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# THE SPACE OF ENTIRE SEQUENCES OF FUZZY NUMBERS DEFINED BY INFINITE MATRICES

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Abstract. This paper is devoted to the study of the general properties of entire sequence space of fuzzy numbers by using infinite matrices.

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## 1. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh[18] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming.

In this paper we introduce and examine the concepts of Orlicz space of entire sequence of fuzzy numbers generated by infinite matrices.

Let $C(R^n) = \{A \subset R^n : A\, compact\, and\, convex\}$. The space $C(R^n)$ has linear structure induced by the operations $A + B = \{a + b : a \in A, b \in B\}$ and $\lambda A = \{\lambda a : a \in A\}$ for $A, B \in C(R^n)$ and $\lambda \in R$. The Hausdorff distance between $A$ and $B$ of $C(R^n)$ is defined as

$$\delta_\infty (A, B) = max \{sup_{a \in A} inf_{b \in B} \|a - b\|, sup_{b \in B} inf_{a \in A} \|a - b\|\}.$$

It is well known that $(C(R^n), \delta_\infty)$ is a complete metric space.
The fuzzy number is a function $X$ from $R^n$ to [0,1] which is normal, fuzzy convex, upper semi-continuous and the closure of $\{x \in R^n : X(x) > 0\}$ is compact. These properties imply that for each $0 < \alpha \leq 1$, the $\alpha-$level set $[X]^\alpha = \{x \in R^n : X(x) \geq \alpha\}$ is a nonempty compact convex subset of $R^n$, with support $X^c = \{x \in R^n : X(x) > 0\}$. Let $L(R^n)$ denote the set of all fuzzy numbers. The linear structure of $L(R^n)$ induces the addition $X + Y$ and scalar multiplication $\lambda X, \lambda \in R$, in terms of $\alpha-$ level sets, by $|X + Y|^\alpha = |X|^\alpha + |Y|^\alpha, |\lambda X|^\alpha = \lambda |X|^\alpha$ for each $0 \leq \alpha \leq 1$. Define, for each $1 \leq q < \infty$,

$$d_q(X,Y) = \left(\int_0^1 \delta_\infty (X^\alpha, Y^\alpha)^q \, d\alpha\right)^{1/q} \quad and \quad d_\infty = sup_{0\le\alpha\le1} \delta_\infty (X^\alpha, Y^\alpha),$$

where $\delta_\infty$ is the Hausdorff metric. Clearly $d_\infty(X,Y) = lim_{q\to\infty} d_q(X,Y)$ with $d_q \le d_r$, if $q \le r$ [11]. Throughout the paper, $d$ will denote $d_q$ with $1 \le q \le \infty$. A complex sequence, whose $k^{th}$ term $x_k$ is denoted by $\{x_k\}$ or simply $x$. Let $\phi$ be the set of all finite sequences. Let $\ell_\infty, c, c_0$ be the sequence spaces of bounded, convergent and null sequences $x = (x_k)$ respectively. In respect of $\ell_\infty, c, c_0$ we have $\|x\| = \overset{sup}{k} |x_k|$, where $x = (x_k) \in c_0 \subset c \subset \ell_\infty$. A sequence $x = \{x_k\}$ is said to be analytic if $sup_k |x_k|^{1/k} < \infty$. The vector space of all analytic sequences will be denoted by $\Lambda$. A sequence $x$ is called entire sequence if $lim_{k\to\infty} |x_k|^{1/k} = 0$. The vector space of all entire sequences will be denoted by $\Gamma$. Given a sequence $x = \{x_k\}$ its $n^{th}$ section is the sequence $x^{(n)} = \{x_1, x_2, ..., x_n, 0, 0, ...\}$, $\delta^{(n)} = (0, 0, ..., 1, 0, 0, ...)$, 1 in the $n^{th}$ place and zeros elsewhere.

## 2.Definitions and Preliminaries

Let $w$ denote the set of all fuzzy complex sequences $x = (x_k)_{k=1}^\infty$. Consider
$\Gamma = \left\{x \in w : lim_{k\to\infty}\left(|x_k|^{1/k}\right) = 0\right\}$ and
$\Lambda = \left\{x \in w : sup_k\left(|x_k|^{1/k}\right) < \infty\right\}$.
The space $\Gamma$ and $\Lambda$ is a metric space with the metric

$$d(x, y) = inf\left\{sup_k\left(|x_k - y_k|^{1/k}\right) \le 1\right\} \tag{1}$$

for all $x = \{x_k\}$ and $y = \{y_k\}$ in $\Gamma$.

