ON NONSTATIONARY PERIODICALLY CORRELETED PROCESSES IN COMPLETE CORRELATED ACTIONS

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Abstract. The aim of the paper is to construct the general framework for the study of periodically correlated processes in the general context of a complete correlated action and to find relations with some attached stationary processes which can help in the prediction of a given periodically correlated process. Some used tools are extended to the T-variate case of some appropriate correlated actions and some specific results are obtained in the view of the best estimation.

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1. Introduction

Starting with the results of Gladyshev [3] , the study of periodically correlated processes is a very atractive one, either for mathematicians, who analysed various concrete or abstract casses (univariate, multivariate), or for engineers due to the applications in signal processing, especially in communications systems.

In this paper, the general framework of the study for periodically correlated processes in the general context of a complete correlated action is constructed and some relations with the attached stationary processes which can help in the prediction of a given periodically correlated process are obtained. To do this, a briefly recalling of the fundamental tools introduced in [11]-[14] for the study of stationary processes in complete correlated actions is needed. Also, some used tools must be extended to the T-variate case of some appropriate correlated actions. Then an extension of some known results to the context of a complete correlated action is given and some specific results are analysed.

2. Preliminaries

Let $\mathcal{E}$ be a separable Hilbert space and, as usually by $\mathcal{L}(\mathcal{E})$ will be denoted the C^*-algebra of all linear bounded operators on $\mathcal{E}$. Let $\mathcal{H}$ be a right $\mathcal{L}(\mathcal{E})$-module. An *action* of $\mathcal{L}(\mathcal{E})$ on $\mathcal{H}$ is a map from $\mathcal{L}(\mathcal{E}) \times \mathcal{H}$ into $\mathcal{H}$ given by $(A, h) \longrightarrow Ah$, where $Ah := hA$.

A *correlation of the action* of $\mathcal{L}(\mathcal{E})$ on $\mathcal{H}$ is a map Γ from $\mathcal{H} \times \mathcal{H}$ into $\mathcal{L}(\mathcal{E})$ given by

$$(f, g) \longrightarrow \Gamma[f, g] \tag{2.1}$$

with the properties:

(i) $\Gamma[h, h] \geq 0$, $\Gamma[h, h] = 0$ implies $h = 0$

(ii) $\Gamma[g, h] = \Gamma[h, g]^*$

(iii) $\Gamma[\Sigma_i A_i h_i, \Sigma_j B_j g_j] = \Sigma_{i,j} A_i^* \Gamma[h_i, g_j] B_j$

for finite summs of actions of $\mathcal{L}(\mathcal{E})$ on $\mathcal{H}$.

The triplet $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$ with the previous properties was called [12] the *correlated action* of $\mathcal{L}(\mathcal{E})$ onto $\mathcal{H}$. The Hilbert space $\mathcal{E}$ is the *parameter space* , and the right $\mathcal{L}(\mathcal{E})$-module $\mathcal{H}$ is the *state space* .

To each correlated action $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$, another Hilbert space $\mathcal{K}$ (the *measuring space*) is attached (see [12]) by the positively definite Aronsjain reproducing kernel

$$\langle \gamma_{\lambda_1}, \gamma_{\lambda_2} \rangle = \Big(\Gamma[h_2, h_1] a_1, a_2 \Big)_{\mathcal{E}} \tag{2.2}$$

where $\lambda_i = (a_i, h_i) \in \mathcal{E} \times \mathcal{H}$.

By Theorem 9 from [13], up to a unitary equivalence, there exists a unique algebraic imbedding of $\mathcal{H}$ into $\mathcal{L}(\mathcal{E}, \mathcal{K})$, $h \longrightarrow V_h$ given by

$$V_h a = \gamma_{(a,h)} \tag{2.3}$$

and

$$\Gamma[h_1, h_2] = V_{h_1}^* V_{h_2}. \tag{2.4}$$

The correlated action $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$ is a *complete correlated action* , if the map $h \longrightarrow V_h$ of $\mathcal{H}$ into $\mathcal{L}(\mathcal{E}, \mathcal{K})$ is surjective.

It is easy to see that $\mathcal{L}(\mathcal{E}, \mathcal{K})$ is an $\mathcal{L}(\mathcal{E})$ -module. Considering the action of $\mathcal{L}(\mathcal{E})$ on $\mathcal{L}(\mathcal{E}, \mathcal{K})$ given by $AV = VA$, where VA is the usual operator composing, it follows that $\{\mathcal{E}, \mathcal{L}(\mathcal{E}, \mathcal{K}), \Gamma\}$, where Γ is given by (2.4), is a correlated action. This is called the *operator model.* Due to the imbedding $h \longrightarrow V_h$ given by (2.3), each abstract correlated action $\{\mathcal{E}, \mathcal{H}, \Gamma\}$ can be imbedded into the operator model, and in the complete correlated case, onto the operator model.

A *stationary process* is a sequence $\{f_t\}_{t \in G}$ in $\mathcal{H}$, where G is a locally compact group or hypergroup, such that $\Gamma[f_t, f_s]$ depends only on the difference $s - t$ and not on s and t separately. The function $\Gamma : G \longrightarrow \mathcal{L}(\mathcal{E})$ given by

$$\Gamma(t) = \Gamma[f_0, f_t] \tag{2.5}$$

is the *correlation function* of the process $\{f_t\}_{t \in G}$

For operator valued stationary processes in complete correlated actions, where G is $\mathbb{Z}$ or $\mathbb{R}$, a study was done (e.g. [12]-[14] and [16]-[18]) and the prediction problems which arisen in these cases was solved, including the Wiener filter for prediction and an estimation of the prediction error operator.

3.Periodically correlated processes.

In the context of a complete correlated action $\{\mathcal{E}, \mathcal{H}, \Gamma\}$, a *periodically* Γ-*correlated process* is a sequence $\{f_t\}_{t \in G}$ in $\mathcal{H}$ with the property that there exists a positive T such that for any t, s in the locally compact group G

$$\Gamma[f_{t+T}, f_{s+T}] = \Gamma[f_t, f_s]. \tag{3.1}$$

The smallest T which verifies (3.1) is called the *period* of the periodically Γ-correlated process $\{f_t\}$.

Let us remark that the *correlation function* of a periodically Γ-correlated process $\Gamma : G \times G \to \mathcal{L}(\mathcal{E})$ has the form

$$\Gamma(t, s) = \Gamma[f_t, f_s] \tag{3.2}$$

and is a periodic function, with the same period T.

