nonlinear term.

3. Application to the coupled Higgs equation

In this section we employ the EFM to the following coupled Higgs equation

$$u_{tt} - u_{xx} + |u|^2 u - 2uv = 0, \tag{7}$$

$$v_{tt} + v_{xx} - (|u|^2)_{xx} = 0.$$

We begin first with the coupled Higgs Eq. (7). Using the wave variables as follow

$$u = e^{i\theta} U(\eta), \quad v = V(\eta), \quad \theta = px + rt, \quad \eta = x + ct, \tag{8}$$

Eq. (7) are carried to ODEs

$$(c^2 - 1)U'' + (p^2 - r^2)U - 2UV + U^3 = 0, \tag{9}$$

$$(c^2 + 1)V'' - 2(U')^2 - 2UU'' = 0.$$

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Integrating the second equation in the system and neglecting the constant of integration we find

$$(c^2 + 1)V = U^2. \tag{10}$$

Substituting Eq. (10) into the first equation of the system we get

$$(c^4 - 1)U'' + (c^2 + 1)(p^2 - r^2)U + (c^2 - 1)U^3 = 0. \tag{11}$$

In order to determine values of c and p, we balance the linear term of the highest order U'' with the highest order nonlinear term U^3 in Eq. (11), to get

$$U'' = \frac{c_1 \exp((c + 3p)\eta) + ...}{c_2 \exp(4p\eta) + ...}, \tag{12}$$

$$U^3 = \frac{c_3 \exp(3c\eta) + ...}{c_4 \exp(3p\eta) + ...} = \frac{c_3 \exp((3c + p)\eta) + ...}{c_4 \exp(4p\eta) + ...}, \tag{13}$$

respectively. Balancing highest order of the Exp–function in (12) and (13), we get

$$c + 3p = 3c + p, \tag{14}$$

which leads to the result c = p. Similarly to determine values of d and q, for the terms U'' and U^3 in Eq. (11) by simple calculation, we have

$$U'' = \frac{... + d_1 \exp(-(d + 3q)\eta)}{... + d_2 \exp(-4q\eta)}, \tag{15}$$

$$U^3 = \frac{... + d_3 \exp(-3d\eta)}{... + d_4 \exp(-3q\eta)} = \frac{... + d_3 \exp(-(3d + q)\eta)}{... + d_4 \exp(-4q\eta)}, \tag{16}$$

respectively. Balancing highest order of the Exp–function in (15) and (16), we obtain

$$-(d + 3q) = -(3d + q), \tag{17}$$

which leads to the result d = q.

Case 1: p = c = 1 **and** q = d = 1.
For simplicity, we set p = c = 1 and d = q = 1. Then Eq. (6) reduces to

$$U(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{18}$$

Substituting (18) into Eq. (11), and by using the well-known Maple software, we have

$$\frac{1}{A}[C_3 \exp(3\eta) + C_2 \exp(2\eta) + C_1 \exp(\eta) + C_0 + C_{-1} \exp(-\eta)+ \tag{19}$$

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$$C_{-2} \exp(-2\eta) + C_{-3} \exp(-3\eta)] = 0,$$

where

$$A = [b_{-1} \exp(-\eta) + b_0 + b_1 \exp(\eta)]^3, \tag{20}$$

and C_n' are coefficients of $\exp(n\eta)$'. Equating the coefficients of $\exp(n\eta)$ to be zero, we obtain the following set of algebraic equations for $a_1, a_0, a_{-1}, b_1, b_0, b_{-1}$ and c, as

$$\begin{cases} C_3 = 0, C_2 = 0, C_1 = 0, \\ C_0 = 0, \\ C_{-3} = 0, C_{-2} = 0, C_{-1} = 0. \end{cases} \tag{21}$$

