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REGULARITY AND NORMALITY ON L-TOPOLOGICAL SPACES (II)

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Abstract. In this paper, we have defined not only S_1 regularity and S_1 normality but also we have defined those strong and weak forms on L-topological spaces. we investigate some of their properties and the relations between them.

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1. Introduction

The concept of fuzzy topology was first defined in 1968 by chang [2] and later redefined in a somewhat different way by Hutton and Reilly and others. A new definition of fuzzy topology introduced by Badard [1] under the name of "smooth topology". The smooth topological space was rediscovered by Ramadan[5].

In the present paper, we shall study strong S_1 regularity, S_1 regularity, weak S_1 regularity, strong S_1 normality, S_1 normality and weak S_1 normality on L-topological spaces. Also we shall investigate some of their properties and the relations between them on the L-topological spaces.

2. Preliminaries

Throughout this paper, L, L' represent two completely distributive lattice with the smallest element 0 (or $\perp$) and the greatest element 1 (or $\top$), where $0 \neq 1$. Let $P(L)$ be the set of all non-unit prime elements in L such that $a \in P(L)$ iff $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. Finally, let X be a non-empty usual set, and L^X be the set of all L-fuzzy sets on X. For each $a \in L$, let $\underline{a}$ denote constant-valued L-fuzzy set with a as its value. Let $\underline{0}$ and $\underline{1}$ be the smallest element and the greatest element in L^X, respectively. For the empty set $\emptyset \subset L$, we define $\wedge\emptyset = 1$ and $\vee\emptyset = 0$.

Definition 2.1. (Wang [7]) *Suppose that $a \in L$ and $A \subseteq L$.*

(1) A is called a maximal family of a if
(a) $infA = a$*,*
(b) $\forall B \subseteq L, infB \leq a$ *implies that* $\forall x \in A$ *there exists* $y \in B$ *such that* $y \leq x$*.*
(2) A is called a minimal family of a if
(a) $supA = a$*,*
(b) $\forall B \subseteq L, supB \geq a$ *implies that* $\forall x \in A$ *there exists* $y \in B$ *such that* $y \geq x$*.*

Remark 2.1. Hutton [4] proved that if L is a completely distributive lattice and $a \in L$, then there exists $B \subseteq L$ such that
(i) $a = \bigvee B$, and
(ii) if $A \subseteq L$ and $a = \bigvee A$, then for each $b \in B$ there is a $c \in A$ such that $b \leq c$.

However, if $\forall a \in L$, and if there exists $B \subseteq L$ satisfying (i) and (ii), then in general L is not a completely distributive lattice. To this end, Wang [7] introduced the following modification of condition (ii),
(ii') if $A \subseteq L$ and $a \leq \bigvee A$, then for each $b \in B$ there is a $c \in A$ such that $b \leq c$.
Wang proved that a complete lattice L is completely distributive if and only if for each element a in L, there exists $B \subseteq L$ satisfying (i) and (ii'). Such a set B is called a minimal set of a by Wang [7]. The concept of maximal family is the dual concept of minimal family, and a complete lattice L is completely distributive if and only if for each element a in L, there exists a maximal family $B \subseteq L$.

Let $\alpha(a)$denote the union of all maximal families of a. Likewise, let $\beta(a)$ denote the union of all minimal sets of a. Finally, let $\alpha^*(a) = \alpha(a) \wedge M(L)$. one can easily see that both $\alpha(a)$ and $\alpha^*(a)$ are maximal sets of a. likewise, both $\beta(a)$ and $\beta^*(a)$ are minimal sets of a. Also, we have $\alpha(1) = \emptyset$ and $\beta(0) = \emptyset$.