Also we can see that the *matrix of the correlations* associated to a discrete Γ-stationary process $\{f_n\}_{n \in \mathbb{Z}}$ given by $(\Gamma[f_n, f_m])_{n,m \in \mathbb{Z}}$ has the form

$$\begin{bmatrix}
A_{11} & A_{12} & \dots & A_{1T} & A_{1,T+1} & \dots & A_{1,2T} & \\
A_{21} & A_{22} & \dots & A_{2T} & \vdots & & \vdots & \\
 & & & & & & & \dots \\
\vdots & & & \vdots & \vdots & & \vdots & \\
A_{T1} & A_{T2} & \dots & A_{TT} & A_{T,T+1} & \dots & A_{T,2T} & \\
A^*_{1,T+1} & \dots & & A^*_{T,T+1} & A_{11} & \dots & A_{1T} & \\
\vdots & & & \vdots & \vdots & & \vdots & \\
 & & & & & & & \dots \\
\vdots & & & \vdots & \vdots & & \vdots & \\
A^*_{1,2T} & \dots & & A^*_{T,2T} & A_{T_1} & \dots & A_{TT} & \\
 & \vdots & & & & \vdots & & \\
 & \vdots & & & & \vdots & & \ddots
\end{bmatrix}$$

Therefore the correlation matrix of a periodic stationary process $\{f_n\}$ of the period T can be seen as

$$\begin{bmatrix}
A & B & C & D & \dots \\
B^* & A & B & C & \ddots \\
C^* & B^* & A & B & \ddots \\
D^* & C^* & B^* & A & \ddots \\
\vdots & & \ddots & \ddots & \ddots
\end{bmatrix} \tag{3.3}$$

where $A, B, C, \dots$ are positive T by T bloc matrices with the elements from $\mathcal{L}(\mathcal{E})$. In the case of a stationary process, $A, B, C, \dots$ are simply operators from $\mathcal{L}(\mathcal{E})$.

As in the stationary case, to a periodically Γ-correlated process the prediction spaces can be attached. So, the *past and present* of $\{f_t\}$ will be the submodule of $\mathcal{H}$ given by

$$\mathcal{H}_t^f = \Big\{\sum_k A_k f_k; \quad A_k \in \mathcal{L}(\mathcal{E}); \quad \mathrm{k} \leq \mathrm{t}\Big\} \tag{3.4}$$

the *remote past*

$$\mathcal{H}^f_{-\infty} = \bigcap_n \mathcal{H}^f_n \tag{3.5}$$

and the *space generated* by the process

$$\mathcal{H}^f_\infty = \Big\{ \sum_k A_k f_k; \quad A_k \in \mathcal{L}(\mathcal{E}) \Big\}. \tag{3.6}$$

Also, due to the imbedding $h \to V_h$ of $\mathcal{H}$ into $\mathcal{L}(\mathcal{E}, \mathcal{K})$, the past and present can be seen as

$$\mathcal{K}^f_t = \bigvee_{k<t} V_{f_k}\mathcal{E} \quad \text{or} \quad \mathcal{K}^f_t = \bigvee_{h \in \mathcal{H}^f_t} \mathcal{V}_h \mathcal{E}, \tag{3.4$'$}$$

the remote past

$$\mathcal{K}^f_{-\infty} = \bigcap_n \mathcal{K}^f_n \tag{3.5$'$}$$

and the space generated by the process

$$\mathcal{K}^f_\infty = \bigvee_{-\infty}^{\infty} V_{f_t}\mathcal{E}. \tag{3.6$'$}$$

In the Γ-stationary case, for each process $\{f_t\}$ there exists an *attached shift* (group of unitary operators in the continuous case, and a unitary operator in the discrete case, the relation between the two casses is given by the cogenerator). The attached shift play an important role in in prediction problems. In the periodically Γ-correlated case, to each process a unitary group of operators of T-shift type can be obtained [19] putting $UV_{f_t} = V_{f_{t+T}}$.

Due to the fact that the T-shift act on $\{f_t\}$ with a lag T, it is not possible to use it directely for prediction purposes, like in the stationary case, but, as we'll see, there are strong connections with the shifts attached to the associated stationary processes.

To make prediction on a process it means to obtain the best information on some step, knowing the past and the present of the process. Usually, the best estimator, or the best linear prediction is done by the linear least squares method, using the orthogonal projection on the past and present space. In the context of a complete correlated action, the state space is only a right $\mathcal{L}(\mathcal{E})$-module and we have not a clossness of submodules. Similarly like in the stationary case [14], in the periodically correlated case we will construct an appropriate 'Γ-orthogonal' projection and some specific properties will be studied, using operator theory methods. First of all, due the periodicity, even in the scalar case, the best estimation can not be obtained directly, but with the analysis of some associated stationary processes. Similar things will arise in the complete correlated case and specific properties will be analysed.

In the following we will consider the case when $\{f_t\}_{t\in G} \subset \mathcal{H}$ where G is $\mathbb{Z}$ (the discrete case), or G is $\mathbb{R}$ (the continuous case), but the study can be abstractly done on a locally compact group or hypergroup G. In this paper the discrete case will be esspecially analysed, and in a subsequent paper the continuous case will be studied.

Let $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$ be a complete correlated action, and $\mathcal{H}^T = \mathcal{H} \times \mathcal{H} \times \cdots \times \mathcal{H}$ be the cartesian product of T copies of the right $\mathcal{L}(\mathcal{E})$-module $\mathcal{H}$. An element X of $\mathcal{H}^T$ will be seen as a column vector, taking the transpose $(h_1, \ldots, h_T)^t$ of the vector $(h_1, \ldots, h_T)$. On $\mathcal{H}^T$ it is possible to have the action of $\mathcal{L}(\mathcal{E})$ on the components, with the same operator $A \in \mathcal{L}(\mathcal{E})$, or on each component with a different $A_i \in \mathcal{L}(\mathcal{E})$. Also we can consider various correlations on $\mathcal{H}^T$. From the prediction of periodically correlated processes poin of view, two equivalent correlations on $\mathcal{H}^T$ will be considered as follows:

$$\Gamma_1[X, Y] = \sum_{k=0}^{T-1} \Gamma[x_k, y_k], \tag{3.7}$$

where $X = \{x_k\}, \quad Y = \{y_k\}, \quad k \in \{0, 1, \ldots, T-1\}$ and

$$\Gamma_T[X, Y] = \Big(\Gamma[x_i, y_j]\Big)_{i,j\in\{0,1,\ldots,T-1\}}. \tag{3.8}$$

Let us remark that $\Gamma_1 : \mathcal{H}^T \times \mathcal{H}^T \to \mathcal{L}(\mathcal{E})$ and $\Gamma_T : \mathcal{H}^T \times \mathcal{H}^T \to \mathcal{L}(\mathcal{E})^{\mathcal{T}\times\mathcal{T}}$, or in the set of $T \times T$ matrix with elements from $\mathcal{L}(\mathcal{E})$.

