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COMMON FIXED POINT THEOREM FOR MAPS SATISFYING A GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE

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Abstract. In this paper, we prove a common fixed point theorem for maps satisfying a general contractive condition of integral type with compatibility conditions of type (I) and of type (II).

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1. Introduction

There are a lot of generalization of Banach contraction principle in the literature. One of the most interesting generalization of it is the Branciari's [7] fixed point result. Branciari was proved a fixed point theorem for a single mapping satisfying an analogue of Banach contraction principle for an integral type inequality. After then the authors in [1], [3], [4] , [5] , [10] , [11] , [17] , [25] , [27] and [29] proved some fixed point theorems involving more general contractive conditions. Also in [26], Suzuki shows that Meir-Keeler contractions of integral type are still Meir-Keeler contractions. In this paper, we establish a fixed point theorem for single valued maps satisfying a general contractive inequality of integral type with compatibility condition of type (I) and of type (II).

Jungck [12] initiated and provided a soothing machinery for obtaining common fixed points in metric spaces by using commuting mappings. Inspired by the above work, many authors developed much weaker conditions. Let (X, d) be a metric spaces.

Definition 1. *Mappings $S, T : X \to X$ are said to be*
(a) *compatible* [13] *if* $\lim d(STx_n, TSx_n) = 0$,
(b) *compatible of type* (A) [15] *if*

$$\lim d(STx_n, TTx_n) = 0 \text{ and } \lim d(TSx_n, SSx_n) = 0,$$

(c) *compatible of type* (B) [21] *if*

$$\lim d(STx_n, TTx_n) \leq \frac{1}{2}[\lim d(STx_n, St) + \lim d(St, SSx_n)]$$

and

$$\lim d(TSx_n, SSx_n) \leq \frac{1}{2}[\lim d(TSx_n, Tt) + \lim d(Tt, TTx_n)],$$

(d) *compatible of type* (P) [22] *if*

$$\lim d(SSx_n, TTx_n) = 0,$$

whenever $\{x_n\}$ *is a sequence in* X *such that* $\lim Sx_n = \lim Tx_n = t$ *for some* $t \in X$.

We can find some examples, propositions and lemmas about the above definitions in [13] , [15] , [21] , [22].

Lemma 1 ([13] **resp.**[15] , [21] , [22]**).** *Let* S *and* T *be a compatible (resp. compatible of type* (A)*, compatible of type* (B)*, compatible of type* (P)*) self mapping of a metric space* (X, d)*. If* $Sx = Tx$ *for some* $x \in X$*, then* $STx = TSx$*.*

In 1996 Jungck [14] defines S and T to be weakly compatible if $Sx = Tx$ implies $STx = TSx$. By Lemma 1, follows that every compatible (compatible of type (A), compatible of type (B), compatible of type (P)) pair of mapping is weakly compatible. There is an example in [23] shows that implication is not reversible. Many fixed point results have been obtained for weakly compatible mappings (see [8] , [9] , [16] , [24] and [27])

Now we give the definitions of compatible of type (I) and of type (II) mappings, which were given in [20].

Definition 2. *Let* $S, T : X \to X$ *be mappings. The pair* (S, T) *is said to be compatible of type* (I) *if*

$$d(t, Tt) \leq \overline{\lim} d(t, STx_n),$$

whenever $\{x_n\}$ *is a sequence in* X *such that* $\lim Sx_n = \lim Tx_n = t$ *for some* $t \in X$. *The pair* (S, T) *is said to be compatible of type* (II) *if and only if* (T, S) *is compatible of type* (I).

Again many fixed point results have been obtained for maps satisfying compatibility condition of type (I) and of type (II) (see [2] , [6] , [20] , [28]).

Now we give some examples which shows that the concepts of weakly compatible maps and compatible maps of type (I) and of type (II) are independent from each other.

Example 1. Let $X = [0, \infty)$ be with the usual metric. Define $S, T : X \to X$ by

$$Sx = \begin{cases} 2 & \text{if} \quad x \in [0, 2] \\ 2 + x & \text{if} \quad x \in (2, \infty) \end{cases} \quad \text{and } Tx = \begin{cases} 2 + x & \text{if} \quad x \in [0, 2) \\ 4 + x & \text{if} \quad x \in [2, \infty) \end{cases}.$$

Note that 2 is a fixed point of S, then the pair (S, T) is compatible of type (II) but the mappings S and T are not weak compatible because $S0 = 2 = T0$ but $ST0 = 2 \neq 6 = TS0$.