Definition 2.2. *An L-fuzzy topology on X is a map* $\tau : L^X \to L$ *satisfying the following three axioms:*
1) $\tau(\underline{\top}) = \top$*;*
2) $\tau(A \wedge B) \geq \tau(A) \wedge \tau(B)$ *for every* $A, B \in L^X$*;*
3) $\tau(\vee_{i\in\triangle} A_i) \geq \bigvee_{i\in\triangle} \tau(A_i)$ *for every family* $\{A_i | i \in \triangle\} \subseteq L^X$*.*
The pair (X, τ) is called an L-fuzzy topological space. For every $A \in L^X, \tau(A)$ is called the degree of openness of the fuzzy subset A.

Lemma 2.1. *(Shi [6] and Wang [7]). For* $a \in L$ *and a map* $\tau : L^X \to L$*, we define*

$$\tau^{[a]} = \left\{ A \in L^X \mid a \notin \alpha(\tau(A)) \right\}.$$

Let τ be a map from L^X to L and $a, b \in L$. Then

(1) $a \in \alpha(b) \Rightarrow \tau^{[a]} \subseteq \tau^{[b]}$.

(2) $a \leq b \Leftrightarrow \beta(a) \subseteq \beta(b) \Leftrightarrow \beta^*(a) \subseteq \beta^*(b) \Leftrightarrow \alpha(b) \subseteq \alpha(a) \Leftrightarrow \alpha^*(b) \subseteq (a)$.

(3) $\alpha(\bigwedge_{i\in I} a_i) = \bigcup_{i\in I} \alpha(a_i)$ and $\beta(\bigvee_{i\in I} a_i) = \bigcup_{i\in I} \beta(a_i)$ for any sub-family $\{a_i\}_{i\in I} \subseteq L$.

The family of all fuzzy sets on X will be denoted by L^X.

Definition 2.3. *A smooth topological space (sts) [3] is an ordered pair (X, τ), where X is a non-empty set and $\tau : L^X \to L'$ is a mapping satisfying the following properties :*

(O1) $\tau(\underline{0}) = \tau(\underline{1}) = 1_{L'}$,

(O2) $\forall A_1, A_2 \in L^X, \tau(A_1 \cap A_2) \geq \tau(A_1) \wedge \tau(A_2)$,

(O3) $\forall I, \tau(\bigcup_{i\in I} A_i) \geq \bigwedge_{i\in I} \tau(A_i)$.

Definition 2.4. *A smooth cotopology is defined as a mapping $\Im : L^X \to L'$ which satisfies*

(C1) $\Im(\underline{0}) = \Im(\underline{1}) = 1_{L'}$,

(C2) $\forall B_1, B_2 \in L^X, \Im(B_1 \cup B_2) \geq \Im(B_1) \wedge \Im(B_2)$,

(C3) $\forall I, \Im(\bigcap_{i\in I} B_i) \geq \bigwedge_{i\in I} \Im(B_i)$.

In this paper we suppose $L' = L$.

The mapping $\Im_t : L^X \to L'$, defined by $\Im_t(A) = \tau(A^c)$ where τ is a smooth topology on X, is smooth cotopology on X. Also $\tau_\Im : L^X \to L'$, defined by $\tau_\Im(A) = \Im(A^c)$ where $\Im$ is a smooth cotopology on X, is a smooth topology on X where A^c denotes the complement of A [5].

Definition 2.5. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a mapping ; then [10], f is smooth continuous iff $\Im_{\tau_2}(A) \leq \Im_{\tau_1}(f^{-1}(A)), \forall A \in L^Y$.*

Definition 2.6. *A map $f : X \to Y$ is called smooth open (resp. closed) with respect to the smooth topologies τ_1 an τ_2 (resp. cotopologies $\Im_1$ and $\Im_2$), respectively, iff for each $A \in L^X$ we have $\tau_1(A) \leq \tau_2(f(A))$ (resp. $\Im_1(A) \leq \Im_2(f(A))$), where*

$$f(C)(y) = sup\,\{C(x) : x \in f^{-1}(\{y\})\},\ \textit{if}\quad f^{-1}(\{y\}) \neq \emptyset,$$
$$\textit{and } f(C)(y) = 0 \quad \textit{if otherwise}.$$