Solving the system of algebraic equations with the help of Maple gives the following set of non-trivial solutions
(I) The first set is:

$$a_1 = 0, \quad a_{-1} = a_{-1}, \quad p = p, \quad b_0 = b_0, \quad b_{-1} = -\frac{a_{-1}b_0}{a_0}, \quad b_1 = 0, \quad c^2 + 1 = c^2 + 1,$$
$$\tag{22}$$

$$a_0 = a_0, \quad r = r, \quad c^4 - 1 = 2(c^2 + 1)(p^2 - r^2), \quad c^2 - 1 = -\frac{b_0^2(c^2 + 1)(p^2 - r^2)}{a_0^2},$$

$$U_1(x, t) = \frac{a_0}{b_0} \frac{a_{-1} \exp(-x + ct) + a_0}{-a_{-1} \exp(-x + ct) + a_0}, \quad c = \sqrt{1 + 2(p^2 - r^2)}.$$

If we choose $a_0 = a_{-1}$, then we can obtain

$$u_1(x, t) = i\sqrt{1 + (p^2 - r^2)}e^{i(px+rt)} \coth\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right),$$

$$v_1(x, t) = -\frac{1}{2} \coth^2\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right).$$

(II) The second set is:

$$a_1 = 0, \quad a_{-1} = 0, \quad p = p, \quad b_1 = b_1, \quad b_{-1} = b_{-1}, \quad a_0 = a_0, \quad c^2 + 1 = c^2 + 1,$$
$$\tag{23}$$

$$b_0 = 0, \quad r = r, \quad c^4 - 1 = (c^2 + 1)(r^2 - p^2), \quad c^2 - 1 = -\frac{8(c^2 + 1)b_1b_{-1}(p^2 - r^2)}{a_0^2},$$

$$U_2(x, t) = \frac{a_0}{b_{-1} \exp(-x + ct) + b_1 \exp(x - ct)}, \quad c = \sqrt{1 + (r^2 - p^2)}.$$

If we choose $b_1 = b_{-1}$, then we can obtain

$$u_2(x, t) = \sqrt{2(2 + (r^2 - p^2))}e^{i(px+rt)}\text{sech}\left(x + \sqrt{1 + (r^2 - p^2)}t\right),$$

$$v_2(x,t) = 2\operatorname{sech}^2\left(x + \sqrt{1 + (r^2 - p^2)}t\right).$$

(III) The third set is:

$$a_1 = a_1, \quad a_0 = 0, \quad p = p, \quad a_{-1} = a_{-1}, \quad b_{-1} = b_{-1}, \quad b_1 = -\frac{b_{-1}a_1}{a_{-1}}, \quad c^2 + 1 = c^2 + 1,$$

(24)

$$b_0 = 0, \ r = r, \ c^4 - 1 = \frac{1}{2}(c^2 + 1)(p^2 - r^2), \ c^2 - 1 = -\frac{(c^2 + 1)b_{-1}^2(p^2 - r^2)}{a_{-1}^2},$$

$$U_3(x,t) = \frac{a_{-1}}{b_{-1}}\frac{a_{-1}\exp(-x - ct) + a_1\exp(x + ct)}{a_{-1}\exp(-x - ct) - a_1\exp(x + ct)}, \quad c = \sqrt{1 + \frac{p^2 - r^2}{2}}.$$

If we choose $a_1 = a_{-1}$, then we can obtain

$$u_3(x,t) = -i\sqrt{4 + p^2 - r^2}\,e^{i(px+rt)}\coth\left(x + \sqrt{1 + \frac{p^2 - r^2}{2}}t\right),$$

$$v_3(x,t) = -2\coth^2\left(x + \sqrt{1 + \frac{p^2 - r^2}{2}}t\right).$$

(IV) The fourth set is:

$$a_1 = -\frac{4a_{-1}b_1^2}{b_0^2}, \quad b_0 = b_0, \quad p = p, \quad a_{-1} = a_{-1}, \quad b_{-1} = \frac{1}{4}\frac{b_0^2}{b_1}, \quad b_1 = b_1, \quad c^2 + 1 = c^2 + 1,$$