Example 2 ([19]). Let $X = [0, \infty)$ be with the usual metric. Define $S, T : X \to X$ by

$$Sx = 2x + 1 \text{ and } Tx = x^2 + 1.$$

Then at $x = 0$, $Sx = Tx$. Also $STx = 3$ and $TSx = 2$, which shows that S and T are not weak compatible. Now suppose that $\{x_n\}$ be a sequence in X such that $\lim Sx_n = \lim Tx_n = t$ for some $t \in X$. By definition of S and T, $t = 1$. For this value we have $d(t, Tt) = 1 \leq 2 = \overline{\lim} d(t, STx_n)$, which shows that the pair (S, T) is compatible mappings of type (I).

Example 3 ([19]). Let $X = [0, \infty)$ be with the usual metric. Define $S, T : X \to X$ by

$$Sx = \begin{cases} \cos x & \text{if} \quad x \neq 1 \\ 0 & \text{if} \quad x = 1 \end{cases} \quad \text{and } Tx = \begin{cases} e^x & \text{if} \quad x \neq 1 \\ 0 & \text{if} \quad x = 1 \end{cases}.$$

Then it is clear that $Sx = Tx$ if and only if $x = 0$ and $x = 1$. Also at these points $STx = TSx$. It means that S and T are weakly compatible. Now suppose that $\{x_n\}$ be a sequence in X such that $\lim Sx_n = \lim Tx_n = t$ for some $t \in X$. By definition of S and T, $t = 1$. For this value we have $d(t, Tt) = 1$ and $\overline{\lim} d(t, STx_n) = (1 - \cos x) < 1$. Therefore the pair (S, T) is not compatible mappings of type (I).

Preposition 1 ([20]). *Let $S, T : X \to X$ be such that the pair (S, T) is compatible of type (I) (resp. type (II)) and $Sp = Tp$ for some $p \in X$. Then $d(Sp, TTp) \leq d(Sp, STp)$ (resp. $d(Tp, SSp) \leq d(Tp, TSp)$).*

Lemma 2 ([18]). *Let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ be a right continuous function such that $\psi(t) < t$ for every $t > 0$, then $\lim_{n \to \infty} \psi^n(t) = 0$, where ψ^n denotes the n-times repeated composition of ψ with itself.*

2. Main result

Let (X, d) be a metric space and let A, B, S and T be self-maps defined on X. We consider the following:

(i) $S(X) \subseteq B(X)$, $T(X) \subseteq A(X)$,

(ii) for all $x, y \in X$, there exists a right continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}^+, \psi(0) = 0$ and $\psi(s) < s$ for $s > 0$ such that

$$\int_0^{d(Sx,Ty)} \varphi(t)dt \leq \psi\left(\int_0^{M(x,y)} \varphi(t)dt\right),$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesque integrable mapping which is summable, non-negative and such that

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \text{ for each } \varepsilon > 0, \tag{1}$$

and

$$M(x,y) = \frac{1}{2}\max\{2d(Ax,By), d(Sx,Ax), d(Ty,By), d(Sx,By), d(Ty,Ax)\},$$

(iii) A or B is continuous and the pairs (S, A) and (T, B) are compatible of type (I),

(iv) S or T is continuous and the pairs (S, A) and (T, B) are compatible of type (II).

Now we prove the following theorem.

Theorem 1. *Let (X, d) be a complete metric space, A, B, S and T be self-maps defined on X satisfying the conditions $(i), (ii)$ and any one of (iii) or (iv), then A, B, S and T have a unique common fixed point.*

Proof. Let $x_0 \in X$ be an arbitrary point of X. From (i) we can construct a sequence $\{y_n\}$ in X as follows:

$$y_{2n+1} = Sx_{2n} = Bx_{2n+1} \text{ and } y_{2n+2} = Tx_{2n+1} = Ax_{2n+2}$$

for all $n = 0, 1, ...$ Define $d_n = d(y_n, y_{n+1})$, then, by (ii),

$$\int_0^{d(Sx_{2n},Tx_{2n+1})} \varphi(t)dt \leq \psi\left(\int_0^{M(x_{2n},x_{2n+1})} \varphi(t)dt\right) \tag{2}$$

where

$$\begin{aligned} M(x_{2n}, x_{2n+1}) &= \frac{1}{2}\max\{2d(Ax_{2n}, Bx_{2n+1}), d(Sx_{2n}, Ax_{2n}), d(Tx_{2n+1}, Bx_{2n+1}), \\ &\quad d(Sx_{2n}, Bx_{2n+1}), d(Tx_{2n+1}, Ax_{2n})\} \\ &= \max\{d_{2n}, \frac{d_{2n+1}}{2}, \frac{d(y_{2n}, y_{2n+2})}{2}\} \\ &\leq \max\{d_{2n}, d_{2n+1}\} \end{aligned}$$