So, starting with the correlated action $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$ of $\mathcal{L}(\mathcal{E})$ on $\mathcal{H}$, we obtain two associated correlated actions of $\mathcal{L}(\mathcal{E})$ on $\mathcal{H}^T$, namely $\{\mathcal{E}, \mathcal{H}^T, \Gamma_1\}$ and $\{\mathcal{E}, \mathcal{H}^T, \Gamma_T\}$. In the following, to avoid the confusion, various correlations will be prefixed with Γ, Γ_1 and Γ_T.

To a periodically Γ-correlated process $\{f_n\}_{n\in\mathbb{Z}}$ from $\mathcal{H}$ we can attach at least two types of stationary processes in $\mathcal{H}^T$ as follows:

1) taking sequences of consecutive T terms starting with f_n, namely the column vector

$$X_n = \Big(f_n, f_{n+1}, \ldots, f_{n+T-1}\Big)^t, \tag{3.9}$$

or

2) taking consecutive blocks of length T

$$X_n^T = \Big(f_{nT}, f_{nT+1}, \ldots, f_{nT+T-1}\Big)^t. \tag{3.10}$$

It is easy to see that $\{X_n\}$ and $\left\{X_n^T\right\}$ are respectively Γ_1 and Γ_T stationary processes in $\mathcal{H}^T$. From prediction point of view and the study of the periodically Γ-correlated process $\{f_n\}_{n\in\mathbb{Z}}$ from $\mathcal{H}$, the Γ_1-correlation of $\{X_n\}$ and Γ_T-correlation of $\left\{X_n^T\right\}$ are equivalent, as can be seen from the following Proposition.

Proposition 3.1 Let $\{f_n\}_{n\in\mathbb{Z}}$ from $\mathcal{H}$, an integer $T \geq 2$ and $\{X_n\}$, $\left\{X_n^T\right\}$ defined by (3.9) and (3.10). The following are equivalent:

(i) $\{f_n\}$ is periodically Γ-correlated in $\mathcal{H}$, with the period T.

(ii) $\{X_n\}$ is stationary Γ_1-correlated in $\mathcal{H}^T$.

(iii) $\left\{X_n^T\right\}$ is stationary Γ_T-correlated in $\mathcal{H}^T$.

Proof. $(i) \Rightarrow (ii)$. Having $\{f_n\}$ periodically Γ-correlated , i.e., $\Gamma[f_n, f_m] = \Gamma[f_{n+T}, f_{m+T}]$, it follows that

$$\Gamma_1[X_n, X_m] = \sum_{k=0}^{T-1} \Gamma[f_{n+k}, f_{m+k}] = \Gamma[f_n, f_m] + \sum_{k=1}^{T-1} \Gamma[f_{n+k}, f_{m+k}] =$$

$$= \Gamma[f_{n+T}, f_{m+T}] + \sum_{k=1}^{T-1} \Gamma[f_{n+k}, f_{m+k}] = \sum_{k=1}^{T} \Gamma[f_{n+k}, f_{m+k}] =$$

$$= \sum_{j=0}^{T-1} \Gamma[f_{(n+1)+j}, f_{(m+1)+j}] = \Gamma_1[X_{n+1}, X_{m+1}] \ .$$

Therefore $\{X_n\}_{n\in\mathbb{Z}}$ is stationary Γ_1 -correlated in $\mathcal{H}^T$.

Conversely $(ii) \Rightarrow (i)$. The process $\{X_n\}$ being stationary Γ_1 -correlated in $\mathcal{H}^T$ we have succesively:

$$\Gamma_1[X_{n+1}, X_{m+1}] = \Gamma_1[f_n, f_m]$$

$$\sum_{k=0}^{T-1} \Gamma[f_{n+1+k}, f_{m+1+k}] = \sum_{k=0}^{T-1} \Gamma[f_{n+k}, f_{m+k}]$$

$$\sum_{j=1}^{T} \Gamma[f_{n+j}, f_{m+j}] = \sum_{k=0}^{T-1} \Gamma[f_{n+k}, f_{m+k}]$$

$$\sum_{j=1}^{T-1}\Gamma[f_{n+j},f_{m+j}]+\Gamma[f_{n+T},f_{m+T}]=\Gamma[f_n,f_m]+\sum_{k=1}^{T-1}\Gamma[f_{n+k},f_{m+k}].$$

It follows that $\Gamma[f_{n+T},f_{m+T}]=\Gamma[f_n,f_m]$, i.e., $\{f_n\}_{n\in\mathbf{Z}}$ is periodically Γ-correlated in $\mathcal{H}$.

$(i)\Rightarrow(iii)$. Taking account that $\{f_n\}$ from $\mathcal{H}$ is periodically Γ-correlated with the period T, we have

$$\Gamma_T[X_n^T,X_m^T]=\Big(\Gamma[f_{nT+i},f_{mT+j}]\Big)_{i,j\in\{0,1,\ldots,T-1\}}=\Big(\Gamma[f_{nT+i+T},f_{mT+j+T}]\Big)_{i,j}=$$

$$=\Big(\Gamma[f_{(n+1)T+i},f_{(m+1)T+j}]\Big)_{i,j}=\Gamma_T[X_{n+1}^T,X_{m+1}^T]$$

and $\{X_n^T\}$ is stationary Γ_T-correlated in $\mathcal{H}^T$.

$(iii)\Rightarrow(i)$. If $\{X_n^T\}$ is stationary Γ_T-correlated in $\mathcal{H}^T$, then for each $n,m\in\mathbb{Z}$

$$\Gamma_T[X_n^T,X_m^T]=\Gamma_T[X_{n+1}^T,X_{m+1}^T],$$

i.e., the matrix equality

$$\Big(\Gamma[f_{nT+i},f_{mT+j}]\Big)_{0\le i,j\le T-1}=\Big(\Gamma[f_{(n+1)T+i},f_{(m+1)T+j}]\Big)_{0\le i,j\le T-1}.$$

It follows that for each $n,m\in\mathbb{Z}$ and $0\le i,j\le T-1$ we have

$$\Gamma[f_{nT+i},f_{mT+j}]=\Gamma[f_{nT+i+T},f_{mT+j+T}]. \tag{3.11}$$

Taking first $n=m=0$ in (3.11) obtain that for $0\le i,j\le T-1$

$$\Gamma[f_i,f_j]=\Gamma[f_{i+T},f_{j+T}].$$

Then for various other combinations of n and m, denotting $nT+i=p\in\mathbb{Z}$ and $mT+j=q\in\mathbb{Z}$, it follows that for each $p,q\in\mathbb{Z}$ we have

$$\Gamma[f_i,f_j]=\Gamma[f_{i+T},f_{j+T}]$$

i.e., the process $\{f_n\}\subset\mathcal{H}$ is periodically Γ-correlated.