(25)

$$a_0 = 0, \ r = r, \ c^4 - 1 = 2(c^2 + 1)(p^2 - r^2), \ c^2 - 1 = -\frac{1}{16}\frac{(c^2 + 1)b_0^4(p^2 - r^2)}{b_1^2 a_{-1}^2},$$

$$U_4(x,t) = \frac{4a_{-1}b_1}{b_0^2}\frac{b_0^2\exp(-x - ct) - 4b_1^2\exp(x + ct)}{b_0^2\exp(-x - ct) + 4b_0b_1 + 4b_1^2\exp(x + ct)}, \quad c = \sqrt{1 + 2(p^2 - r^2)}.$$

If we choose $b_0 = 2b_1$, then we can obtain

$$u_4(x,t) = -i\sqrt{1 + (p^2 - r^2)}\,e^{i(px+rt)}\tanh\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right),$$

$$v_4(x,t) = -\frac{1}{2}\tanh^2\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right).$$

(V) The fifth set is:

$$a_1 = a_1, \quad b_{-1} = -\frac{1}{4}\frac{-a_1^2 b_0^2 + b_1^2 a_0^2}{a_1^2 b_1}, \quad c^2 + 1 = c^2 + 1, \quad a_{-1} = \frac{1}{4}\frac{-a_1^2 b_0^2 + b_1^2 a_0^2}{a_1 b_1^2},$$

(26)

$$p = p, \quad b_0 = b_0, \quad r = r, \quad a_0 = a_0, \quad c^4 - 1 = 2(c^2 + 1)(p^2 - r^2),$$

$$c^2 - 1 = -\frac{(c^2 + 1)b_1^2(p^2 - r^2)}{a_1^2}, \quad b_1 = b_1, \quad a_{-1} = 0,$$

$$U_5(x, t) = \frac{1}{b_1}\frac{-a_1 b_{-1}\exp(-x - ct) + a_0 b_1 + a_1 b_1 \exp(x + ct)}{b_{-1}\exp(-x - ct) + b_0 + b_1 \exp(x + ct)}, \quad c = \sqrt{1 + 2(p^2 - r^2)}.$$

If we choose $b_1 = b_{-1}$, $b_0 = 2b_1$ and $a_0 = 4a_1$ then we can obtain

$$u_5(x, t) = i\sqrt{1 + (p^2 - r^2)}e^{i(px+rt)}\left[\tanh\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right)\right.$$

$$\left. +\operatorname{sech}^2\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right)\right],$$

$$v_5(x, t) = -\frac{1}{2}\left[\tanh\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right) + \operatorname{sech}^2\left(\frac{x + \sqrt{1 + 2(p^2 - r^2)}t}{2}\right)\right]^2.$$

4. APPLICATION TO THE MACCARI SYSTEM

We next apply the EFM to the Maccari system

$$iu_t + u_{xx} + uv = 0, \tag{27}$$

$$v_t + v_y + (|u|^2)_x = 0.$$

We begin first with the Maccari system Eq. (27). Using the wave variables as follow

$$u = e^{i\theta}U(\eta), \quad v = V(\eta), \quad \theta = px + qy + rt, \quad \eta = x + y + ct, \tag{28}$$

Eq. (27) are carried to ODEs

$$U'' - (r + p^2)U + UV = 0, \tag{29}$$

$$(c + 1)V' + 2UU' = 0.$$

Integrating the second equation in the system and neglecting the constant of integration we find

$$-(c+1)V = U^2. \tag{30}$$

Substituting Eq. (30) into the first equation of the system we obtain

$$(c+1)U'' - (c+1)(r - p^2)U - U^3 = 0. \tag{31}$$

Case 1: $p = c = 1$ **and** $q = d = 1$.
If we set $p = c = 1$ and $d = q = 1$. Then Eq. (6) reduces to