Definition 2.7. *Let $\tau : L^X \to L$ be an sts, and $A \in L^X$, the τ-smooth closure of A, denoted by $\overline{A}$, is defined by*

$$\overline{A} = A, \quad \textit{if } \Im_\tau(A) = 1_L,$$
$$\textit{and } \overline{A} = \bigcap \left\{F : F \in L^X, F \supseteq A, \Im_\tau(F) > \Im_\tau(A)\right\}, \quad \textit{if } \Im_\tau(A) \neq 1_L.$$

Definition 2.7. *A map $f : X \to Y$ is called L-preserving (resp. strictly L-preserving) with respect to the L-topologies $\tau_1^{[a]}$ and $\tau_2^{[a]}$, for each $a \in L$ respectively, iff for every $A, B \in L^Y$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$, we have*

$$\tau_2(A) \geq \tau_2(B) \Rightarrow \tau_1(f^{-1}(A)) \geq \tau_1(f^{-1}(B))$$
$$\text{(resp. } \tau_2(A) > \tau_2(B) \Rightarrow \tau_1(f^{-1}(A))) > \tau_1(f^{-1}(B)).$$

Let $f : X \to Y$ be a strictly L-preserving and continuous map with respect to the L-topologies $\tau_1^{[a]}$ and $\tau_2^{[a]}$, respectively, then for every $A \in L^Y$ with $a \notin \alpha(\tau(A)), f^{-1}(\overline{A}) \supseteq \overline{f^{-1}(A)}$.

3.Relationship between the different Regularity and normality notions On L-fts

Definition 3.1. *An L-topology space $(X, \tau^{[a]})$ for each $a \in L$ is called*

(a) strong s_1 regular (resp. strong S_2 regular) space iff for each $C \in L^X$, satisfying $\Im_\tau(C) > 0$, and each $x \in X$ satisfying $x \notin suppC$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $x \in suppA$ (resp. $x \in supp(A \setminus \overline{B}), \tau(A) \geq A(x), C \subseteq B, \tau(B) \geq \Im_\tau(C)$ and $\overline{A} \cap \overline{B} = \underline{0}$ (resp. $\overline{A} \subseteq (\overline{B})^c$),

(b) s_1 regular (resp. S_2 regular) space iff for each $C \in L^X$, satisfying $\Im_\tau(C) > 0$, and each $x \in X$ satisfying $x \notin suppC$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $x \in suppA$ (resp. $x \in supp(A \setminus B), \tau(A) \geq A(x), C \subseteq B, \tau(B) \geq \Im_\tau(C)$ and $A \cap B = \underline{0}$ (resp. $A \subseteq (B)^c$),

(c) weak s_1 regular (resp. weak S_2 regular) space iff for each $C \in L^X$, satisfying $\Im_\tau(C) > 0$, and each $x \in X$ satisfying $x \notin suppC$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $x \in suppA \setminus suppB^\circ$ (resp. $x \in supp(A \setminus B^\circ), \tau(A) \geq A(x), C \subseteq B, \tau(B) \geq \Im_\tau(C)$ and $A^\circ \cap B^\circ = \underline{0}$ (resp. $A^\circ \subseteq (B^\circ)^c$).

Definition 3.2. *An L-topology space $(X, \tau^{[a]})$ for each $a \in L$ is called*

(a) strong S_1 normal (resp. strong S_2 normal) space iff for each $C, D \in L^X$ such that $C \subseteq (D^c)$ (resp. $C \cap D = \underline{0}), \Im_\tau(C) > 0$ and $\Im_\tau(D) > 0$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $C \subseteq A, \tau(A) \geq \Im_\tau(C), D \subseteq B, \tau(B) \geq \Im_\tau(D)$ and $\overline{A} \cap \overline{B} = \underline{0}$ (resp. $\overline{A} \subseteq (\overline{B})^c$),