Thus from (2) we have

$$\int_0^{d_{2n+1}} \varphi(t)dt \leq \psi\left(\int_0^{\max\{d_{2n},d_{2n+1}\}} \varphi(t)dt\right). \tag{3}$$

Now, if $d_{2n+1} \geq d_{2n}$ for some n, then, from (3) we have

$$\int_0^{d_{2n+1}} \varphi(t)dt \leq \psi\left(\int_0^{d_{2n+1}} \varphi(t)dt\right) < \int_0^{d_{2n+1}} \varphi(t)dt$$

which is a contradiction. Thus $d_{2n} > d_{2n+1}$ for all n, and so, from (3) we have

$$\int_0^{d_{2n+1}} \varphi(t)dt \leq \psi\left(\int_0^{d_{2n}} \varphi(t)dt\right).$$

Similarly,

$$\int_0^{d_{2n}} \varphi(t)dt \leq \psi\left(\int_0^{d_{2n-1}} \varphi(t)dt\right).$$

In general, we have for all $n = 1, 2, ...$,

$$\int_0^{d_n} \varphi(t)dt \leq \psi\left(\int_0^{d_{n-1}} \varphi(t)dt\right). \tag{4}$$

From (4), we have

$$\begin{aligned}
\int_0^{d_n} \varphi(t)dt &\leq \psi\left(\int_0^{d_{n-1}} \varphi(t)dt\right) \\
&\leq \psi^2\left(\int_0^{d_{n-2}} \varphi(t)dt\right) \\
&\;\;\vdots \\
&\leq \psi^n\left(\int_0^{d_0} \varphi(t)dt\right),
\end{aligned}$$

and, taking the limit as $n \to \infty$ and using Lemma 2, we have

$$\lim_{n\to\infty} \int_0^{d_n} \varphi(t)dt \leq \lim_{n\to\infty} \psi^n\left(\int_0^{d_0} \varphi(t)dt\right) = 0,$$

which, from (1), implies that

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} d(y_n, y_{n+1}) = 0. \tag{5}$$

We now show that $\{y_n\}$ is a Cauchy sequence. For this it is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ such that for each even integer $2k$ there exist even integers $2m(k) > 2n(k) > 2k$ such that

$$d(y_{2n(k)}, y_{2m(k)}) \geq \varepsilon. \tag{6}$$

For every even integer $2k$, let $2m(k)$ be the least positive integer exceeding $2n(k)$ satisfying (6) such that

$$d(y_{2n(k)}, y_{2m(k)-2}) < \varepsilon. \tag{7}$$

Now

$$\begin{aligned} 0 &< \delta := \int_0^{\varepsilon} \varphi(t)dt \\ &\leq \int_0^{d(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt \\ &\leq \int_0^{d(y_{2n(k)}, y_{2m(k)-2}) + d_{2m(k)-2} + d_{2m(k)-1}} \varphi(t)dt. \end{aligned}$$

Then by (5), (6) and (7) it follows that

$$\lim_{k\to\infty} \int_0^{d(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt = \delta. \tag{8}$$

Also, by the triangular inequality,

$$\left| d(y_{2n(k)}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)}) \right| \leq d_{2m(k)-1}$$

and

$$\left| d(y_{2n(k)+1}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)}) \right| \leq d_{2m(k)-1} + d_{2n(k)}$$

and so

$$\int_0^{\left| d(y_{2n(k)}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)}) \right|} \varphi(t)dt \leq \int_0^{d_{2m(k)-1}} \varphi(t)dt,$$

and

$$\int_0^{\left| d(y_{2n(k)+1}, y_{2m(k)-1}) - d(y_{2n(k)}, y_{2m(k)}) \right|} \varphi(t)dt \leq \int_0^{d_{2m(k)-1} + d_{2n(k)}} \varphi(t)dt.$$