We now give the following new definitions which will be needed in the sequel.

**Definition 2.1** *Let $X = (X_k)$ be a sequence of fuzzy numbers. The fuzzy number $X_n$ denotes the value of the function at $n \in \mathbb{N}$ and is called the $n^{th}$ term of the sequence. We denote $w(F)$ be the set of all sequences $X = (X_k)$ of fuzzy numbers.*

**Definition 2.2** *Let $X = (X_k)$ be a sequence of fuzzy numbers. Then the set of all $X = (X_k)$ the entire sequence space of fuzzy numbers converge to zero and is written as $\left(|X_k|^{1/k}\right) \to 0$ as $k \to \infty$. It is defined by $\left[d\left(|X_k|^{1/k}\right) \to 0 \, as \, k \to \infty\right]$. We denote the set of all entire sequence space of fuzzy numbers by $\Gamma(F)$. The $\Gamma(F)$ is a metric space with the metric $\rho(X,Y) = sup_k d(X_k, Y_k) = sup_k d\left(|X_k - Y_k|^{1/k}\right)$*

**Definition 2.2** *Let $X = (X_k)$ be a sequence of fuzzy numbers. Then the set of all $X = (X_k)$ sequences of fuzzy numbers are said to be analytic sequence if the set $\left\{\left(|X_k|^{1/k}\right) : k \in \mathbb{N}\right\}$ of fuzzy numbers are bounded.*

By $\Lambda$, we shall denote the set of all analytic sequence space of fuzzy numbers.

Let $A = (a_{nk})$ be an infinite matrix of fuzzy numbers and let $(p_k)$ be a bounded sequence of positive real numbers, then $A_k(X) = \sum_{k=1}^{\infty} a_{nk}x_k$ (provided that the series converge for each $k = 1, 2, \cdots .$) is called the $A-$ transform of $X$. We write $AX = A_k(X)$.

**Definition 2.3** *Let $X = (X_k)$ be a sequence of fuzzy numbers. Then we define*
$\Gamma(F, A, p) = \left\{X \in w(F) : \left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \to 0 \quad as \quad k \to \infty\right\}$
$\Lambda(F, A, p) = \left\{X \in w(F) : sup_k \left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} < \infty\right\}.$
*If $A = I$, the unit matrix, then we get*
$\Gamma(F, A, p) = \Gamma(F, p) = \left\{X \in w(F) : \left[d\left(|X_k|^{1/k}\right)\right]^{p_k} \to 0 \quad as \quad k \to \infty\right\}$
$\Lambda(F, A, p) = \Lambda(F, p) = \left\{X \in w(F) : sup_k \left[d\left(|X_k|^{1/k}\right)\right]^{p_k} < \infty\right\}.$
*If $A$ is an infinite matrix as above $p_k = p$ for all $k$, then we get*
$\Gamma(A, p) = (\Gamma)_A(F) = \{X \in w(F) : AX \in \Gamma(F)\}$
$\Lambda(A, p) = (\Lambda)_A(F) = \{X \in w(F) : AX \in \Lambda(F)\}.$
Suppose that $p_k$ is a constant for all $k$, then $\Gamma(F, A, p) = \Gamma(F, A)$. A metric $d$ on $L(R)$ is said to be translation invariant if $d(X + Z, Y + Z) = d(X, Y)$ for all $X, Y, Z \in L(R)$.

In this paper we study the spaces $\Gamma(F)$, $\Lambda(F)$, $\Gamma(F, A, p)$ and $\Lambda(F, A, p)$ respectively, by applying the infinite matrix $A = (a_{nk})(n, k = 1, 2, 3, \cdots)$.

## 3.Results

**Proposition 3.1** *If $d$ is a translation invariant metric on $L(R)$, then (i)$d(X + Y, 0) \leq d(X, 0) + d(Y, 0)$ (ii)$d(\lambda X, 0) \leq |\lambda| d(X, 0), |\lambda| > 1$. If $d$ is a translation invariant, we have the following straight forward results.*

**Proposition 3.2** *Let $X = (X_k)$ and $Y = (Y_k)$ be a sequence of fuzzy numbers, then $\Gamma(A, p)$ is linear set over the set of complex numbers $C$.*

*Proof:* It is easy. Therefore the proof is omitted.