$$U(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{32}$$

Substituting (32) into Eq. (31), we have

$$\frac{1}{A}[C_3 \exp(3\eta) + C_2 \exp(2\eta) + C_1 \exp(\eta) + C_0 + C_{-1} \exp(-\eta) \tag{33}$$

$$+ C_{-2} \exp(-2\eta) + C_{-3} \exp(-3\eta)] = 0,$$

where

$$A = (b_{-1} \exp(-\eta) + b_0 + b_1 \exp(\eta))^3, \tag{34}$$

and C_n' are coefficients of $\exp(n\eta)$'. Equating the coefficients of $\exp(n\eta)$ to be zero, we have

$$\begin{cases} C_3 = 0, C_2 = 0, C_1 = 0, \\ C_0 = 0, \\ C_{-3} = 0, C_{-2} = 0, C_{-1} = 0. \end{cases} \tag{35}$$

Solving the system of algebraic equations we get

(I) The first set is:

$$a_1 = 0, \quad a_{-1} = 0, \quad b_0 = 0, \quad c + 1 = -\frac{1}{8}\frac{a_0^2}{b_1 b_{-1}}, \quad b_{-1} = b_{-1}, \quad b_1 = b_1, \quad r = p^2 - 1, \tag{36}$$

$$p = p, \quad U_1(x, y, t) = \frac{a_0}{b_{-1} \exp(-x - y - ct) + b_1 \exp(x + y + ct)}, \quad c = -1 - \frac{a_0^2}{8b_1^2}.$$

If we choose $a_0 = 2b_1 = 2b_{-1}$, then we can obtain

$$u_1(x, y, t) = e^{i[px + qy + (p^2 - 1)t]} \text{sech}\left(x + y - \frac{3}{2}t\right),$$

$$v_1(x, y, t) = 2\mathrm{sech}^2\left(x + y - \frac{3}{2}t\right).$$

(II) The second set is:

$$a_1 = a_1, \quad a_{-1} = 0, \quad p = p, \quad b_1 = -\frac{a_1 b_0}{a_0}, \quad b_{-1} = 0, \quad a_0 = a_0, \quad c + 1 = \frac{2a_0^2}{b_0^2},$$

(37)

$$b_0 = b_0, \quad r = \frac{1}{2} + p^2, \quad U_2(x, y, t) = -\frac{a_0}{b_0}\frac{a_0 + a_1\exp(x + y + ct)}{-a_0 + a_1\exp(x + y + ct)}, \quad c = \frac{2a_0^2}{b_0^2} - 1.$$

If we choose $a_0 = b_0 = a_1$, then we can obtain

$$u_2(x, y, t) = -e^{\frac{1}{2}i(2px + 2qy + (2p^2 + 1)t)}\coth\left(\frac{x + y + t}{2}\right),$$

$$v_2(x, y, t) = -\frac{1}{2}\coth^2\left(\frac{x + y + t}{2}\right).$$

(III) The third set is:

$$a_1 = \frac{1}{4}\frac{b_{-1}^2 a_0^2 - b_0^2 a_{-1}^2}{b_{-1}^2 a_{-1}}, \quad a_0 = a_0, \quad p = p, \quad a_{-1} = a_{-1}, \quad b_{-1} = b_{-1},$$

(38)

$$b_1 = -\frac{1}{4}\frac{b_{-1}^2 a_0^2 - b_0^2 a_{-1}^2}{b_{-1} a_{-1}^2}, \quad c + 1 = \frac{2a_{-1}^2}{b_{-1}^2}, \quad b_0 = b_0, \quad r = \frac{1}{2} + p^2,$$

$$U_3(x, y, t) = \frac{a_{-1}b_{-1}\exp(-x - y - ct) + a_0 b_{-1} - a_{-1}b_1\exp(x + y + ct)}{b_{-1}^2\exp(-x - y - ct) + b_0 b_{-1} + b_1 b_{-1}\exp(x + y + ct)}, \quad c = \frac{2a_{-1}^2}{b_{-1}^2} - 1.$$