Using (8), we get

$$\int_0^{d(y_{2n(k)}, y_{2m(k)-1})} \varphi(t)dt \to \delta \tag{9}$$

and
$$\int_0^{d(y_{2n(k)+1},y_{2m(k)-1})} \varphi(t)dt \to \delta \tag{10}$$
as $k \to \infty$. Thus
$$\begin{aligned} d(y_{2n(k)}, y_{2m(k)}) &\leq d_{2n(k)} + d(y_{2n(k)+1}, y_{2m(k)}) \\ &\leq d_{2n(k)} + d(Sx_{2n(k)}, Tx_{2m(k)-1}), \end{aligned}$$
and so
$$\int_0^{d(y_{2n(k)},y_{2m(k)})} \varphi(t)dt \leq \int_0^{d_{2n(k)}+d(Sx_{2n(k)},Tx_{2m(k)-1})} \varphi(t)dt.$$
Letting $k \to \infty$ on both sides of the last inequality, we have
$$\begin{aligned} \delta &\leq \lim_{k\to\infty} \int_0^{d(Sx_{2n(k)},Tx_{2m(k)-1})} \varphi(t)dt \\ &\leq \lim_{k\to\infty} \psi\left(\int_0^{M(x_{2n(k)},x_{2m(k)-1})} \varphi(t)dt\right), \end{aligned} \tag{11}$$
where
$$\begin{aligned} M(x_{2n(k)}, x_{2m(k)-1}) &= \frac{1}{2}\max\{2d(y_{2n(k)}, y_{2m(k)-1}), d_{2n(k)}, d_{2m(k)-1}, \\ & d(y_{2n(k)+1}, y_{2m(k)-1}), d(y_{2n(k)}, y_{2m(k)})\}. \end{aligned}$$
Combining (5), (6), (7), (8), (9) and (10), yields the following contradiction from (11):
$$\delta \leq \psi(\delta) < \delta.$$
Thus $\{y_{2n}\}$ is a Cauchy sequence and so $\{y_n\}$ is a Cauchy sequence. Since X is complete it converges to a point z in X. Since $\{Sx_{2n}\}, \{Bx_{2n+1}\}, \{Tx_{2n+1}\}$ and $\{Ax_{2n+2}\}$ are subsequences of $\{y_n\}$, then $Sx_{2n}, Bx_{2n+1}, Tx_{2n+1}, Ax_{2n+2} \to z$ as $n \to \infty$.

Now, suppose that the condition (iii) holds with B is continuous. Then, since the pair (T, B) is compatible of type (I) and B is continuous, we have
$$d(z, Bz) \leq \overline{\lim} d(z, TBx_{2n+1}),\ BBx_{2n+1} \to Bz. \tag{12}$$
Now setting $x = x_{2n}$ and $y = Bx_{2n+1}$ in (ii), we obtain
$$\int_0^{d(Sx_{2n},TBx_{2n+1})} \varphi(t)dt \leq \psi\left(\int_0^{M(x_{2n},Bx_{2n+1})} \varphi(t)dt\right), \tag{13}$$

where

$$\begin{aligned} M(x_{2n}, Bx_{2n+1}) &= \frac{1}{2}\max\{2d(Ax_{2n}, BBx_{2n+1}), d(Sx_{2n}, Ax_{2n}), \\ & d(TBx_{2n+1}, BBx_{2n+1}), d(Sx_{2n}, BBx_{2n+1}), d(TBx_{2n+1}, Ax_{2n})\}. \end{aligned}$$

We claim that $\overline{\lim} d(z, TBx_{2n+1}) = 0$, Suppose $\overline{\lim} d(z, TBx_{2n+1}) > 0$. Now, by letting the limit superior on both sides of (13), we have

$$\begin{aligned} \int_0^{\overline{\lim} d(z,TBx_{2n+1})} \varphi(t)dt &= \overline{\lim} \int_0^{d(Sx_{2n},TBx_{2n+1})} \varphi(t)dt \\ &\leq \overline{\lim}\psi\left(\int_0^{M(x_{2n},Bx_{2n+1})} \varphi(t)dt\right) \\ &\leq \psi\left(\int_0^{\max\{d(z,Bz), \frac{\overline{\lim} d(TBx_{2n+1},Bz)}{2}, \frac{\overline{\lim} d(TBx_{2n+1},z)}{2}\}} \varphi(t)dt\right) \\ &\leq \psi\left(\int_0^{\overline{\lim} d(TBx_{2n+1},z)} \varphi(t)dt\right) \\ &< \int_0^{\overline{\lim} d(TBx_{2n+1},z)} \varphi(t)dt, \end{aligned}$$