## 4.Inclusion Relations

**Proposition 4.1** *If $X = (X_k)$ be a sequence of fuzzy numbers. Let $0 \leq p_k \leq q_k$ and let $\left\{\frac{q_k}{p_k}\right\}$ be bounded. Then $\Gamma(A, q) \subset \Gamma(A, p)$.*

*Proof: The proof is clear.*

**Proposition 4.2** *Let $X = (X_k)$ be a sequence of fuzzy numbers.*
*(a) Let $0 < inf p_k \leq p_k \leq 1$. Then $\Gamma(A, p) \subset \Gamma(A)$;*
*(b) Let $1 \leq p_k \leq sup p_k < \infty$. Then $\Gamma(A) \subset \Gamma(A, p)$.*

*Proof:*(a) Let $X \in \Gamma(A, p)$. Then

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \to 0 \quad as \quad k \to \infty \tag{2}$$

Since $0 < inf p_k \leq p_k \leq 1$,

$$\left[d\left(|A_k(X)|^{1/k}\right)\right] \leq \left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \tag{3}$$

From (2) and (3) it follows that $X \in \Gamma(A)$. Thus $\Gamma(A,p) \subset \Gamma(A)$. We have thus proved (a).

*Proof: (b)* Let $p_k \geq 1$ for each $k$ and $supp_k < \infty$. Let $X \in \Gamma(A)$.

$$\left[d\left(|A_k(X)|^{1/k}\right)\right] \to 0 \quad as \quad k \to \infty \tag{4}$$

Since $1 \leq p_k \leq supp_k < \infty$, we have

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \leq \left[d\left(|A_k(X)|^{1/k}\right)\right]. \tag{5}$$

Hence $\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \to 0 \quad as \quad k \to \infty$ [by using eq(4)]. Therefore $X \in \Gamma(A,p)$. This completes the proof.

**Proposition 4.3** *If* $X = (X_k)$ *be a sequence of fuzzy numbers. Let* $0 < p_k \leq q_k < \infty$ *for each* $k$. *Then* $\Gamma(A,p) \subseteq \Gamma(A,q)$.

*Proof:*Let $X \in \Gamma(A,p)$. Hence

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \to 0 \quad as \quad k \to \infty. \tag{6}$$

This implies that $\left[d\left(|A_k(X)|^{1/k}\right)\right] \leq 1$ for sufficiently large $k$. We get

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{q_k} \leq \left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k}. \tag{7}$$

$\Rightarrow \left[d\left(|A_k(X)|^{1/k}\right)\right]^{q_k} \to 0 \quad as \quad k \to \infty$ by using eq(6). We get $X \in \Gamma(A,q)$. Hence $\Gamma(A,p) \subseteq \Gamma(A,q)$. This completes the proof.

**Proposition 4.4** If $lim\,inf_k \left(\frac{p_k}{q_k}\right) > 0$ then $\Gamma(A,q) \subset \Gamma(A,p)$.

*Proof:*Suppose that $lim\,inf_k \left(\frac{p_k}{q_k}\right)$ holds. Let $X \in \Gamma(A,q)$. Then there is $\beta > 0$ such that $p_k > \beta q_k$ for large $k$ such that

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \leq \left[\left[d\left(|A_k(X)|^{1/k}\right)\right]^{q_k}\right]^{\beta}. \tag{8}$$

Since $\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} \leq 1$ for each $k, X \in \Gamma(A,p)$. This completes the proof.

## 5.Paranormed spaces

If $E$ is a linear space over the filed $C$, then a paranorm on $E$ is a function $g : E \to R$ which satisfies the following axioms; for $X, Y \in E$,
(P.1)$g(\theta) = 0$, (P.2)$g(X) \geq 0$ for all $X \in E$, (P.3)$g(-X) = g(X)$ for all $X \in E$, (P.4)$g(X+Y) \leq g(X) + g(Y)$ for all $X, Y \in E$, (P.5)If $(\lambda_n)$ is a sequence of scalars with $\lambda_n \to \lambda\,(n \to \infty)$ and $(X_n)$ is a sequence of the elements of $E$ with $X_n \to X$ imply $\lambda_n X_n \to \lambda X$, where $\lambda_n, \lambda \in C$ and $X_n, X \in E$. In other words $|\lambda_n - \lambda| \to 0, g(X_n - X) \to 0$ imply $g(\lambda_n X_n - \lambda X) \to 0\,(n \to \infty)$. A paranormed space is a linear space $E$ with a paranorm $g$ and is written as $(E, g)$.