(b) S_1 normal (resp. S_2 normal) space iff for each $C, D \in L^X$ such that $C \subseteq (D^c)$ (resp. $C \cap D = \underline{0}), \Im_\tau(C) > 0$ and $\Im_\tau(D) > 0$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $C \subseteq A, \tau(A) \geq \Im_\tau(C), D \subseteq B, \tau(B) \geq \Im_\tau(D)$ and $A \cap B = \overline{0}$ (resp. $A \subseteq (B)^c$),

(c) weak S_1 normal (resp. weak S_2 normal) space iff for each $C, D \in L^X$ such that $C \subseteq (D^c)$ (resp. $C \cap D = \underline{0}), \Im_\tau(C) > 0$ and $\Im_\tau(D) > 0$, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $C \subseteq A, \tau(A) \geq \Im_\tau(C), D \subseteq B, \tau(B) \geq \Im_\tau(D)$ and $A^\circ \cap B^\circ = \underline{0}$ (resp. $A^\circ \subseteq (B^\circ)^c$).

Remark 3.1. Definitions 3.1 and 3.2 also satisfy for each $a \in P(L)$.

Lemma 3.1. *Let $(X, \tau^{[a]})$ be an L-topology space for each $a \in L, A, B \in L^X$ and $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$. Then the following properties hold:*
(i) $suppA \setminus suppB \subseteq supp(A \setminus B)$,
(ii) $suppA \setminus supp\overline{B} \subseteq suppA \setminus suppB \subseteq suppA \setminus suppB^{\circ}$,
(iii) $A \setminus \overline{B} \subseteq A \setminus B \subseteq A \setminus B^{\circ}$,
(iv) $A \cap B = \underline{0} \Rightarrow A \subseteq B^c$.

Proof. (i) Consider $x \in suppA \setminus suppB$. Then we obtain $A(x) > 0$ and $B(x) = 0$. Hence, $min(A(x), 1 - B(x)) = A(x) > 0$, i.e., $x \in supp(A \setminus B)$. The reverse inclusion in (i) is not true as can be seen from the following counterexample. Let $X = \{x_1, x_2\}, A(x_1) = 0.5, B(x_1) = 0.3$. Then we have $x_1 \in supp(A \setminus B)$ and $x_1 \notin suppA \setminus suppB$.
(ii) and (iii) easily follow from $B^{\circ} \subseteq B \subseteq \overline{B}$.
(iv) See [3].

Remark 3.2. The Lemma 3.1 also satisfies for each $a \in P(L)$.

Proposition 3.1. *Let $(X, \tau^{[a]})$ be an L-topology space for each $a \in L$. Then the relationships as shown in Fig. 1 hold.*

Proof. All the implications in Fig. 1 are straightforward consequences of Lemma 3.1 As an example we prove that strong S_1 normal implies strong S_2 normal. Suppose that the space $(X, \tau^{[a]})$ is strong S_1 normal, so there exist $C, D \in L^X$ such that $C \cap D = \underline{0}, \Im_{\tau}(C) > 0$ and $\Im_{\tau}(D) > 0$. From Lemma 3.1 (iv) it follows that $C \subseteq D^C$. Since $(X, \tau^{[a]})$ is strong S_1 normal, there exist $A, B \in L^X$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $C \subseteq A$, $\tau(A) \geq \Im_{\tau}(C), D \subseteq B$, $\tau(B) \geq \Im_{\tau}(D)$ and $\overline{A} \cap \overline{B} = \underline{0}$. from Lemma 3.1 $\overline{A} \subset (\overline{B})^C$, hence $(X, \tau^{[a]})$ is strong S_2 normal.

$$\begin{array}{ccccc}
\text{strong } S_1 \text{ regular} & \Rightarrow & S_1 \text{ regular} & \Rightarrow & \text{weak } S_1 \text{ regular} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{strong } ST_2 \text{ regular} & \Rightarrow & ST_2 \text{ regular} & \Rightarrow & \text{weak } ST_2 \text{ regular} \\
\text{strong } S_1 \text{ normal} & \Rightarrow & S_1 \text{ normal} & \Rightarrow & \text{weak } S_1 \text{ normal} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\text{strong } S_2 \text{ normal} & \Rightarrow & S_2 \text{ normal} & \Rightarrow & \text{weak } S_2 \text{ normal}
\end{array}$$

Fig. 1 Relationship between the different regularity and normality notions.