If we choose $b_1 = b_{-1}$, $a_0 = a_{-1}$ and $b_0 = 2\sqrt{b_1 b_{-1}}$ then we can obtain

$$u_3(x, y, t) = \sqrt{\frac{c + 1}{2}}e^{\frac{1}{2}i(2px + 2qy + (2p^2 + 1)t)}\left[-\tanh\left(\frac{x + y + ct}{2}\right) + \mathrm{sech}^2\left(\frac{x + y + ct}{2}\right)\right],$$

$$v_3(x, y, t) = -\frac{1}{2}\left[-\tanh\left(\frac{x + y + ct}{2}\right) + \mathrm{sech}^2\left(\frac{x + y + ct}{2}\right)\right]^2.$$

(IV) The fourth set is:

$$a_1 = a_1, \quad b_0 = 0, \quad p = p, \quad a_{-1} = a_{-1}, \quad b_{-1} = b_{-1}, \quad b_1 = -\frac{b_{-1}a_1}{a_{-1}}, c + 1 = \frac{1}{2}\frac{a_{-1}^2}{b_{-1}^2},$$

(39)

$$a_0 = 0, \quad r = p^2 + 2, \quad U_4(x, y, t) = -\frac{a_{-1}}{b_{-1}}\frac{a_{-1}\exp(-x - y - ct) + a_1\exp(x + y + ct)}{-a_{-1}\exp(-x - y - ct) + a_1\exp(x + y + ct)},$$

If we choose $a_1 = a_{-1} = b_{-1}$, then we can obtain

$$u_4(x, y, t) = -e^{i(px+qy+(p^2+2)t)} \coth\left(\frac{2x + 2y - t}{2}\right),$$

$$v_4(x, y, t) = -2 \coth^2\left(\frac{2x + 2y - t}{2}\right).$$

Case 2: $p = c = 2$ **and** $q = d = 2$.

Since the values of c and d can be freely chosen, we set $p = c = 2$ and $d = q = 2$. Then the trial function, Eq. (6) becomes

$$U(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta) + a_{-2} \exp(-2\eta)}{b_2 \exp(2\eta) + b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta) + b_{-2} \exp(-2\eta)}. \tag{40}$$

According to above procedure, substituting (40) into Eq. (31), we obtain

$$\frac{1}{A}[C_6 \exp(6\eta) + C_5 \exp(5\eta) + C_4 \exp(4\eta) + C_3 \exp(3\eta) + C_2 \exp(2\eta) + C_1 \exp(\eta)$$

$$\tag{41}$$

$$+C_0 + C_{-1} \exp(-\eta) + C_{-2} \exp(-2\eta) + C_{-3} \exp(-3\eta)$$

$$+C_{-4} \exp(-4\eta) + C_{-5} \exp(-5\eta) + C_{-6} \exp(-6\eta)] = 0,$$

where

$$A = (b_{-2} \exp(-2\eta) + b_{-1} \exp(-\eta) + b_0 + b_1 \exp(\eta) + b_2 \exp(2\eta))^3, \tag{42}$$

and C_n' are coefficients of $\exp(n\eta)$'. Equating the coefficients of $\exp(n\eta)$ to be zero, we have

$$\begin{cases} C_6 = 0, C_5 = 0, C_4 = 0, C_3 = 0, C_2 = 0, C_1 = 0, \\ C_0 = 0, \\ C_{-6} = 0, C_{-5} = 0, C_{-4} = 0, C_{-3} = 0, C_{-2} = 0, C_{-1} = 0. \end{cases} \tag{43}$$

By the same manipulation as illustrated above, we obtain

(I) The first set is:

$$a_1 = a_1, \quad a_{-1} = 0, \quad a_{-2} = 0, \quad b_0 = -\frac{1}{8} \frac{a_1^2}{(c+1)b_2}, \quad c+1 = c+1, \quad b_{-1} = 0, \tag{44}$$

