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k-MEAN LABELING OF SOME DISCONNECTED GRAPHS

B.Gayathri, R.Gopi

Abstract. A graph G with p vertices and q edges is called a **k-mean labeling (k-ML)** if there is an injective function f from the vertices of G to $\{0, 1, 2, \ldots, k+q-1\}$ such that when each edge uv is labeled with $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ then the resulting edge labels $\{k, k+1, k+2, \ldots, k+q-1\}$ are all distinct.
A graph that admits k-mean labeling is called a **k-mean graph(k-MG)**

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Keywords: k-mean Labeling,k-mean graph.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of[5]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications[1].

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967[6] .

Labeled graphs serve as useful models for a broad range of applications such as X-ray crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [2].

S. Somasundaram and R. Ponraj,introduced the mean labeling. A graph G with p vertices and q edges is called a mean labeling if there is an injective function f from the vertices of G to $f : V(G) \to \{0, 1, \ldots, q\}$ such that when each edge uv is label with $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ then the resulting edge labels are distinct. A

graph which admits mean labeling is called **mean graph.** Mean labeling of graphs was discussed in[4, 3].
B.Gayathri and R.Gopi,introduce the concept of A graph G with p vertices and q edges is called a **k-mean labeling (k-ML)** if there is an injective function f from the vertices of G to $\{0, 1, 2, \ldots, k + q - 1\}$ such that when each edge uv is labeled with $f^*(uv) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil$ then the resulting edge labels $\{k, k + 1, k + 2, \ldots, k + q - 1\}$ are all distinct.

A graph that admits k-mean labeling is called a **k-mean graph(k-MG)**.

In this paper, we have proved the k-mean labeling of Some Disconnected graphs.

Illustration

1. 1-mean labeling and 2-mean labeling of $K_{1,3}$ are shown in Figure 1 & 2 respectively.

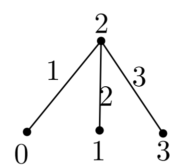

Figure 1:1-mean labeling of $K_{1,3}$

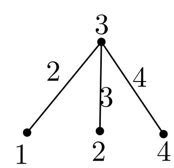

Figure 2: 2-mean labeling of $K_{1,3}$

2. We now give an example of a graph which is not a 1-mean but 2-mean.We know that $K_{1,4}$ is not a 1-mean graph. 2-mean labeling of $K_{1,4}$ is shown in Figure 3.

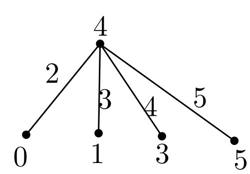

Figure 3:2-mean labeling of $K_{1,4}$

2. Main Results

In [7],it has been proved that $P_m \cup P_n$ is not a mean graph. We now investigate its k-meanness for $k > 1$.

Theorem 1. *The graph $P_2 \cup P_n (n \geq 5)$ is a k-mean graph for any $k > 1$.*

Proof. Let $\{u_1, u_2, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a, b_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 4.

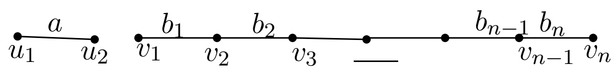

Figure 4: Ordinary labeling of $P_2 \cup P_n$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k+q-1\}$ by
Case(i): $n \equiv 0 (mod \quad 4)$
$f(u_1) = k-2$ $\qquad f(u_2) = k+1$
$f(v_1) = k-1$ $\qquad f(v_2) = k+3$
$f(v_3) = k+2$
For $4 \leq i \leq \frac{n}{2}$, $f(v_i) = k + 2(i-1)$
For $\frac{n+2}{2} \leq i \leq n-2$, $f(v_i) = k+1+2(n-i)$
$f(v_{n-1}) = k+4$ $\qquad f(v_n) = k$
Then the induced edge labels are:
$f^*(a) = k$ $\qquad f^*(b_1) = k+1$
$f^*(b_2) = k+3$ $\qquad f^*(b_3) = k+4$
For $4 \leq i \leq \frac{n}{2}$, $f^*(b_i) = k + 2i - 1$
For $\frac{n+2}{2} \leq i \leq n-3$, $f^*(b_i) = k + 2(n-i)$
$f^*(b_{n-2}) = k+5$ $\qquad f^*(b_{n-1}) = k+2$
Case(ii): $n \equiv 1, 3 (mod \quad 4)$
$f(u_1) = k-2$ $\qquad f(u_2) = k+1$
$f(v_1) = k-1$
For $2 \leq i \leq \frac{n+1}{2}$, $f(v_i) = k + 2(i-1)$
For $\frac{n+3}{2} \leq i \leq n-1$, $f(v_i) = k+1+2(n-i)$
$f(v_n) = k$
Then the induced edge labels are:
$f^*(a) = k$