It is known [12] that the right $\mathcal{L}(\mathcal{E})$-module $\mathcal{H}$ can be uniquely (up to a unitary equivalence) imbedded into the right $\mathcal{L}(\mathcal{E})$-module $\mathcal{L}(\mathcal{E},\mathcal{K})$, where $\mathcal{K}$

is the measuring space attached to the correlated action $\{\mathcal{E}, \mathcal{H}, \Gamma\}$. Therefore the study of a process in $\mathcal{H}$ can be done in the operator model $\{\mathcal{E}, \mathcal{L}(\mathcal{E}, \mathcal{K}), \Gamma\}$.

For the study of the attached column vectors stationary processes from $\mathcal{H}^T$, the corresponding operator model is needed.

Proposition 3.2. There exists a unique (up to a unitary equivalence) imbedding $X \to W_X$ of $\mathcal{H}^T$ into $\mathcal{L}(\mathcal{E}, \mathcal{K}^T)$ such that

$$\Gamma_1[X, Y] = W_X^* W_Y = \sum_{i=1}^{T} V_{x_i}^* V_{y_i} \tag{3.12}$$

where $X = (x_1, \ldots, x_T)^t$, $Y = (y_1, \ldots, y_T)^t$.

The subset $\{W_X a; X \in \mathcal{H}^T, a \in \mathcal{E}\}$ is dense in $\mathcal{K}^T$.

Proof. Taking account of the imbedding $h \to V_h$ of $\mathcal{H}$ into $\mathcal{L}(\mathcal{E}, \mathcal{K})$ given by (2.3) and (2.4), if we take

$$W_X = (V_{x_1}, V_{x_2}, \ldots, V_{x_T})^t, \tag{3.13}$$

then for $a, b \in \mathcal{E}$ we have $W_X a = (\gamma_{(a,x_1)}, \ldots, \gamma_{(a,x_T)})$ and

$$\Big(\Gamma_1[X, Y]a, b\Big)_{\mathcal{E}} = \Big(\sum_{i=1}^{T} \Gamma[x_i, y_i]a, b\Big)_{\mathcal{E}} = \Big(\sum_{i=1}^{T} V_{x_i}^* V_{y_i} a, b\Big)_{\mathcal{E}}.$$

By the fact that the usual scalar product on $\mathcal{K}^T$ is the sum of scalar products on components, it follows that

$$(W_X^* W_Y a, b)_{\mathcal{E}} = (W_Y a, W_X b)_{\mathcal{K}^T} = \sum_{i=1}^{T} (\gamma_{(a,y_i)}, \gamma_{(b,x_i)})_{\mathcal{K}} = \sum_{i=1}^{T} (V_{y_i} a, V_{x_i} b)_{\mathcal{K}} =$$

$$= \sum_{i=1}^{T} (V_{x_i}^* V_{y_i} a, b)_{\mathcal{E}}$$

and (3.12) is proved. Also,

$$\|W_X a\|_{\mathcal{K}^T}^2 = (W_X a, W_X a)_{\mathcal{K}^T} = \sum_{i=1}^{T} (V_{x_i}^* V_{x_i} a, a)_{\mathcal{E}} \le \sum_{i=1}^{T} \|V_{x_i}\|^2 \cdot \|a\|,$$

and W_X is a linear bounded operator from $\mathcal{E}$ into $\mathcal{K}^T$.

If we consider another imbedding W' of $\mathcal{H}^T$ into $\mathcal{L}(\mathcal{E}, \mathcal{K}^T)$ having the property (3.12), then, if we take $\Phi : \mathcal{K}^T \to \mathcal{K}^T$ given by $\Phi W'_X a = W_X a$, we have

$$\|\Phi W'_X\|^2_{\mathcal{K}^T} = \|W_X a\|^2_{\mathcal{K}^T} = \Big(\sum_{i=1}^{T} V^*_{x_i} V_{x_i} a, a \Big)_{\mathcal{E}} = \|W'_X a\|^2$$

. Then for $a, b \in \mathcal{E}$, $W_X a = (\gamma_{(a,x_1)}, \ldots, \gamma_{(a,x_T)})^t$ and

$$\Big(\Gamma_1[X,Y]a,b\Big)_{\mathcal{E}} = \Big(\sum_{i=1}^{T} \Gamma[x_i,y_i]a,b\Big)_{\mathcal{E}} = \Big(\sum_{i=1}^{T} V^*_{x_i} V_{y_i} a,b\Big)_{\mathcal{E}}.$$

Also, $\|W_X a\|^2_{\mathcal{K}^T} = (W_X a, W_X a)_{\mathcal{K}^T}$, i.e., Φ is a unitary operator on $\mathcal{K}^T$. So, the imbedding $X \to W_X$ of $\mathcal{H}^T$ into $\mathcal{L}(\mathcal{E}, \mathcal{K}^T)$ is unique (up to a unitary equivalence.

For prediction purposes, we are interested to find the best estimation of an element from $\mathcal{H}^T$ with elements from a subset $\mathcal{M} = \mathcal{H}_1 \times \mathcal{H}_2 \times \cdots \times \mathcal{H}_T \subset \mathcal{H}^T$. To do this, we need the following Proposition.

Proposition 3.3. Let $\mathcal{M}$ be a subset of $\mathcal{H}^T$. If we take

$$\mathcal{K}^T_1 = \bigvee_{Z \in \mathcal{M}} V_Z \mathcal{E}, \tag{3.14}$$

then for each $X \in \mathcal{H}^T$ there exists a unique element X' in $\mathcal{H}^T$ such that for each $a \in \mathcal{E}$ we have

$$W_{X'} a \in \mathcal{K}^T_1 \text{ and } W_{X-X'} a \in (\mathcal{K}^T_1)^\perp. \tag{3.15}$$

Moreover,

$$\Gamma_1[X - X', X - X'] = \inf_{Z \in \mathcal{M}} \Gamma_1[X - Z, X - Z] \tag{3.16}$$

where the infimum is taken in the set of all positive operators from $\mathcal{L}(\mathcal{E})$.