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$$b_1 = 0, \quad r = p^2 - 1, \quad b_{-2} = 0, \quad p = p, \quad a_2 = 0, \quad b_2 = b_2,$$

$$U_1(x, t) = \frac{8a_1 b_2 (c + 1) \exp(x + y + ct)}{-a_1^2 + 8b_2^2 (c + 1) \exp(2x + 2y + 2ct)}, \quad c = -1 - \frac{a_1^2}{8b_0 b_2}.$$

If we choose $\frac{a_1}{b_2} = 2\sqrt{2(c + 1)}$, then we can obtain

$$u_1(x, t) = \sqrt{2(c + 1)} e^{i(px + qy + (p^2 - 1)t)} \operatorname{csch}(x + y + ct),$$

$$v_1(x, t) = 2\operatorname{csch}^2(x + y + ct).$$

(II) The second set is:

$$a_1 = a_1, \quad a_{-1} = -\frac{a_1 b_{-1}}{b_1}, \quad a_{-2} = 0, \quad b_0 = 0, \quad c + 1 = \frac{1}{2} \frac{a_1^2}{b_1^2}, \quad b_{-1} = b_{-1}, \quad (45)$$

$$b_1 = b_1, \quad r = p^2 + 2, \quad b_{-2} = 0, p = p, \quad a_2 = 0, \quad b_2 = 0,$$

$$U_2(x, t) = \frac{a_1}{b_1} \frac{-b_{-1} \exp(-x - y - ct) + b_1 \exp(x + y + ct)}{b_{-1} \exp(-x - y - ct) + b_1 \exp(x + y + ct)}, \quad c = \frac{1}{2} \frac{a_1^2}{b_1^2} - 1.$$

If we choose $b_1 = b_{-1}$, then we can obtain

$$u_2(x, t) = \sqrt{2(c + 1)} e^{i(px + qy + (p^2 + 2)t)} \tanh(x + y + ct),$$

$$v_2(x, t) = -2\tanh^2(x + y + ct).$$

(III) The third set is:

$$a_1 = 0, \quad a_{-1} = 0, \quad a_{-2} = 0, \quad b_0 = 0, \quad c + 1 = -\frac{1}{32} \frac{a_0^2}{b_2 b_{-2}}, \quad b_{-1} = 0,$$

$$(46)$$

$$b_1 = 0, \quad r = p^2 - 4, \quad b_{-2} = b_{-2}, \quad p = p, \quad a_2 = 0, \quad b_2 = b_2,$$

$$U_3(x, t) = \frac{a_0}{b_{-1} \exp(-x - y - ct) + b_1 \exp(x + y + ct)}, \quad c = -1 - \frac{1}{32} \frac{a_0^2}{b_2 b_{-2}}.$$

If we choose $b_2 = b_{-2}$, then we can obtain

$$u_3(x, t) = -2i\sqrt{2(c + 1)} e^{i(px + qy + (p^2 - 4)t)} \operatorname{sech}(2x + 2y + 2ct),$$

$$v_3(x, t) = 8\operatorname{sech}^2(2x + 2y + 2ct).$$

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(IV) The fourth set is:

$$a_1 = 0, \quad a_{-1} = 0, \quad a_{-2} = -\frac{b_{-2}a_2}{b_2}, \quad b_0 = 0, \quad c + 1 = \frac{1}{8}\frac{a_2^2}{b_2^2}, \; b_{-1} = 0,$$

(47)

$$b_1 = 0, \quad r = p^2 + 8, \quad b_{-2} = b_{-2}, p = p, \quad a_2 = a_2, \quad b_2 = b_2,$$

$$U_4(x,t) = \frac{a_2}{b_2}\frac{-b_{-2}\exp(-2x-2y-2ct) + b_2\exp(2x+2y+2ct)}{b_{-2}\exp(-2x-2y-2ct) + b_2\exp(2x+2y+2ct)}, \quad c = \frac{1}{8}\frac{a_2^2}{b_2^2} - 1.$$