For $1 \leq i \leq \frac{n-1}{2}$, $f^*(b_i) = k + 2i - 1$
For $\frac{n+1}{2} \leq i \leq n-1$, $f^*(b_i) = k + 2(n-i)$
Case(iii): $n \equiv 2(mod \quad 4)$
$f(u_1) = k-2$ $\quad$ $f(u_2) = k+1$
$f(v_1) = k-1$ $\quad$ $f(v_2) = k+3$
$f(v_3) = k+2$
For $4 \leq i \leq \frac{n+2}{2}$, $f(v_i) = k + 2i - 3$
For $\frac{n+4}{2} \leq i \leq n-2$, $f(v_i) = k + 2 + 2(n-i)$
$f(v_{n-1}) = k+4$ $\quad$ $f(v_n) = k$
Then the induced edge labels are:
$f^*(a) = k$ $\quad$ $f^*(b_1) = k+1$
$f^*(b_2) = k+3$
For $3 \leq i \leq \frac{n}{2}$, $f^*(b_i) = k + 2(i-1)$
For $\frac{n+2}{2} \leq i \leq n-2$, $f^*(b_i) = k + 2(n-i) + 1$
$f^*(b_{n-1}) = k+2$
The above defined function f provides k-mean labeling of the graph. So, $P_2 \cup P_n (n \geq 5)$ is a k-mean graph for any $k > 1$.
2-mean labeling of $P_2 \cup P_8$ is shown in Figure 5.

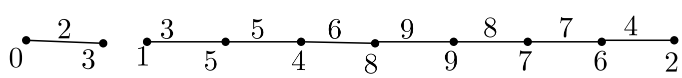

Figure 5:2-mean labeling of $P_2 \cup P_8$

Theorem 2. *The graph $P_3 \cup P_n (n \geq 4)$ is a k-mean graph for any $k > 1$.*

Proof. Let $\{u_1, u_2, u_3, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_1, a_2, b_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 6.

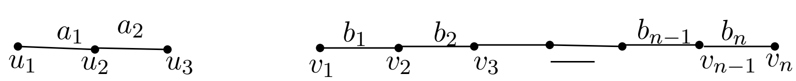

Figure 6: Ordinary labeling of $P_3 \cup P_n$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + q - 1\}$ by
Case(i): $n \equiv 0, 2(mod \quad 4)$

$f(u_1) = k - 2$ $\quad f(u_2) = k + 2$
$f(u_3) = k - 1$ $\quad f(v_1) = k + 1$
For $2 \leq i \leq \frac{n}{2}$, $f(v_i) = k + 2i$
For $\frac{n+2}{2} \leq i \leq n - 1$, $f(v_i) = k + 1 + 2(n - i)$
$f(v_n) = k$
Then the induced edge labels are:
$f^*(a_1) = k$ $\quad f^*(a_2) = k + 1$
For $1 \leq i \leq \frac{n-2}{2}$, $f^*(b_i) = k + 2i + 1$
For $\frac{n}{2} \leq i \leq n - 1$, $f^*(b_i) = k + 2(n - i)$
Case(ii): $n \equiv 1, 3(mod \quad 4)$
$f(u_1) = k - 2$ $\quad f(u_2) = k + 2$
$f(u_2) = k - 1$ $\quad f(v_1) = k$
For $2 \leq i \leq \frac{n+1}{2}$, $f(v_i) = k + 2i - 1$
For $\frac{n+3}{2} \leq i \leq n - 1$, $f(v_i) = k + 2 + 2(n - i)$
$f(v_n) = k + 1$
Then the induced edge labels are:
$f^*(a_1) = k$ $\quad f^*(a_2) = k + 1$
For $1 \leq i \leq \frac{n-1}{2}$, $f^*(b_i) = k + 2i$
For $\frac{n+1}{2} \leq i \leq n - 1$, $f^*(b_i) = k + 1 + 2(n - i)$
The above defined function f provides k-mean labeling of the graph. So, $P_3 \cup P_n (n \geq 4)$ is a k-mean graph for any $k > 1$.
2-mean labeling of $P_3 \cup P_6$,3-mean labeling of $P_3 \cup P_8$ are shown in Figure 7 & 8 respectively.