Proposition 3.2. *The S_i (i = 1, 2) regularity (resp. normality) property is a topological property. when $f : (X, \tau_1) \to (Y, \tau_2)$ be an smooth homeomorphism or $f : (X, {\tau_1}_{[a]}) \to (Y, {\tau_2}_{[a]})$ be an homeomorphism for each $a \in M(L)$ or $f : (X, \tau_1^{[a]}) \to (Y, \tau_2^{[a]})$ be an homeomorphism for each ($a \in L$ or $a \in P(L)$).*

Proof. As an example we give the proof for S_2 normality when $f : X \to Y$ be a homeomorphism from S_2 normal space $(X, \tau_1^{[a]})$ onto a space $(Y, \tau_2^{[a]})$ for each $a \in P(L)$. Let $C, D \in L^Y$ such that $C \cap D = \underline{0}, \Im_{\tau_2}(C) > 0$ and $\Im_{\tau_2}(D) > 0$. Since f is bijective and continuous, from $C' \in \tau_2^{[a]}$ we have $f^{-1}(C') \in \tau_1^{[a]}$. From here, $a \notin \alpha(\tau_2(C'))$ then $a \notin \alpha(\tau_1(f^{-1}(C')))$. Hence $\alpha(\tau_1(f^{-1}(C'))) \subseteq \alpha(\tau_2(C'))\tau_1(f^{-1}(C')) \geq \tau_2(C')$, so $\tau_2(C') \leq \tau_1(f^{-1}(C'))$. it follows that , $\tau_1((f^{-1}(C))') \geq \tau_2(C') > 0$.

Now we obtain that $\Im_{\tau_1}(f^{-1}(C)) \geq \Im_{\tau_2}(C) > 0$. Similarly, $\Im_{\tau_1}(f^{-1}(D)) \geq \Im_{\tau_2}(D) > 0$. we know that $f^{-1}(C) \cap f^{-1}(D) = f^{-1}(C \cap D) = f^{-1}(\underline{0}) = \underline{0}$. Since $(X, \tau_{1_{[a]}})$ is S_2 normal, there exist $A, B \in L^X$ with $a \notin \alpha(\tau_1(A)), a \notin \alpha(\tau_1(B))$ such that $f^{-1}(C) \subseteq A, \tau_1(A) \geq \Im_{\tau_1}(f^{-1}(C)), f^{-1}(D) \subseteq B, \tau_1(B) \geq \Im_{\tau_1}(f^{-1}(D))$ and $A \subseteq B^c$. Since f is L-open and L-closed, it follows that$\tau_2(f(A)) \geq \tau_1(A), \tau_2(f(B)) \geq \tau_1(B), \Im_{\tau_2}(C) \geq \Im_{\tau_1}(f^{-1}(C))$ and $\Im_{\tau_2}(D) \geq \Im_{\tau_1}(f^{-1}(D))$, and hence, $\tau_2(f(A)) \geq \Im_{\tau_1}(f^{-1}(C)) = \Im_{\tau_2}(C), \tau_2(f(B)) \geq \Im_{\tau_1}(f^{-1}(D)) = \Im_{\tau_2}(D), C \subseteq f(A), D \subseteq f(B)$ and $f(A) \subseteq f(B^c) = (f(B))^c$. So $(Y, \tau_2^{[a]})$ is S_2 normal.