**Theorem 5.1** *If $X = (X_k)$ be a sequence of fuzzy numbers. Then $\Gamma(A,p)$ is complete with respect to the topology generated by the paranorm $h$ defined by*

$$h(X) = sup_k \left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k}, where\, d\, is\, translation\, invariant. \tag{9}$$

*Proof:* Clearly $h(\theta) = 0, h(-X) = h(X)$. It can also be seen easily that $h(X+Y) \leq h(X) + h(Y)$ for $X = (X_k), Y = (Y_k) \in \Gamma(A,p)$, since $d$ is a translation invariant. Now for any scalar $\lambda$, we have $|\lambda|^{p_k} < max\{1, |\lambda|\}$, so that $h(\lambda X) < max\{1, |\lambda|\}, h(X)$ on $\Gamma(A,p)$. Hence $\lambda \to 0, X \to \theta$ implies $\lambda X \to \theta$ and also $X \to \theta, \lambda$ fixed implies $\lambda X \to \theta$. Now let $\lambda \to 0, X$ fixed. For $|\lambda| < 1$ we have

$$\left[d\left(|A_k(X)|^{1/k}\right)\right]^{p_k} < \epsilon \text{ for } n > N(\epsilon).$$

Also, for $1 \leq k \leq N$, since $\left[d\left(|A_k(X)|^{1/i}\right)\right]^{p_k} < \epsilon$, there exists $m$ such that $\left(\sum_{i=m}^{\infty}\left[d\left(|\lambda a_{k,i} X_i|^{1/i}\right)\right]^{p_i}\right) < \epsilon$. Taking $\lambda$ small enough then we have $\left(\sum_{i=m}^{\infty}\left[d\left(|\lambda a_{k,i} X_i|^{1/i}\right)\right]^{p_i}\right) < 2\,\epsilon$, for all $i$. Hence $h(\lambda X) \to 0 \quad as \quad \lambda \to 0$. Therefore $h$ is a paranorm on $\Gamma(A,p)$.

To show the completeness , let $\left(X^{(i)}\right)$ be a Cauchy sequence in $\Gamma(A,p)$. Then for a given $\epsilon > 0$ there is $r \in \mathbb{N}$ such that

$$\left[d\left(\left|A_k\left(X^{(i)} - X^{(j)}\right)\right|^{1/k}\right)\right]^{p_k} < \epsilon\, for\, all\, i, j > r. \tag{10}$$

Since $d$ is a translation invariant, so (10) implies that

$$\left(\sum_s a_{ks}\, d\left(\left|X_k^{(i)} - X_k^{(j)}\right|^{1/k}\right)\right) < \epsilon\, for\, all\, i, j > r\, and\, each\, k. \tag{11}$$

Hence $d\left(\left|X_k^{(i)}-X_k^{(j)}\right|^{1/k}\right)<\epsilon\, for\, all\, i,j>r$. Therefore $\left(X^{(i)}\right)$ is a Cauchy sequece in $L\left(R\right)$. Since $L\left(R\right)$ is complete, $lim_{j\to\infty}X_k^j=X_k$, say. Fixing $r_0\geq r$ and letting $j\to\infty$, we obtain (12) that

$$\left(\sum_s a_{ks}\, d\left(\left|X_k^{(i)}-X_k\right|^{1/k}\right)\right)<\epsilon\, for\, all\, r_0>r. \tag{12}$$

(i.e) $d\left(\sum_s a_{ks}\, d\left(\left|X_k^{(i)}-X_k\right|^{1/k}\right)\right)<\epsilon\, for\, all\, r_0>r$, since $d$ is a translation invariant. Hence $\left[d\left(\left|A_k\left(X^{(i)}-X\right)\right|^{1/k}\right)\right]^{p_k}<\epsilon$. (i.e) $X^{(i)}\to X$ in $\Gamma\left(A,p\right)$. It is easy to see that $X\in\Gamma\left(A,p\right)$. Hence $\Gamma\left(A,p\right)$ is complete. This completes the proof.
Similarly we can prove the following:

**Theorem 5.2** *If $X=(X_k)$ be a sequence of fuzzy numbers, then $\Lambda\left(A,p\right)$ is a complete paranormed space with the paranorm given by (9) if $inf p_k>0$.*

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