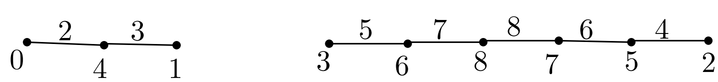

Figure 7:2-mean labeling of $P_3 \cup P_6$

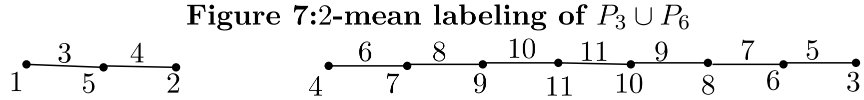

Figure 8:3-mean labeling of $P_3 \cup P_8$

Theorem 3. *The graph* $P_4 \cup P_n (n \geq 5)$ *is a* k*-mean graph for any* $k > 1$.

Proof. Let $\{u_1, u_2, u_3, u_4, v_i, 1 \leq i \leq n\}$ be the vertices and $\{a_1, a_2, a_3, b_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 9.

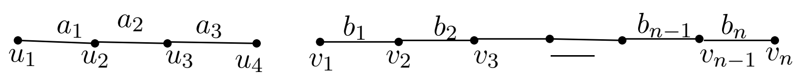

Figure 9: Ordinary labeling of $P_3 \cup P_n$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k+q-1\}$ by
Case(i): $n \equiv 0, 2 (mod \quad 4)$
$f(u_1) = k-1$ $\qquad f(u_2) = k$
$f(u_3) = k+3$ $\qquad f(u_4) = k-2$
$f(v_1) = k+1$
For $2 \leq i \leq \frac{n}{2}$, $f(v_i) = k + 2i$
For $\frac{n+2}{2} \leq i \leq n-1$, $f(v_i) = k + 3 + 2(n-i)$
$f(v_n) = k+2$
Then the induced edge labels are:
$f^*(a_1) = k$ $\qquad f^*(a_3) = k+1$
For $1 \leq i \leq \frac{n}{2}$, $f^*(b_i) = k + 2i + 1$
For $\frac{n+2}{2} \leq i \leq n-1$, $f^*(b_i) = k + 2 + 2(n-i)$
Case(ii): $n \equiv 1, 3 (mod \quad 4)$
$f(u_1) = k-1$ $\qquad f(u_2) = k$
$f(u_3) = k+3$ $\qquad f(u_4) = k-2$
$f(v_1) = k+1$ $\qquad f(v_2) = k+4$
For $3 \leq i \leq \frac{n+1}{2}$, $f(v_i) = k + 2i - 1$
For $\frac{n+3}{2} \leq i \leq n-1$, $f(v_i) = k + 4 + 2(n-i)$
$f(v_n) = k+2$
Then the induced edge labels are:
$f^*(a_1) = k$ $\qquad f^*(a_2) = k+2$ $\qquad f^*(a_3) = k+1$
$f^*(b_1) = k+3$ $\qquad f^*(b_1) = k+5$
For $3 \leq i \leq \frac{n+1}{2}$, $f^*(b_i) = k + 2i$
For $\frac{n+3}{2} \leq i \leq n-2$, $f^*(b_i) = k + 3 + 2(n-i)$
$f^*(b_{n-1}) = k+4$ The above defined function f provides k-mean labeling of the graph. So, $P_4 \cup P_n (n \geq 5)$ is a k-mean graph for any $k > 1$.
2-mean labeling of $P_4 \cup P_8$,4-mean labeling of $P_4 \cup P_{10}$ are shown in Figure 10 & 11

respectively.

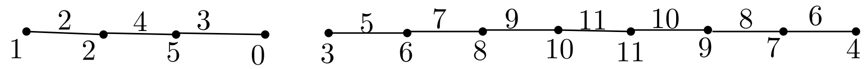

Figure 10:2**-mean labeling of** $P_4 \cup P_8$

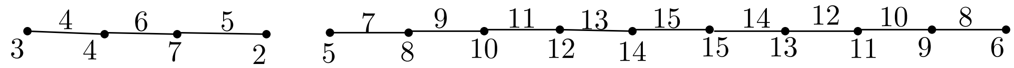

Figure 11:4**-mean labeling of** $P_4 \cup P_{10}$

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