Proposition 3.3. *Let $f : X \to Y$ be an injective, L-closed, L-continuous map with respect to the L-topologies $\tau_1^{[a]}$ and $\tau_2^{[a]}$ respectively for each $a \in L$. If $(Y, \tau_2^{[a]})$ is S_i (i = 1, 2) regular (resp. normality); then so is $(X, \tau_1^{[a]})$.*

Proof. As an example we give the proof for S_1 regularity. Let $C \in L^X$, satisfy $\Im_{\tau_1}(C) > 0$ and let $x \in X$ be such that $x \notin suppC$. Since f is injective and L-closed we have $f(x) \notin suppf(C)$ and $\Im_{\tau_2}(f(C)) \geq \Im_{\tau_1}(C) > 0$. Since $(Y, \tau_2^{[a]})$ is S_1 regular, there exist $A, B \in L^Y$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $f(x) \in suppA, \tau_2(A) \geq A(f(x)), f(C) \subseteq B, \tau_2(B) \geq \Im_{\tau_2}(f(C))$ and $A \cap B = \underline{0}$. Since f is injective and L-continuous, if $A \in \tau_2^{[a]}$ then $f^{-1}(A) \in \tau_1^{[a]}$. Hence when $a \notin \alpha\tau_2(A)$ then $a \notin \alpha(\tau_1(f^{-1}(A)))$. Thus $\tau_1(f^{-1}(A)) \geq \tau_2(A) \geq A(f(x)) = f^{-1}(A)(x)$. Similarly, $\tau_1(f^{-1}(B)) \geq \tau_2(B) \geq \Im_{\tau_1}(C)$. we know that $C \subseteq (f^{-1}(B)), f^{-1}(A)(x) = A(f(x)) > 0$, i.e., $x \in suppf^{-1}(A)$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\underline{0}) = \underline{0}$. Hence $(X, \tau_1^{[a]})$ is S_1 regular.

Proposition 3.4. *Let $f : X \to Y$ be a strictly L-preserving, injective, L-closed and L-continuous map with respect to the L-topologies $\tau_1^{[a]}$ and $\tau_2^{[a]}$ respectively for each $a \in L$. If $(Y, \tau_2^{[a]})$ is strong S_i (i = 1, 2) regular (resp. normal) ; then so is $(X, \tau_1^{[a]})$.*

Proof. As an example we proof the strong S_2 regularity. Let $C \in L^X$, satisfying $\Im_{\tau_1}(C) > 0$ and let $x \in X$ such that $x \notin suppC$. Since f is injective and L-closed we have $f(x) \notin suppf(C)$ and $\Im_{\tau_2}(f(C)) \geq \Im_{\tau_1}(C) > 0$. Since

$(Y, \tau_2^{[a]})$ is S_2 regular, there exist $A, B \in L^Y$ with $a \notin \alpha(\tau(A)), a \notin \alpha(\tau(B))$ such that $f(x) \in supp(A \setminus \overline{B}), \tau_2(A) \geq A(f(x)), f(C) \subseteq B, \tau_2(B) \geq \Im_{\tau_2}(f(C))$ and $\overline{A} \subseteq (\overline{B})^c$. As f is injective, L-continuous and strictly L-preserving it follows that $\tau_1(f^{-1}(A)) \geq \tau_2(A) \geq A(f(x)) = f^{-1}(A)(x), \tau_1(f^{-1}(B)) \geq \Im_{\tau_1}(C), C \subseteq (f^{-1}(B)), [f^{-1}(A) \setminus \overline{f^{-1}(B)}](x) = [f^{-1}(A) \cap (\overline{f^{-1}(B)})^c](x) \geq (A) \cap f^{-1}(\overline{B})^c](x) = f^{-1}(A \cap (\overline{B})^c)(x) = f^{-1}(A \setminus \overline{B})(x) = (A \setminus \overline{B}) f(x) > 0$, i.e., $x \in supp(f^{-1}(A) \setminus \overline{f^{-1}(B)})$ and $\overline{f^{-1}(A)} \subseteq f^{-1}(\overline{A}) \subseteq f^{-1}(\overline{B})^c \subseteq (\overline{f^{-1}(B)})^c$, and hence $(X, \tau_1^{[a]})$ is strong S_2 regular.

Remark 3.3. All the Proposition 3.1, 3.2, 3.3 and 3.4 also satisfy for each $a \in P(L)$.

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