which is a contradiction. Thus $\overline{\lim} d(z, TBx_{2n+1}) = 0$ and so from (12) $Bz = z$. Again replacing x by x_{2n} and y by z in (ii), we have

$$\int_0^{d(Sx_{2n},Tz)} \varphi(t)dt \leq \psi\left(\int_0^{M(x_{2n},z)} \varphi(t)dt\right),$$

where

$$\begin{aligned} M(x_{2n}, z) &= \frac{1}{2}\max\{2d(Ax_{2n}, Bz), d(Sx_{2n}, Ax_{2n}), d(Tz, Bz), \\ & d(Sx_{2n}, Bz), d(Tz, Ax_{2n})\} \\ &= \frac{1}{2}\max\{d(Ax_{2n}, z), d(Sx_{2n}, Ax_{2n}), d(Tz, z), d(Sx_{2n}, z), d(Tz, Ax_{2n})\} \end{aligned}$$

and letting $n \to \infty$, we have

$$\int_0^{d(z,Tz)} \varphi(t)dt \leq \psi\left(\int_0^{\frac{d(z,Tz)}{2}} \varphi(t)dt\right) < \int_0^{\frac{d(z,Tz)}{2}} \varphi(t)dt$$

which is a contradiction if $d(z,Tz) > 0$. Thus $d(z,Tz) = 0$, i.e. $Tz = z$. Since $T(X) \subseteq A(X)$, there is a point $u \in X$ such that $Tz = Au = z$. From (ii), we have,

$$\begin{aligned}
\int_0^{d(Su,z)} \varphi(t)dt &= \int_0^{d(Su,Tz)} \varphi(t)dt \\
&\leq \psi\left(\int_0^{d(Su,z)} \varphi(t)dt\right) \\
&< \int_0^{d(Su,z)} \varphi(t)dt,
\end{aligned}$$

which is a contradiction if $d(Su,z) > 0$. Thus $Su = z = Au$. By Proposition 1, we have $d(Su, AAu) \leq d(Su, SAu)$ and so $d(z, Az) \leq d(z, Sz)$. Again from (ii), we have

$$\begin{aligned}
\int_0^{d(Sz,z)} \varphi(t)dt &= \int_0^{d(Sz,Tz)} \varphi(t)dt \\
&\leq \psi\left(\int_0^{\frac{d(Sz,z)}{2}} \varphi(t)dt\right) \\
&< \int_0^{\frac{d(Sz,z)}{2}} \varphi(t)dt,
\end{aligned}$$

which is a contradiction if $d(Sz,z) > 0$. This shows that $Sz = z = Az = Bz = Tz$ and z is a common fixed point of A, B, S and T.

If we suppose that A is continuous instead of B, similarly we can show that z is a common fixed point of A, B, S and T.

The other case (iv) can be disposed from a similar argument as above.

It is easy to see that the common fixed point of A, B, S and T is unique.

Remark 1. *By Theorem 1, we have a different version of Theorem 2.1 of* [5]*, since we use compatibility condition of type* (I) *and of type* (II) *for mappings.*

If we use $d(Ax, By)$ instead of $M(x,y)$ in Theorem 1, we have the following corollary.

Corollary 1. *Let* (X,d) *be a complete metric space,* A, B, S *and* T *be self-maps defined on* X *satisfying the conditions* (i) *and*

(ii^*) *for all* $x, y \in X$*, there exists a right continuous function* $\psi : \mathbb{R}^+ \to \mathbb{R}^+, \psi(0) = 0$ *and* $\psi(s) < s$ *for* $s > 0$ *such that*

$$\int_0^{d(Sx,Ty)} \varphi(t)dt \leq \psi\left(\int_0^{d(Ax,By)} \varphi(t)dt\right),$$

where φ is as in Theorem 1. If (iii) or (iv) holds, then A, B, S and T have a unique common fixed point.

If we choose $\varphi(t) \equiv 1$ and $\psi(s) = \alpha s, 0 < \alpha < 1$ in Corollary 1, we have Corollary 3.1 of [20].

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