If we choose $b_2 = b_{-2}$, then we can obtain

$$u_4(x,t) = -\sqrt{8(c+1)}e^{i(px+qy+(p^2+8)t)}\coth\left(2x+2y+2ct\right),$$

$$v_4(x,t) = -8\coth^2\left(2x+2y+2ct\right).$$

(V) The fifth set is:

$$a_1 = a_1, \quad a_{-1} = 0, \quad a_{-2} = -\frac{a_1 b_{-2}}{b_1}, \quad b_0 = 0, \quad c + 1 = \frac{2}{9}\frac{a_1^2}{b_1^2}, \quad b_{-1} = 0,$$

(48)

$$b_1 = b_1, \quad r = p^2 + \frac{9}{2}, \quad b_{-2} = b_{-2}, \quad p = p, \quad a_2 = 0, \; b_2 = 0,$$

$$U_5(x,t) = \frac{a_1}{b_1}\frac{-b_{-2}\exp(-2x-2y-2ct) + b_1\exp(x+y+ct)}{b_{-2}\exp(-2x-2y-2ct) + b_1\exp(x+y+ct)}, \quad c = \frac{2}{9}\frac{a_1^2}{b_1^2} - 1.$$

If we choose $b_1 = b_{-2}$, then we can obtain

$$u_5(x,t) = 3\sqrt{\frac{(c+1)}{2}}e^{\frac{1}{2}i(2px+2qy+(2p^2+9)t)}\tanh\left(\frac{3x+3y+3ct}{2}\right),$$

$$v_5(x,t) = -\frac{9}{2}\tanh^2\left(\frac{3x+3y+3ct}{2}\right).$$

(VI) The sixth set is:

$$a_1 = \frac{a_2 b_1}{b_2}, \quad a_{-1} = \frac{a_{-2}b_2}{b_1}, \quad a_{-2} = a_{-2}, \quad b_0 = 0, \quad c + 1 = \frac{2}{9}\frac{a_2^2}{b_2^2}, \; b_{-1} = \frac{b_2^2 a_{-2}}{b_1 a_2},$$

(49)

$$b_1 = b_1, \quad r = p^2 + \frac{9}{2}, \quad b_{-2} = -\frac{a_{-2}b_2}{a_2}, \quad p = p, \quad a_2 = a_2, \quad b_2 = b_2,$$

$$U_6(x,t) = \frac{a_2}{b_2}\frac{a_{-2}b_2\exp(-2x-2y-2ct) + a_2 b_1\exp(x+y+ct)}{-a_{-2}b_2\exp(-2x-2y-2ct) + a_2 b_1\exp(x+y+ct)}, \quad c = \frac{2}{9}\frac{a_2^2}{b_2^2} - 1.$$

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If we choose $\frac{a_2 b_1}{a_{-2} b_2} = 1$, then we can obtain

$$u_6(x, t) = 3\sqrt{\frac{(c+1)}{2}} e^{\frac{1}{2}i(2px+2qy+(2p^2+9)t)} \coth\left(\frac{3x + 3y + 3ct}{2}\right),$$

$$v_6(x, t) = -\frac{9}{2} \coth^2\left(\frac{3x + 3y + 3ct}{2}\right).$$

The results obtained in the above are exact solutions of the couple Higgs equation and the Maccari system. In this article we investigated two systems of two complex coupled equations. The Exp-function method has been successfully applied to obtain some new generalized solitonary solutions to the couple Higgs equation and the Maccari system. Here, we have used the EFM to derive exact solutions with distinct physical structures. Some of these results are in agreement with the results reported specially by [**?**, **?**]. Comparing our results and Bekir's and Zhao's results then it can be seen that the results are same. Also, new results are formally developed in this article. It can be concluded that the Exp-function method is a very powerful and efficient technique in finding exact solutions for wide classes of problems.

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