Proof. Let $W_{X'} = P_{\mathcal{K}^T_1} W_X$ where $P_{\mathcal{K}^T_1}$ is the orthogonal projection of $\mathcal{K}^T$ on its closed subset $\mathcal{K}^T_1$. For each $a \in \mathcal{E}$ we have $W_{X'} a \in \mathcal{K}^T_1$ and

$$W_{X-X'} a = (\gamma_{(a,x_1 - x'_1}, \ldots, \gamma_{(a,x_T - x'_T})^t = (\gamma_{(a,x_1} - \gamma_{(a,x'_1}, \ldots, \gamma_{(a,x_T} - \gamma_{(a,x'_T})^t$$

$$= W_X a - W_{X'} a = W_X a - P_{\mathcal{K}^T_1} W_X a = (I - P_{\mathcal{K}^T_1}) W_X a \in (\mathcal{K}^T_1)^\perp.$$

If there exists $X"$ with the property (3.15), then for each $a \in \mathcal{E}$ we have $W_X a = W_{X"} a + W_{X-X"} a$. It follows that $W_{X"} a = P_{\mathcal{K}_1^T} W_X a = W_{h'} a$, i.e., $X" = X'$.

Moreover,

$$(\Gamma_1[X-X', X-X']a, a) = (W^*_{X-X'} W_{X-X'} a, a) = \|W_{X-X'} a\|^2 = \|(I-P_{\mathcal{K}_1^T}) W_X a\|^2 =$$

$$= \inf_{K \in \mathcal{K}_1^T} \|W_X a - K\|^2 = \inf_{\sum_1^n W_{X_j} a_j} \|W_X a - \sum_{i=1}^n W_{X_j} a_j\|^2 =$$

$$= \inf_{\sum_1^n W_{X_j} a_j} \|W_X a - W_{\sum_{j=1}^n X_j} a_j\|^2 =$$

$$= \inf(\Gamma_1[X - \sum_{j=1}^n A_j X_j, X - \sum_{j=1}^n A_j X_j]a, a) = \inf_{Z \in \mathcal{M}} (\Gamma_1[X - Z, X - Z]a, a),$$

where for each finite systems $\{a_1, \ldots, a_n\}$ of elements from $\mathcal{E}$ we choose $A_1, \ldots, A_n \in \mathcal{L}(\mathcal{E})$ such that $A_j a = a_j, \quad j = 1, 2, \ldots, n$.

If we denote by $\mathcal{P}_{\mathcal{M}}$ the endomorphism of $\mathcal{H}^T$ defined by $\mathcal{P}_{\mathcal{M}} X = X'$,then we have

$$W_{\mathcal{P}^2_{\mathcal{M}} X} = W_{\mathcal{P}_{\mathcal{M}} X'} = P_{\mathcal{K}_1^T} W_{X'} = P^2_{\mathcal{K}_1^T} W_X = P_{\mathcal{K}_1^T} W_X = W_{\mathcal{P}_{\mathcal{M}} X}$$

and also,

$$\Gamma_1[\mathcal{P}_{\mathcal{M}} X, Y] = W^*_{\mathcal{P}_{\mathcal{M}} X} W_Y = (P_{\mathcal{K}_1^T} W_X)^* W_Y = W^*_X P_{\mathcal{K}_1^T} W_Y = W^*_X W_{\mathcal{P}_{\mathcal{M}} Y} =$$

$$= \Gamma_1[X, \mathcal{P}_{\mathcal{M}} Y]$$

Hence $\mathcal{P}^2_{\mathcal{M}} X = \mathcal{P}_{\mathcal{M}} X$ and $\Gamma_1[\mathcal{P}_{\mathcal{M}} X, Y] = \Gamma_1[X, \mathcal{P}_{\mathcal{M}} Y]$. Therefore we can interpret $\mathcal{P}_{\mathcal{M}}$ as a "orthogonal" projection on $\mathcal{M}$, and this will be called the Γ_1-*orthogonal projection* of $\mathcal{H}^T$ on $\mathcal{M} \subset \mathcal{H}^T$.

As we have seen, to each periodically correlated process $\{f_n\}$ from $\mathcal{H}$ we can attach its T-shift, a unitary operator U_f on $\mathcal{K}^f_\infty$ such that $U_f V_{f_n} = V_{f_{n+T}}$, where $h \to V_h$ is the usual imbedding of $\mathcal{H}$ into $\mathcal{L}(\mathcal{E}, \mathcal{K})$. Then it is easy to see that the unitary operator on $(\mathcal{K}^f_\infty)^T$ defined by

$$U_T(V_{f_1}, V_{f_2}, \ldots, V_{f_T})^t = (U_f V_{f_1}, U_f V_{f_2}, \ldots, U_f V_{f_T})^t \tag{3.17}$$

is the shift operator attached to the statoinary Γ_T-correlated process $\{X_n^T\}$ defined by (3.10). Indeed,

$$U_T W_{X_n^T} = U_T(V_{f_{nT}}, V_{f_{nT+1}}, \dots, V_{f_{nT+T-1}})^t =$$
$$= (U_f V_{f_{nT}}, U_f V_{f_{nT+1}}, \dots, U_f V_{f_{nT+T-1}})^t = (V_{f_{nT+T}}, V_{f_{nT+1+T}}, \dots, V_{f_{nT+T-1+T}})^t =$$
$$= (V_{f_{(n+1)T}}, V_{f_{(n+1)T+1}}, \dots, V_{f_{(n+1)T+T-1}})^t = W_{X_{n+1}^T}.$$

It follows that $W_{X_n^T} = U^n W_{X_0^T}$.

On the other part, let us remark that we can identify $\mathcal{H}$ as the subset $\mathcal{N} = \mathcal{H} \times \{0\} \times \cdots \times \{0\}$ in $\mathcal{H}^T$. From (3.13) it follows that $W_{(h,0,\dots,0)} = (V_h, 0, \dots, 0)^t$ and the corresponding subspace from $\mathcal{K}^T$ for $\mathcal{N}$ will be

$$\mathcal{K}_{\mathcal{N}}^T = \bigvee_{Z \in \mathcal{M}} W_Z \mathcal{E} = \mathcal{K} \times \{0\} \times \cdots \times \{0\} \subset \mathcal{K}^T$$

Considering the stationary Γ_1-correlated process $\{X_n\} \subset \mathcal{H}^T$ given by (3.9), $X_n = \Big(f_n, f_{n+1}, \dots, f_{n+T-1}\Big)^t$, we have

$$P_{\mathcal{K}_{\mathcal{N}}^T} W_{X_n} = P_{\mathcal{K}_{\mathcal{N}}^T} (V_{f_n}, \dots, V_{f_{n+T-1}})^t = V_{f_n}$$

and follows that f_n can be identified with $f_n = \mathcal{P}_{\mathcal{N}} X_n$, i.e. the periodically Γ-correlated process from $\mathcal{H}$ admits a stationary Γ_1-correlated *dilation* $\{X_n\}$ in $\mathcal{H}^T$.

Also, using this type of identification of $\mathcal{H}$ in $\mathcal{H}^T$ it is possible to find informations and prediction facts about the periodically Γ-correlated process $\{f_n\}$ from the study of stationary Γ_1-correlated process $\{X_n\}$. As we will see, there are strong connections among the corresponding spectral distributions of $\{f_n\}$, $\{X_n\}$ and Γ_T-correlated stationary process $\left\{X_n^T\right\}$. So, prediction error operator and the linear Wiener filter for prediction can be found. This will be done in a separate paper, the aim of this paper being to construct the general frame for the study of periodically correlated processes in the context of a complete correlated action.

In the following, some results obtained by Gladyshev [3] for the scalar case will be extended to the case of a complete correlated action $\{\mathcal{E}, \mathcal{H}, \mathit{\Gamma}\}$ and some specific results will be obtained.

Let $\{f_n\}$ be a periodically Γ-correlated process in the right $\mathcal{L}(\mathcal{E})$ -module $\mathcal{H}$ and its imbedding in the operator model $\{V_{f_n}\} \subset \mathcal{L}(\mathcal{E}, \mathcal{K})$ given by (2.3). The Γ-correlation function of the process $\{f_n\}$ given by (3.2) is an $\mathcal{L}(\mathcal{E})$-valued periodic function with the same period $T \geq 2$.

$$\Gamma(m,n) = \Gamma[f_m, f_n] = V_{f_m}^* V_{f_n} \ ; \quad m,n \in \mathbb{Z}$$

Following the notations from [3], let us define the $\mathcal{L}(\mathcal{E})$-valued function $B(n,t)$ on $\mathbb{Z}\times\mathbb{Z}$ by

$$B(n,t)=\Gamma(n+t,n). \tag{3.18}$$

Then $B(n,t)$ is a periodic function in n with the same period T and has a Fourier series representation

$$B(n,t)=\sum_{k=0}^{T-1}B_k(t)\exp(2\pi ikn/T), \tag{3.19}$$

where $B_k(t)$ are the $\mathcal{L}(\mathcal{E})$-valued coefficients for $k=0,1,\dots,T-1$. For convenience $B_k(t)$ can be completed to $\mathbb{Z}$ by the equality $B_k(t)=B_{k+T}(t)$.

In the following the attached function $B(n,t)$ will be called the *covariance function* of the periodically Γ-correlated process $\{f_n\}$.

A simple computation on (3.19) shows that

$$B_l(t)=\frac{1}{T}\sum_{n=0}^{T-1}B(n,t)\exp(-2\pi inl/T). \tag{3.20}$$

As a remark, the correlation function and the covariance function of an arbitrary Γ-correlated process $\{f_n\}$ are positive definite functions. Indeed, for any $A_1,\dots,A_n\in\mathcal{L}(\mathcal{E})$; $t_1,t_2,\dots,t_n\in\mathbb{Z}$ and $a_1,\dots,a_n\in\mathcal{E}$ we have, taking account that for a finite system of elements $a_k\in\mathcal{E}$ there exist a system of operators $S_k\in\mathcal{L}(\mathcal{E})$ such that $a_k=S_ka$

$$\sum_{p,q=1}^{n}(A_p^*\Gamma(t_p,t_q)A_qa_q,a_p)_{\mathcal{E}}=\sum_{p,q=1}^{n}(A_p^*\Gamma[f_{t_p},f_{t_q}]A_qa_q,a_p)=$$

$$=\sum_{p,q=1}^{n}(\Gamma[A_pf_{t_p},A_qf_{t_q}]a_q,a_p)=\sum_{p,q=1}^{n}(\Gamma[A_pf_{t_p},A_qf_{t_q}]S_qa,S_pa)=$$

$$=\Big(\Gamma[\sum_{p=1}^{n}S_pA_pf_{t_p},\sum_{q=1}^{n}S_qA_qf_{t_q}]a,a\Big)_{\mathcal{E}}=(\Gamma[h,h]a,a)=$$

$$=(V_h^*V_ha,a)_{\mathcal{E}}=\|V_ha\|_{\mathcal{K}}^2\geq 0.$$

Or equivalent for the covariance function

$$\sum_{p,q=1}^{n}\Big(A_p^*B(t_q,t_p-t_q)A_qa_q,a_p\Big)_{\mathcal{E}}\geq 0. \tag{3.21}$$

The following theorem is a generalization to the complete correlated action case of the Theorem 1 from [3].

Theorem 3.4 (*Gladyshev*). A function $B(n,t)$ defined by (3.19) will be the covariance function of some periodically Γ-correlated process in $\mathcal{H}$ with the period T, where the correlation function $\Gamma(s,n) = B(n, s-n)$, if and only if the $T \times T$ matrix valued function

$$B(t) = \Big(B_{jk}(t) \Big)_{j,k=0,1,\ldots,T-1} \tag{3.22}$$

is the operator correlation function of some stationary Γ_T-correlated process from $\mathcal{H}^T$, where

$$B_{jk}(t) = B_{k-j}(t) \exp(2\pi ijt/T) \tag{3.23}$$

and the Γ_T-correlation on $\mathcal{H}^T$ is given by (3.8).

Proof. As in [3] we have to show that satisfaction of the inequality

$$\sum_{p,q=1}^{n} A_p^* B(t_q, t_p - t_q) A_q \geq 0 \tag{3.24}$$

in the sense of (3.21), is equivalent to satisfaction of the inequality

$$\sum_{p,q=1}^{n} A_p^* B_{k_p k_q}(t_p - t_q) A_q \geq 0 \tag{3.25}$$

for arbitrary $A_1, \ldots, A_n \in \mathcal{L}(\mathcal{E})$; $t_1, t_2, \ldots, t_n \in \mathbb{Z}$ and $k_1, \ldots, k_n \in \{0, 1, \ldots, T-1\}$.

For the first implication we have

$$\sum_{p,q=1}^{n} A_p^* B_{k_p k_q}(t_p - t_q) A_q = \sum_{p,q=1}^{n} A_p^* B_{k_q - k_p}(t_p - t_q) \exp(2\pi i k_p (t_p - t_q)/T) A_q =$$

$$= \sum_{p,q=1}^{n} A_p^* \frac{1}{T} \sum_{m=0}^{T-1} B(m, t_p - t_q) \exp(-2\pi i m (k_q - k_p)/T) \cdot \exp(2\pi i k_p (t_p - t_q)/T) A_q =$$

$$= \frac{1}{T} \sum_{p,q=1}^{n} \sum_{m=0}^{T-1} A_p^* B(m, t_p - t_q) \exp[-2\pi i (m k_q - m k_p - t_p k_p + t_q k_q)/T] A_q =$$

$$= \frac{1}{T} \sum_{p,q=1}^{n} \sum_{m=0}^{T-1} A_p^* B(m, t_p - t_q) \exp(-2\pi i m k_q)/T) \cdot \exp[2\pi i (m - t_q + t_p) k_p / T] A_q =$$

$$= \frac{1}{T} \sum_{p,q=1}^{n} \sum_{s=0}^{T-1} A_p^* B(s+t_q, t_p - t_q) \exp[-2\pi i(s+t_q)k_q/T] \cdot \exp[2\pi i(s+t_p)k_p/T) A_q =$$

$$= \frac{1}{T} \sum_{p,q=1}^{n} \sum_{s=0}^{T-1} [A_p \exp(-2\pi i(s+t_p)k_p/T)]^* B(s+t_q, t_p - t_q) A_q \exp(-2\pi i(s+t_q)k_q/T) =$$

$$= \frac{1}{T} \sum_{s=0}^{T-1} \sum_{p,q=1}^{n} [A_p \exp(-2\pi i(s+t_p)k_p/T)]^* \Gamma(s+t_p, s+t_q) A_q \exp(-2\pi i(s+t_q)k_q/T) =$$

$$= \frac{1}{T} \sum_{s=0}^{T-1} \Gamma\Big[\sum_{p=1}^{n} \exp(-2\pi i(s+t_p)k_p/T) A_p f_{s+t_p}, \sum_{q=1}^{n} \exp(-2\pi i(s+t_q)k_q/T) A_q f_{s+t_q}\Big] \geq 0.$$

Conversely, if (3.25) is true for any $A_1, \ldots, A_n \in \mathcal{L}(\mathcal{E})$; $t_1, t_2, \ldots, t_n \in \mathbb{Z}$ and $k_1, \ldots, k_n \in \{0, 1, \ldots, T-1\}$, then

$$\sum_{p,q=1}^{n} A_p^* B(t_q, t_p - t_q) A_q = \sum_{p,q=1}^{n} A_p^* \sum_{l=0}^{T-1} B_l(t_p - t_q) \exp(2\pi i l t_q/T) A_q =$$

$$= \sum_{p,q=1}^{n} A_p^* \sum_{j,k=0}^{T-1} B_{k-j}(t_p - t_q) \exp(2\pi i(k-j)t_q/T) A_q =$$

$$= \sum_{p,q=1}^{n} \sum_{j,k=0}^{T-1} A_p^* B_{jk}(t_p - t_q) \exp(2\pi i j(t_p - t_q)/T) \exp(2\pi i(k-j)t_q/T) A_q =$$

$$= \sum_{p,q=1}^{n} \sum_{j,k=0}^{T-1} A_p^* B_{jk}(t_p - t_q) \exp(2\pi i(k t_q - j t_p)/T) A_q =$$

$$= \sum_{p,q=1}^{n} \sum_{j,k=0}^{T-1} [\exp(2\pi i j t_p) A_p]^* B_{jk}(t_p - t_q) [\exp(2\pi i k t_q) A_q] \geq 0,$$

and the theorem is proved.

As a remark, Gladyshev's theorem affirm the existence of some stationary Γ_T-correlated process, not necessary our stationary process $\{X_n^T\}$ introduced by (3.10). But, as we will see, this Γ_T-correlated process contains a lot of informations about the periodically Γ-correlated process $\{f_n\}$, or the attached stationary Γ_1-correlated process $\{X_n\}$ given by (3.9).

Recall that [16], if $\{f_n\}$ is not a Γ-stationary process in $\mathcal{H}$, but there exists an $\mathcal{L}(\mathcal{E})$-valued semispectral measure on bitorus such that

$$\Gamma[f_n, f_m] = \int_{\mathbb{T}^2} \chi^n(t)\overline{\chi^m}(t) dK(t,s),$$

where $\chi^n(t) = e^{-int}$ and $\mathbb{T}$ is the unit torus from the complex plane $\mathbb{C}$, then the process $\{f_n\}$ is Γ-*harmonizable*. Also, (see [16]) any Γ-harmonizable process has a spectrum and this is obtained by the restriction of the $\mathcal{L}(\mathcal{E})$-valued semispectral measure K to the diagonal of the bitorus.

In the following we will see that each discrete periodically Γ-correlated process is Γ-harmonizable.

If $\Gamma(m,n)$ is the correlation function of the periodically Γ-correlated process $\{f_n\}$ in $\mathcal{H}$ given by (3.2) and the covariance function $B(n,t) = \Gamma(n+t,n)$ given by (3.18) with the attached matrix $B(t) = \Big(B_{jk}(t)\Big)$ given by (3.22) and (3.23), then by Theorem 3.4, the attached Γ_T-correlated process is stationary and it follows that there exists a unique $\mathcal{L}(\mathcal{E})^{T\times T}$-valued semispectral measure F_T on $\mathbb{T}$

$$F_T(\cdot) = \Big(F_{jk}(\cdot)\Big)_{j,k\in\{0,1,\ldots,T-1\}} \tag{3.26}$$

such that

$$B(n) = \int_0^{2\pi} e^{int} F_T(dt), \quad n\in\mathbb{Z}. \tag{3.27}$$

From (3.23) $B_{jk}(n) = B_{k-j}(n)\exp(2\pi ijn/T)$ and taking account by (3.27) it follows that for $k\in\{0,1,\ldots,T-1\}$ we have

$$B_k(n) = B_{0k}(n) = \int_0^{2\pi} e^{int} F_{0k}(dt), \quad n\in\mathbb{Z}. \tag{3.28}$$

Extending by periodicity $F_{0k} = F_{0(k+T)}$, it follows that for any $k\in\mathbb{Z}$ we have F_{0k} as an $\mathcal{L}(\mathcal{E})$-valued semispectral measure on the unit torus. Using (3.18) and (3.19) it follows that

$$\Gamma(n,m) = B(m,n-m) = \sum_{j=0}^{T-1} B_j(n-m)\exp(2\pi ijm/T) =$$

$$= \sum_{j=0}^{T-1} \int_0^{2\pi} e^{i(n-m)t} F_{0j}(dt)\cdot\exp(2\pi ijm/T) =$$

$$= \int_0^{2\pi} \sum_{j=0}^{T-1} \exp\Big\{ i[nt - m(t - 2\pi j/T)] \Big\} F_{0j}(dt).$$

If we define the $\mathcal{L}(\mathcal{E})$-valued bimeasure K on bitorus putting for σ and ω in $\mathcal{B}(\mathbb{T})$

$$K(\sigma, \omega) = \sum_{j=-T+1}^{T-1} F_{0j}(\sigma \cap \omega - 2\pi j/T) \tag{3.29}$$

then

$$\Gamma(n, m) = \int_{\mathbb{T}^2} e^{i(nt-ms)} K(dt, ds) \tag{3.30}$$

and it follows that $\{f_n\}$ is Γ-harmonizable.

As a remark, the support of the $\mathcal{L}(\mathcal{E})$-valued bimeasure K attached to a periodically Γ-correlated process with the period T is concentrated on $2T-1$ equidistant straight line segments $v = u - 2\pi k/T, \quad k \in \{0, \pm 1, \ldots, \pm(T-1)\}$ parallel to the diagonal of the square $[0, 2\pi] \times [0, 2\pi]$. Actually, as in the scalar case, the Γ-harmonizable processes generalize the Γ-stationary and periodically one. If the support of the bimeasure K is concentrated only on the diagonal of the square, the harmonizable process is stationary and when the support is concentrated on $2T-1$ equidistant straight line segments parallel to the diagonal of the square, the harmonizable process becomes a periodically process with the period T. So we can sumarize the following

Corolary 3.5 If $\{f_n\}$ is a periodically Γ-correlated process in $\mathcal{H}$ with the period T, then it is Γ-harmonizable, having the support concentrated on $2T-1$ equidistant straight line segments parallel to the diagonal of the square $[0, 2\pi] \times [0, 2\pi]$.

As we have seen, the Γ_1-correlated process $\{X_n\}$ in $\mathcal{H}^T$ attached by (3.9) to the periodically Γ-correlated process $\{f_n\}$ from $\mathcal{H}$ is a stationary dilation for $\{f_n\}$. Also we have seen that the correlation function of the periodically Γ-correlated process $\{f_n\}$ has the form

$$\Gamma(n, m) = \int_0^{2\pi} \sum_{j=0}^{T-1} \exp\Big\{ i[nt - m(t - 2\pi j/T)] \Big\} F_{0j}(dt).$$

In the following we can see that the semispectral measure attached to the stationary Γ_1-correlated process $\{X_n\}$ is expressed with F_{00} the 00-component of the correlation matrix F_T of some stationary Γ_T-correlated process.

Proposition 3.6. Let $\{X_n\} \subset \mathcal{H}^T$ be the stationary Γ_1-correlated dilation of the periodically Γ-correlated process $\{f_n\} \subset \mathcal{H}$ of period T. Then the $\mathcal{L}(\mathcal{E})$-valued semispectral measure F_X attached to $\{X_n\}$ is given by

$$F_X = T \cdot F_{00} \tag{3.31}$$

where F_{00} is the 00-component of the matrix $\mathcal{L}(\mathcal{E})^{T\times T}$-valued semispectral measure F_T given by (3.26).

Proof. For each $n \in \mathbb{Z}$ we have $X_n = \Big(f_n, f_{n+1}, \ldots, f_{n+T-1}\Big)^t$ and taking account by (3.19) and (3.28) we have

$$\Gamma_1(n) = \Gamma_1[X_0, X_n] = \sum_{j=0}^{T-1} \Gamma[f_j, f_{n+j}] = \sum_{j=0}^{T-1} \Gamma(j, n+j) =$$

$$= \sum_{j=0}^{T-1} B(n+j, -n) = \sum_{j=0}^{T-1}\sum_{k=0}^{T-1} B_k(-n) \exp(2\pi ik(n+j)/T) =$$

$$= \sum_{k=0}^{T-1}\sum_{j=0}^{T-1} \exp(2\pi ik(n+j)/T) \int_0^{2\pi} e^{-int} F_{0k}(dt) = T \int_0^{2\pi} e^{-int} F_{00}(dt).$$

It follows that the $\mathcal{L}(\mathcal{E})$-valued semispectral measure F_X attached to the stationary dilation $\{X_n\}$ from $\mathcal{H}^T$ of the periodically Γ-correlated process $\{f_n\}$ is $F_X = T \cdot F_{00}$, and the proof is finished.

It is known [13] that for each $\mathcal{L}(\mathcal{E})$-valued semispectral measure F on $\mathbb{T}$ there exists a unique operator valued L^2-bounded outer function $\{\mathcal{E}, \mathcal{F}_1, \Theta_1(\lambda)\}$ which has the property that its attached semispectral measure F_{Θ_1} is dominated by F and is maximal between all semispectral measures attached to L^2-bounded analytic functions $\Theta(\lambda)$ with the property $F_\Theta \le F$. So, to each $\mathcal{L}(\mathcal{E})$-valued semispectral measure F on $\mathbb{T}$, a *maximal* L^2-bounded outer function is attached.

For stationary processes in complete correlated actions a complete study was done (see [11]-[14]), the Wiener filter for prediction and the prediction error operator was obtained in terms of its attached maximal functions. Now, the general framework for the prediction of a periodically correlated process in the context of a complete correlated action $\{\mathcal{E}, \mathcal{H}, \Gamma\}$ is done. As was seen, a Γ_1-projection $\mathcal{P}_\mathcal{M}$ of $\mathcal{H}^T$ on a submodule $\mathcal{M} \subset \mathcal{H}^T$ was constructed and some spectral connections between the periodically Γ-correlated process and its stationary Γ_1-correlated dilation was found. Also a stationary Γ_T-correlated process was attached by Gladyshev's theorem, which contains the best possible information about the initial periodically Γ-correlated process in its 00-component. Now, theoreticaly we are ready to obtain the best linear predictor for $\{f_n\}$ as the Γ_1-orthogonal projection on the past and present space of the process, using all the possible knowledge about attached stationary processes $\{X_n\}$ and $\left\{X_n^T\right\}$. To do this, preliminary prediction filters for stationary attached processes are necessary. In a following paper a detailled study will be done and an explicit formula, in terms of its maximal function, for the linear predictor of a periodically Γ-correlated process will be found.

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